Proceedings

The 6th International Congress
of Serbian Society of Mechanics

Tara, June 19-21, 2017

Edited by:

Mihailo Lazarević
Damir Madjarević
Ines Grozdanović
Nemanja Zorić
Aleksandar Tomović

The 6th International Congress of Serbian Society of Mechanics

Editors:
Mihailo P.Lazarević
Damir Madjarević
Ines Grozdanović
Nemanja Zorić
Aleksandar Tomović

Circulation
120 copies

Published by
Serbian Society of Mechanics and
Faculty of Mechanical Engineering,
University of Belgrade,Belgrade

Printed by
Djurdjevdan, Arandjelovac

CIP- Каталогизација у публикацији
Народна библиотека Србије

531/534(082)(0.034.2)

SRPSKO društvo za mehaniku. Međunarodni kongres (6 ; 2017 ; Tara)
Proceedings [Elektronski izvor] / The 6th International Congress of Serbian Society of Mechanics, Tara, June 19-21, 2017 ; edited by Mihailo P. Lazarević ... [et al.]. - Belgrade : Serbian Society of Mechanics : Faculty of Mechanical Engineering, University, 2017 (Aranđelovac : Đurđevdan).
- 1 USB fleš memorija ; 9 x 5 cm (u obliku kartice)

Sistemski zahtevi: Nisu navedeni. - Nasl. sa naslovne strane dokumenta. - Tiraž 120. - Bibliografija uz svaki rad.

ISBN 978-86-909973-6-7

а) Механика - Зборници

COBISS.SR-ID 237139468

Table of Contents

Congress Committees

Scientific Committee

Adhikari, S. (UK), Atanacković, T. (Serbia), Baleanu D. (Turkey), Bauer S. (Russia), Borovac, B. (Serbia), Bulatović, R. (Montenegro),Carpinteri A. (Italy), Chernousko F. (Russia), Charalambakis N. (Greece), Challamel, N. (France), Chen,W. (Hohai,China), Chow, Ch. (United States), Djordjević, V. (Serbia), Dolićanin, Ć., (Serbia), Dragović, V. (Serbia), Filipović, N. (Serbia), Frischmuth, K.(Germany), Gajić, B. (Serbia), Glavardanov, V. (Serbia), Golubović-Bugarski,V. (R. Srpska,BiH), Grillo A. (Italy), Hajdin, N. (Serbia), Hedrih (Stevanović), K. (Serbia), Igić, T. (Serbia), Jarić, J. (Serbia), Jovanović, B. (Serbia),Jovanović J. (Germany), Katsikadelis, J. (Greece), Kienzler, R. (Germany), Kojić, M. (Serbia), Kounadis, A. (Greece), Kraseilnikov P. (Russia), Kuzmanović, D. (Serbia), Lanusse P. (France), Lazarević, M. (Serbia), Melchior P. (France), Malti R. (France), Makris, N. (Greece), Maksimović, S. (Serbia), Manolis, G. (Greece), Maretić, R. (Serbia), Milosavljević, D. (Serbia), Mićunović, M. (Serbia), Mitrović, Z. (Serbia), Müller, I. (Germany), Nedeljković, M. (Serbia), Nigmatullin R. (Russia), Obradović, A. (Serbia), Pavlović, R. (Serbia), Polyzos, D. (Greece),Rega G. (Italia), Ruggeri T. (Italia),Rusov, S. (Serbia), Saccomandi,G.(Italy), Sedmak, A. (Serbia), Seyranian A. (Russia), Simić, S. (Serbia), Shitikova M. (Russia), Spanos P. (USA), Soltakhanov,Sh. (Russia), Spasić, D.T. (Serbia), Stevanović, V. (Serbia), Sun, H.G.,(Hohai,China), Šumarac, D. (Serbia), Tenreiro Machado, J.A. (Portugal),Tikhonov, A.(Russia),Tucker, R. (England), Vignjević, R. (England), Voronkova E. (Russia), Vrcelj Z. (Australia), Zarka, J. (France), Zeković, D. (Serbia), Zloković, Dj. (Serbia)

Organizing Committee

Mihailo Lazarević,(Chairman)
Damir Madjarević
Ines Grozdanović
Nemanja Zorić
Aleksandar Tomović

Foreward

The present volume contains plenary lectures, abstracts and papers of young authors competing for the „Rastko Stojanović'' award at the 6th International Congress of Serbian Society of Mechanics. The objectives of this Congress, to be held at Mountain Tara during the period 19th -21th June 2017, are to review and discuss some of the latest trends in various fields of theoretical and applied mechanics as well as it aims to bring together the scientific communities of theoretical and applied mechanics in an effort to facilitate the exchange of ideas on topics of mutual interests, and to serve as a platform for establishing links between research groups with complementary activities.

We are happy to report that the number of accepted papers to be presented at the 6th Congress is 99. In addition, among them, 7 invited plenary lectures were presented by the authors from Rusia, USA, Greece, Germany, Serbia and BIH, Republika Srpska. These papers were grouped in the following sections General Mechanics, Fluid Mechanics, Mechanics of Solid Bodies, Control and Robotics, and Interdisciplinary and Multidisciplinary Areas. Also, the three Minisymposia were organized with following topics: Nonlinear Dynamics, Turbulence and Bioengineering.

The Editors would like to express their thanks to all participants for their scientific contribution to 6th Congress of Mechanics, as well as colleagues and friends who helped with the organization. Next, to the distinguished invited lecturers who kindly accepted the invitation to come to Congress and helped make it success. We owe great thanks to the reviewers of the papers, to the members of the Scientific and Organizing Committee, and to the organizers of the Mini-symposia on Nonlinear Dynamics, Turbulence and Bioengineering. The support of the members of Steering Committee of Serbian Society of Mechanics in organizing this event is also appreciated. Finally, special thanks are also due to those organizations which supported financially this Congress: Serbian Society of Mechanics, Ministry of Education, Science and Technological Development of the Republic of Serbia, and Faculty of Mechanical Engineering, University of Belgrade, Belgrade. It is our great pleasure to have you with us at the 6th Congress International Congress of Serbian Society of Mechanics.

We would like to wish all participants of this Congress a warm welcome to our country, our Serbian Society of Mechanics and Venue Congress place at Hotel Omorika, Tara, Serbia.

Tara, June, 2017

The Editors
Mihailo P. Lazarević
Damir Madjarević
Ines Grozdanović
Nemanja Zorić
Aleksandar Tomović

Technical program

SUNDAY, June 18, 2017

19:00 *Welcoming Coctail* (*Banquet hall*)

MONDAY, June 19, 2017

8:00 - 9:00 *Registration of participants* (*Main Hall*)

9:00 - 9:15 Welcome address: (*Forum Palace Hall*)
Professor M. P. Lazarević, the President of
Serbian Society of Mechanics

Plenary Lectures (*Forum Palace Hall*)
Chairman: Mihailo P. Lazarević

9:15 - 10:00 P-1 F. Chernousko
LOCOMOTION OF MOBILE ROBOTIC SYSTEMS: DYNAMICS AND OPTIMIZATION

10:00 - 10:45 P-2 P. Spanos
RANDOMLY EXCITED NONLINEAR DYNAMIC SYSTEMS ENDOWED WITH FRACTIONAL DERIVATIVES ELEMENTS

10:45 - 11:10 *Coffee Break* (*Main Hall*)

11:10 - 12:30 Parallel Sessions

Session	G1	S1	C1	F1
Hall	*Forum Palace*	*Banquet hall*	*Idea hall*	*Dialogue hall*
11:10	G1a	S1a	C1a	F1a
11:30	G1b	S1b	C1b	F1b
11:50	G1c	S1c	C1c	F1c
12:10	G1d	S1d	C1d	F1d

12:10 - 13:20 *Lunch* (*Restaurant*)

Plenary Lecture (*Forum Palace Hall*)

Chairman: Srboljub Simić

13:30 - 14:15 P-3 G. Saccomandi
UT VIS SIC TENSIO

14:20 -17:30 *Social program* (*excursion to Mokra Gora and train ride "Sargan Eight"*)

17:40 - 19:20 Parallel Sessions

Session	G2	S2	M1	C2
Hall	*Forum Palace*	*Banquet hall*	*Idea hall*	*Dialogue hall*
17:40	G2a	S2a	M1a	C2a
18:00	G2b	S2b	M1b	C2b
18:20	G2c	S2c	M1c	C2c
18:40	G2d	S2d	M1d	C2d
19:00	G2e	S2e	M1e	

TUESDAY, June 20, 2017

Plenary Lectures (*Forum Palace Hall*)
Chairman: G. Saccomandi

9:00 - 9:45 P-4 J. Jovanović
DESIGN OF THE ANTITURBULENCE SURFACE FOR PRODUCING MAXIMUM DRAG REDUCTION EFFECT

9:45 - 10:30 P-5 T. P. Exarchos et al.
PERSONALIZED SITE-SPECIFIC MODELS OF ATHEROSCLEROTIC PLAQUE PROGRESSION

10:30 - 11:00 *Coffee Break* (*Main Hall*)

11:00 - 13:00 Parallel Sessions

Session	G3	M2	S3	M1
Hall	*Forum Palace*	*Banquet hall*	*Idea hall*	*Dialogue hall*
11:00	G3a	M2a	S3a	M1f
11:20	G3b	M2b	S3b	M1g
11:40	G3c	M2c	S3c	M1h
12:00	G3d	M2d	S3d	M1i
12:20	G3e	M2e	S3e	M1j
12:40	G3f	M2f	S3f	M1k

13:00 - 14:20 *Lunch* (*Restaurant*)

Plenary Lecture (*Forum Palace Hall*)
Chairman: T. P. Exarchos

14:30 - 15:15 P-06 Valentina Golubović-Bugarski et al.
IDENTIFICATION OF DYNAMIC PROPERTIES OF MECHANICAL STRUCTURE FROM MEASURED VIBRATION RESPONSES

15:15 - 15:40 *Coffee Break* (*Main Hall*)

15:40 - 17:20 Parallel Sessions

Session	S4	M2	C3	I1
Hall	*Forum Palace*	*Banquet hall*	*Idea hall*	*Dialogue hall*
15:40	S4a	M2g	C3a	I1a
16:00	S4b	M2h	C3b	I1b
16:20	S4c	M2i	C3c	I1c
16:40	S4d		C3d	I1d
17:00	S4e			I1e

17:20 - 18:00 Round table: CURRICULUM IN ENGINEERING MECHANICS
-modernization of teaching mechanics in higher education
(*Forum Palace Hall*)

18:00 - 19:00 General Assembly Meeting of Serbian Society of Mechanics
(*Forum Palace Hall*)

19:00 - 21:30 *Gala Dinner* (*Restaurant Hotel Omorika*)

WEDNESDAY, June 21, 2017

Plenary Lecture (*Forum Palace Hall*)

Chairman: V. Golubović-Bugarski

9:00 - 9:45 P-7 D. Milosavljević
DYNAMICAL BEHAVIOR OF COMPOSITE MATERIALS REINFORCED
WITH STRONG FIBERS

9:50 - 11:10 Parallel Sessions

Session	M3	S5	I2	
Hall	*Banquet hall*	*Idea hall*	*Dialogue hall*	
9:50	M3a	S5a	I2a	
10:10	M3b	S5b	I2b	
10:30	M3c	S5c	I2c	
10:50	M3d	S5d	I2d	

11:10 - 11:40 *Coffee Break* (*Main Hall*)

11:40 - 12:40 Parallel Sessions

Session	M3	S6		
Hall	*Idea hall*	*Dialogue hall*		
11:40	M3e	S6a		
12:00	M3f	S6b		
12:20	M3g	S6c		
12:40		S6d		

13:00 Closing Ceremony (*Dialogue hall*)

List of Contributions

General Mechanics (G)

G1 *Chair: Katica R. (Stevanović) Hedrih*
Co-Chair: Srboljub Simić

G1a Jovo Jarić, Dragoslav Kuzmanović,
ON LINEAR ANISOTROPIC ELASTICITY DAMAGE TENSOR

G1b Vladimir Dragović, Katarina Kukić,
THE SEPARATION VARIABLES FOR THE GENERALIZED KOWALEVSKI TOP VIA DISCRIMINANTLY SEPARABLE POLYNOMIALS

G1c: Katica R. (Stevanović) Hedrih
EXTENDED CLASSICAL THEORY OF IMPACTS BY KINEMATICS AND DYNAMICS OF TWO ROLLING BODIES IN SKEW COLLISION

G1d Nenad M. Grahovac, Miodrag M. Zigić
ENERGY DISSIPATION ANALYSIS OF A COLUMN LIKE STRUCTURE DURING EARTHQUAKE EXCITATION

G2 *Chair: Vladimir Dragović*
Co-Chair: Miodrag Žigić

G2a: Božidar Jovanović
CONTACT SYMMETRIES AND NOETHER THEOREM FOR TIME-DEPENDENT HOLONOMIC AND NONHOLONOMIC SYSTEMS

G2b: Ljudmila T. Kudrjavceva, Marko D. Topalović, Milan V. Mićunović
RUTTING PROBLEM FOR RUBBER WHEEL MOTION OVER HMA ASPHALT CONCRETE PAVEMENT

G2c: Vladimir Dragović, Borislav Gajić, Božidar Jovanović
INTEGRATION OF $SO(n-2)$ AND $SO(n-3)$ SYMMETRIC TOPS

G2d: Radoslav Radulović, Bojan Jeremić, Aleksandar Obradović, Zoran Stokić
GLOBAL MINIMUM TIME FOR THE BRACHISTOCHRONIC MOTION OF A PARTICLE IN AN ARBITRARY FIELD OF POTENTIAL FORCES

G2e: Aleksandar S. Okuka, Miodrag M. Žigić, Nenad M. Grahovac
ON RHEOLOGICAL BEHAVIOR OF ASPHALT CONCRETES

G3 *Chair: Božidar Jovanović*
Co-Chair: Nenad Grahovac

G3a: Nenad M. Grahovac, Miodrag M. Žigic, S. Goločorbin-Kon, M. Mikov, D.T. Spasić
A DESCRIPTION OF METHOTREXATE DISTRIBUTION AND EXCRETION IN A RAT BY A FRACTIONAL MODEL

G3b: Srdjan Jović
VIBRO-IMPACT SYSTEM BASED ON FORCED OSCILLATIONS OF HEAVY MASS PARTICLE ALONG A ROUGH CICLOID

G3c: Srđan Kostić, Nebojša Vasović, Kristina Todorović, Dragoslav Kuzmanović
STABILITY ANALYSIS OF STATICALLY INDETERMINATE EARTH SLOPES USING FINITE ELEMENT METHOD

G3d: Bojan Medjo, Marko Rakin, Nenad Gubeljak, Walid Musraty, Andrej Likeb, Ivana Cvijović-Alagić, Aleksandar Sedmak
FRACTURE MECHANICS ANALYSIS OF HETEROGENEOUS CYLINDRICAL STRUCTURES USING PIPE-RING NOTCHED BEND SPECIMENS

G3e: Sreten Mastilović
A NOTE ON DAMAGE-FRAGMENTATION TRANSITION OF SLENDER TAYLOR PROJECTILES: SIZE EFFECT AND SCALING BEHAVIOR

G3f: Antonio Rinaldi, Sreten Mastilović
A REMINDER OF THE KRAJCINOVIC APPROACH TO SCALING OF QUASI-BRITTLE FRACTURE

Mechanics of Solid Bodies (S)

S1 *Chair: Ratko Pavlović*
Co-Chair:Dušan Zorica

S1a: Dušan Zorica, Teodor M. Atanacković, Zora Vrcelj, Branislava Novaković
NON-LOCAL AXIALLY LOADED ROD PLACED ON VISCOELASTIC AND PASTERNAK TYPE FOUNDATION: DYNAMIC STABILITY ANALYSIS

S1b: Milan Cajić, Danilo Karličić, Mihailo Lazarević
COMBINED SUB-HARMONIC RESONANCES OF NANOBEAM ON FRACTIONAL VISCO-PASTERNAK TYPE FOUNDATION

S1c:Miloš Jočković, Matthias Baitsch, Marija Nefovska-Danilović
FREE VIBRATION ANALYSIS OF CURVED BERNOULLI-EULER BEAM USING ISOGEOMETRIC APPR OACH

S1d: I. Pavlović, R. Pavlović, P. Kozić,G. Janevski
INFLUENCE OF PASTERNAK VISCOELASTIC LAYER ON TIMOSHENKO BEAMS STOCHASTIC STABILITY

S2 *Chair: Ratko Maretić*
Co-Chair: Ivan Pavlović

S2a: Ratko Maretić, Sanja Ožvat, Armin Berecki, Valentin Glavardanov
BUCKLING OF CIRCULAR GRAPHENE SHEETS IN AN ELASTIC MEDIUM

S2b:Dragan Rakić, Miroslav Živković, Milan Bojović
IMPLICIT STRESS INTEGRATION OF THE ELASTIC-PLASTIC STRAIN HARDENING MODEL BASED ON MOHR-COULOMB

S2c: Vladimir Lj. Dunić, Nenad A. Grujović, Radovan B. Slavković, Nenad M. Busarac, Vukašin R. Slavković
FEM ANALYSIS OF CONCRETE GRAVITY DAM BY DAMAGE PLASTICITY CONSTITUTIVE MODEL

S2d: Dejan B. Momcilović, Ivana D. Atanasovska
APPLICATION OF GRADIENT ELASTICITY ON CORROSION FATIGUE DAMAGE ASSESSMENT

S2e: Vladimir Milovanović, Aleksandar Dišić, Nikola Jovanović, Gordana Jovičić, Miroslav Živković
EXPERIMENTAL STUDY OF DEFORMATION BEHAVIOR AND FATIGUE LIFE OF S355J2+N STEEL GRADE UNDER CYCLIC LOADING

S3 *Chair: Nina Anđelić*
Co-Chair: Živojin M. Stamenković

S3a: Slaviša Šalinić, Aleksandar Nikolić
DETERMINATION OF NATURAL FREQUENCIES OF A PLANAR SERIAL FLEXURE-HINGE MECHANISM USING A NEW PSEUDO-RIGID-BODY MODEL (PRBM) METHOD

S3b: Nina M. Anđelić, Vesna O. Milošević-Mitić, Taško Đ. Maneski, Đorđe D. Đurđević
MINIMUM WEIGHT OF OPEN SECTION THIN-WALLED STRUCTURAL ELEMENTS UNDER STRESS CONSTRAINT

S3c: Aleksandar Tomović
A NOVEL APPROACH TO THE FREE AXIAL-BENDING VIBRATION PROBLEM OF INHOMOGENEOUS ELASTIC BEAMS WITH VARIABLE CROSS-SECTIONAL PROFILES

S3d: Dragan V. Čukanović, Aleksandar B. Radaković, Gordana M. Bogdanović, Dragan I. Milosavljević, Vladimir N. Geroski
BENDING ANALYSIS OF FUNCTIONALLY GRADED PLATE ACCORDING TO HIGH ORDER SHEAR DEFORMATION THEORY

S3e: Nikola Nešić, Milan Cajić, Danilo Karličić
NON-LINEAR PRINCIPAL RESONANCE OF AN ORTHOTROPIC AND MAGNETOELASTIC RECTANGULAR PLATE OSCILLATING ON FRACTIONAL VISCOELASTIC LAYER

S3f: Jelena D. Petrović, Živojin M. Stamenković, Miloš M. Kocić,Jasmina B. Jovanović-Bogdanović, Milica D. Nikodijević
MHD FLOW AND HEAT TRANSFER IN THE POROUS MEDIUM UNDER THE INFLUENCE OF AN EXTERNALLY APPLIED MAGNETIC FIELD AND INDUCED MAGNETIC FIELD

S4 *Chair: Dragan Milosavljević*
Co-Chair: Gordana Kastratović

S4a Aleksandar R. Savić, Marina M. Aškrabić
CORRELATION BETWEEN DYNAMIC MODULUS OF ELASTICITY AND COMPRESSIVE STRENGTH for SELF-COMPACTING MORTARs

S4b Marija M. Rafailović, Miroslav M. Živković, Vladimir P. Milovanović, Jelena M. Živković
E4 AND MITC4+ SHELL FINITE ELEMENT PERFORMANCE ANALYSIS

S4c Gordana Kastratović, N. Vidanović, Aleksandar Grbović
STRESS INTENSITY FACTOR ASSESSMENT FOR MULTIPLE SURFACE CRACKS

S4d Milena Rajić, Dragan B. Jovanović, Dragoljub S. Živković
THERMOELASTIC STRESS AND DEFORMATION STATE IN GAS PIPES OF HOT WATER BOILER UNDER THE STATIONARY AND NON STATIONARY TEMPERATURE FIELD

S4e Marina Ćetković
IMPERFECTION SENSITIVITY OF PLATES IN THERMAL ENVIRONMENT

S5 *Chair: Miroslav Živković*
Co-Chair: Stanko Ćorić

S5a Stanko B. Ćorić
COMPARATIVE BUCKLING ANALYSIS AND DETERMINATION OF THE EFFECTIVE LENGTH FACTORS USING TWO NUMERICAL PROCEDURES

S5b Andrija Radović, Nikola Blagojević, Dragoslav Šumarac
STABILITY ANALYSIS OF VERTICAL CUTS

S5c Zoran Perović, Dragoslav Šumarac
ELASTOPLASTIC ANALYSIS OF FRAME STRUCTURES SUBJECTED TO CYCLIC LOADING

S5d Svetlana M. Kostić, Biljana Deretić-Stojanović
AN EFFICIENT MODEL FOR NONLINEAR ANALYSIS OF CIRCULAR CFT COLUMNS

S6 *Chair: Gordana Kastratović*
Co-Chair: Nataša Trišović

S6a: Janko Radovanović, Dragoslav Šumarac, Marija Lazović

CALCULATION OF THE LIMIT LOAD FOR THE STEEL ARC OF THE BIOCE BRIDGE

S6b: Miroslav S. Marjanović, Dragan M. Kovačević
FREE AND FORCED VIBRATION ANALYSIS OF DELAMINATED COMPOSITE PLATES OF ARBITRARY SHAPE USING TRIANGULAR LAYERED FINITE ELEMENTS

S6c: Nataša Trišović, Wei Li, Aleksandar Sedmak, Ana Petrović, Radivoje Mitrović, Zoran Stokić
ITERATIVE METHODS FOR EIGENSENSITIVITY ANALYSIS - A REVIEW

S6d: Nataša Trišović, Wei Li, Nikola Mladenović, Olivera Jeremić, Ines Grozdanović, Ana Petrović
EIGENSENSITIVITY AND STRUCTURAL OPTIMIZATION WITH ACCENT ON THE REPEATED FREQUENCIES

Fluid Mechanics (F)

F1 *Chair: Dragan Jovanović*
Co-Chair: Damir Mađarević

F1a: Saša M. Milanović, Miloš M. Jovanović, Boban D. Nikolić, Živan T. Spasić
SOLID PARTICLES VELOCITY DISTRIBUTION IN PNEUMATIC TRANSPORT OF GRANULAR MATERIALS IN CHANNELS WITH A NON-CIRCULAR CROSS SECTION TAKING INTO ACCOUNT SECONDARY FLOW

F1b: Miloš M.Jovanović, Boban D.Nikolić, Saša Milanović, Živan Spasić
FORCED RAYLEIGH BENARD CONVECTION SECONDARY INSTABILITY IN PRESENCE OF TEMPERATURE MODULATION ON BOTH PLATES

F1c: Srboljub Simić, Ana Jacinta Soares, and Damir Madjarević MULTI-TEMPERATURE ZND DETONATION MODEL

F1d: Damir Madjarević, Ana Jacinta Soares, and Srboljub Simć
NUMERICAL ANALYSIS OF DETONATION WAVES IN MULTI-TEMPERATURE ZND MODEL

Control and Robotics (C)

C1 *Chair: Dragan T. Spasić*
Co-Chair: Nemanja D. Zorić

C1a: Nemanja D. Zorić, Aleksandar M. Tomović, Miroslav M. Jovanović, Nebojša S. Lukić, Zoran M. Stokić

EFFECT OF PIEZOELECTRIC FIBER-REINFORCED COMPOSITE (PFRC) ACTUATOR ORIENTATION ON CONTROLLABILITY OF ANTISYMMETRIC COMPOSITE PLATES FOR ACTIVE VIBRATION CONTROL

C1b: Jovana Kovačević, Mara Bosnić, Dragan T. Spasić
ON APPROXIMATE ANALYTICAL SOLUTIONS OF EQUATIONS OF MOTION: TODA OSCILLATORS AND AN OPTIMAL ORBITAL TRANSFER

C1c: Miloš M. Živanović
VELOCITY CONTROL OF A MECHANICAL SYSTEM USING SECOND-ORDER DECOMPOSITION PRINCIPLE

C1d: Petar D. Mandić, Mihailo P. Lazarević, Zoran Stokić, Tomislav B. Šekara
DYNAMIC MODELLING AND CONTROL DESIGN OF SEVEN DEGREES OF FREEDOM ROBOTIC ARM

C2 *Chair: Slaviša Šalinić*
Co-Chair: Zoran Perović

C2a: Jelena Vidaković, Mihailo P. Lazarević, Vladimir M. Kvrgić, Maja M. Lutovac Banduka, Stefan M. Mitrović

CONTROL SYSTEM DESIGN OF SPATIAL DISORIENTATION TRAINER

C2b:Marina Bošković, Slaviša Šalinić, Radovan Bulatović, Goran Miodragović
MULTIOBJECTIVE OPTIMIZATION FOR DYNAMIC BALANCING OF FOUR-BAR MECHANISM

C2c: Miloš D. Lukić, Mihailo P. Lazarević, Petar B. Petrović
ANALYSIS AND OPTIMIZATION OF UNDERACTUATED FINGER FOR CMSYSLAB ROBOTIC HAND

C2d: Isak Karabegović, Ermin Husak
BY USING DIGITAL TECHNOLOGY ROBOTIC TECHNOLOGY FOLLOW THE FOURTH INDUSTRIAL REVOLUTION-INDUSTRY 4.0

C3 *Chair: Srdjan Jović*
Co-Chair: Sreten Stojanović

C3a: Vladimir Kvrgić, Jelena Vidaković,Mihailo Lazarević,Goran Pavlović
CALCULATION OF THE ACCELERATION FORCE COMPONENTS AND ROLL AND PITCH LINK ANGLES OF THE CFS AND SDT

C3b: Sreten B Stojanović, Dragutin Lj. Debeljković, Milos M. Stevanović, Milan S. Stojanović

RECIPROCALLY CONVEX APPROACH TO FINITE-TIME STABILITY OF SYSTEMS WITH TIME-VARYING DELAYS

C3c: Sreten B Stojanović, Dragutin Lj. Debeljković, Milos M. Stevanović, Milan S. Stojanović
DELAY DECOMPOSITION APPROACH TO DELAY DEPENDENT STABILITY FOR LINEAR DISCRETE-TIME SYSTEMS WITH TIME-VARYING DELAY

C3d: Mandić Slobodan, Pavić Miloš, Ignjativić Milan,Vlahović Nataša, Ocokoljić Goran
MISSILE ROLL AUTOPILOT WITH AERODYNAMIC INTERCEPTOR

Interdisciplinary Areas (I)

I1 *Chair: Nebojša Vasović*
Co-Chair: Ivan A. Kostić

I1a: Marija Stamenković Atanasov, Predrag Kozić, Aleksandar Atanasov, Nikola Nešić
THERMAL AND MAGNETIC EFFECTS ON THE FORCED VIBRATION OF AN ELASTICALLY CONNECTED NONLOCAL ORTHOTROPIC DOUBLE-NANOPLATE SYSTEM

I1b: Miloš M. Kocić, Živojin M. Stamenković, Jelena D. Petrović, Jasmina B. Jovanović-Bogdanović, Milica D. Nikodijević
FLOW AND HEAT TRANSFER OF TWO IMMISCIBLE MICROPOLAR FLUIDS IN THE PRESENCE OF UNIFORM MAGNETIC FIELD

I1c: Vukašin Slavković, Nenad Grujović, Aleksandar Dišić, Andreja Radovanović
INFLUENCE OF ANNEALING AND PRINTING DIRECTIONS ON MECHANICAL PROPERTIES OF PLA SHAPE MEMORY POLYMER PRODUCED BY FUSED DEPOSITION MODELING

I2d: Iva Guranov, Srđan Kostić, Nebojša Vasović
PERIODIC AND AUTOREGRESSIVE MODELS OF GROUNDWATER LEVEL DYNAMICS

I1e: Ivan A. Kostić, Olivera P. Kostić , Zoran A. Stefanović
COMPUTATIONAL 2D ANALYSES OF SEVERAL JET VANE TYPES AIMED FOR THE ROCKET ENGINE THRUST VECTOR CONTROL

I2 *Chair: Predrag Elek*
Co-Chair: Jelena Svorcan

I2a:Jelena Svorcan, Zorana Trivković, Marija Baltić, Ognjen Peković
COMPARISON OF DIFFERENT NUMERICAL APPROACHES TO COMPUTATION OF WIND TURBINE BLADE AERODYNAMIC PERFORMANCES WITH SPECIAL ATTENTION TO VORTEX METHODS

I2b: Predrag Elek, Slobodan Jaramaz, Dejan Micković, Ivana Todić
ANALYTICAL MODELING OF BALLISTIC PERFORATION IN PLUG FORMATION MODE

I2c: Jelena S. Andrić
NUMERICAL STUDY OF THE EFFECTS OF DRIVING PATTERNS ON ENERGY FLOW AND FUEL CONSUMPTION IN PARALLEL HYBRID ELECTRIC VEHICLES

I2d: Kosta Velimirović, Nemanja Velimirović
FLIGHT PERFORMANCE DETERMINATON OF THE TURBOJET AIRCRAFT ORAO

Mini-symposia - Nonlinear Dynamics (M1)

M1_1 *Chair: Katica (Stevanović) Hedrih*
Co-Chair: Ivana D. Atanasovska

M1a: Katica R. (Stevanović) Hedrih
PHASE PORTRAIT AND ENERGY SURFACES OF NON-LINEAR SYSTEMS WITH ONE DEGREE OF FREEDOM: CHARACTERISTIC EXAMPLES

M1b: P. Krasilnikov, R. Amelin
ON THE SATURN'S PRECESSION

M1c: Vladimir S. Korolev, Elena N. Polyakhova, Irina Yu. Pototskaya, Alexey A. Tikhonov,
PROBLEMS OF MOTION STABILITY OF SOLAR SAIL SPACECRAFT

M1d: Ivana D. Atanasovska, Dejan B. Momcilović
DYNAMICS OF COMPLEX SYSTEMS CONSIST OF DEFORMABLE BODIES IN CONTACT - THE NEW APPROACH

M1e: J. Simonović
NONLINEAR MODELS OF BONE EXTERNALLY EXCITED CELLULAR REMODELING

M1_2 *Chair: P. Krasilnikov*
Co-Chair: J. Simonović

M1f: Nebojša I. Potkonjak, Ljiljana Z. Kolar-Anić, Maja C. Pagnacco, Slobodan R. Anić,
SOME CONSIDERATION ABOUT VOLTAMMOGRAM AS BIFURCATIONS DIAGRAM IN ELECTROCHEMICAL OSCILLATORY SYSTEMS

M1g: Jelena P. Maksimović, Kristina Stevanović, Itana Nuša Bubanja, Ljiljana Z. Kolar-Anić, Slobodan R. Anić, Nebojša I. Potkonjak, Maja C. Pagnacco
THE NON-LINEAR BRIGGS-RAUSCHER REACTIONS AS A MEDIUM FOR INVESTIGATION OF THE CAFFEIC ACID CONCETRATION AND ITS POTENTIAL ANTIRADICAL ACTIVITY

M1h: Željko Čupić, Ana Ivanović-Šašić, Vladimir Marković, Ana Stanojević, Slobodan Anić, Nataša Pejić, Ljiljana Kolar-Anić
BIFURCATION ANALYSIS AS THE METHOD FOR DETERMINATION OF PARAMETERS THAT APPEAR IN THE MODEL OF CONSIDERED PROCESS

M1i: Ljubinko B. Kevac, Mirjana M. Filipović, Ana M. Djurić
ELASTIC S-TYPE CABLE-SUSPENDED PARALLEL ROBOT IN PRESENCE OF SECOND MODE

M1j: Danilo Z. Karličić, Milan S. Cajić
NONLINEAR DYNAMIC STABILITY OF A FUNCTIONALLY GRADED NONLOCAL NANOBEAM IN THERMAL ENVIRONMENT BY USING INCREMENTAL HARMONIC BALANCE METHOD

M1k: Marina Trajković-Milenković, Otto T. Bruhns,Dragoslav Šumarac
NUMERICAL ANALYSIS OF FINITE HYPO-ELASTIC CYCLIC DEFORMATION WITH SMALL AND MODERATE ROTATIONS

M2 Turbulence

Minisymposia - Turbulence (M2)

M2_1 *Chair: Đorđe Čantrak*
Co-Chair: Milan Šekularac

M2a: Aleksandar S. Ćoćić, Milan M. Raković, Dejan B. Ilić, Milan R. Lečić
NUMERICAL COMPUTATIONS OF TURBULENT FLOW THROUGH ORIFICE FLOW METER

M2b: Dejan Cvetinović, Rastko Jovanović, Jiří Vejražka, Jaroslav Tihon, Kazuyoshi Nakabe, Kazuya Tatsumi
POSSIBILITY OF VORTEX STRUCTURES CONTROL BY MODULATION OF THE NOZZLE EXIT VELOCITY USING LOW-AMPLITUDE OSCILLATIONS

M2c: Jelena Svorcan, Zorana Trivković, Toni Ivanov
COMPUTATIONAL ANALYSIS OF HORIZONTAL-AXIS WIND TURBINE BY DIFFERENT RANS TURBULENCE MODELS

M2_2 *Chair: Dejan Cvetinović*
Co-Chair: Živojin Stamenković

M2d: Živojin M. Stamenković, Jasmina B. Jovanović-Bogdanović, Miloš M. Kocić, Jelena D. Petrović
MHD STEADY AND UNSTEADY FLOW PAST A CIRCULAR CYLINDER

M2e: Dejan B. Ilić, Đorđe S. Čantrak, Novica Z. Janković
REYNOLDS NUMBER INFLUENCE ON INTEGRAL AND STATISTICAL CHARACTERISTICS OF THE TURBULENT SWIRL FLOW IN STRAIGHT CONICAL DIFFUSER

M2f: Milan B. Šekularac, Novica Z. Janković
EXPERIMENTAL AND NUMERICAL ANALYSIS OF FLOW FIELD AND VENTILATION PERFORMANCE IN A TRAFFIC TUNNEL VENTILATED BY AXIAL FANS

M2_3 *Chair: Aleksandar Ćoćić*
Co-Chair: Jelena Svorcan

M2g: Djordje M. Novković, Jela M. Burazer, Aleksandar S. Ćoćić, Milan R. Lečić
NUMERICAL RESEARCH OF SWIRL-FREE FLOW IN AZAD DIFFUSER

M2h: Jasmina B. Bogdanović-Jovanović, Živojin M. Stamenković, Miloš M. Kocić, Jelena D. Petrović
CAVITATION IN VENTURI NOZZLE – NUMERICAL PREDICTION OF CAVITATION PARAMETERS

M2i: Jela M. Burazer
NUMERICAL RESEARCH OF ENERGY SEPARATION IN A CYLINDER WAKE

M3 Bionegineering

M3_1 *Chair: Miloš Kojić*
Co-Chair: Miljan Milošević

M3a: Kristina Todorović, I. Franović, N. Vasović, S. Kostić
SPONTANEOUS FORMATION OF SYNCHRONIZATION CLUSTERS IN NEURONAL POPULATIONS INDUCED BY NOISE AND INTERACTION DELAYS

M3b: Miloš Kojić, Miljan Milosević, Vladimir Simić
CONVECTION–DIFFUSION TRANSPORT MODEL USING COMPOSITE SMEARED FINITE ELEMENT

M3c: Andjelka N. Hedrih, Katica (Stevanović) Hedrih
RESONANCE AS POTENTIAL MECHANISM FOR HOMOLOG CHROMOSOMES SEPARATION TROUGH BIOMECHANICAL OSCILLATORY MODEL OF MITOTIC SPINDLE

M3d: Miljan Milosević, Milos Kojić, Vladimir Simić
FIELD OF CORRECTION FACTORS FOR SMEARED FINITE ELEMENT

M3_2 *Chair: Nenad Filipović*
Co-Chair: Velibor Isailović

M3e: Milica M. Nikolić,Nenad D. Filipović
APPLICATION OF DPD METHOD ON MODELLING SEMICIRCULAR CANALS

M3f: Velibor Isailović, Igor Saveljić, Dalibor Nikolić, Zarko Milosević,Dusan Pavlović,Nenad D. Filipović
EYE TRACKING ALGORITHM AND COMPUTATIONAL MODELING IN PREDICTION OF BENIGN PAROXYSMAL POSITIONAL VERTIGO DISEASE

M3g: Nenad Filipović, Velibor Isailović, Žarko Milosević, Dalibor Nikolić, Igor Saveljić, Milica Nikolić, Bojana Ćirković-Andjelković, Nikola Jagić, Exarchos Themis, Dimitris Fotiadis, Gualtiero Pelosi, Oberdan Parodi
COMPUTATIONAL MODELING FOR PLAQUE PROGRESSION AND FRACTIONAL FLOW RESERVE IN THE CORONARY ARTERIES

the subject of following tests: resonant frequency, ultrasonic pulse velocity and compressive strength of mortar. Dynamic modulus of elasticity was calculated using the first two methods mentioned. Results from resonant frequency measurements were then used for calculating compressive strength of the mixtures through different models developed for ordinary concrete and available in the literature. It was shown that the model proposed by Gardner is closest to describing the behavior of mixtures with different component materials, but would need some corrections in order to be used for these purposes.

S4b Marija M. Rafailović, Miroslav M. Živković, Vladimir P. Milovanović, Jelena M. Živković

E4 AND MITC4+ SHELL FINITE ELEMENT PERFORMANCE ANALYSIS

This paper presents a correction of the 4-node MITC (Mixed Interpolation of Tensorial Components) quadrilateral shell finite element, i.e., MITC4 element [1]. Correction refers to the improvement of membrane behavior, where the components of membrane strain in mid-surface of the shell are assumed using the concept of MITC method. In other words, the mid-surface of 4-node quadrilateral shell finite element is subdivided into four triangular domains, in order to construct the assumed membrane strain field by using the membrane strains obtained from those four triangular subdomains of the shell finite element [2]. The purpose is to reduce membrane locking that happen when the MITC4 shell elements are geometrically distorted in curved geometries, while the element retains good membrane behavior. Numerical results obtained with E4 shell finite element [3], which is implemented in software package PAK, are presented at the end of the paper and compared with those obtained with MITC4+ element. This numerical example shows the main problems of E4 element in the case of very thin shells and curved geometries, which should be significantly reduced by using the abovementioned modified element.

S4c Gordana Kastratović, N. Vidanović, Aleksandar Grbović

STRESS INTENSITY FACTOR ASSESSMENT FOR MULTIPLE SURFACE CRACKS

The aim of this paper was to establish and demonstrate the capacity and performances of simple and easy to use approximate method for mode I stress intensity factors calculations in case of multiple surface cracks. This procedure, which is based on principle of superposition, was applied on a configuration with three coplanar semi-elliptical cracks embedded in three dimensional elastic body, subjected to remote uniaxial tensile loading. All cracks are located in the same plane at same distances, in the middle of the body and the applied stress is perpendicular to the cracks plane. For the verification purposes, the stress intensity factors solutions were obtained by using finite element method (FEM) based computer program. The conducted analysis showed that approximate method is, above all, fast and efficient tool for stress intensity factors assessment even in the case of 3D configurations with MSD. The comparison between results also showed, the significance of accurate calculation of SIFs, in order to provide a better understanding and prediction of 3D multiple cracks propagation.

The 6th International Congress of Theoretical and Applied Mechanics

is organized by

Serbian Society of Mechanics

and supported by

Ministry of Education, Science and Technological Development of the Republic Serbia

Faculty of Mechanical Engineering, University of Belgrade, Belgrade

6th International Congress of Serbian Society of Mechanics
Mountain Tara, Serbia, June 19-21, 2017

E4 AND MITC4+ SHELL FINITE ELEMENT PERFORMANCE ANALYSIS

Marija M. Rafailović[1], Miroslav M. Živković[1], Vladimir P. Milovanović[1], Jelena M. Živković[1]

[1] Faculty of Mechanical Engineering,
University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac, Serbia
e-mail: marijarafailovic109@gmail.com, miroslav.zivkovic@kg.ac.rs, vladicka@kg.ac.rs, jelena.zivkovic@kg.ac.rs

Abstract

This paper presents a correction of the 4-node MITC (Mixed Interpolation of Tensorial Components) quadrilateral shell finite element, i.e., MITC4 element [1]. Correction refers to the improvement of membrane behavior, where the components of membrane strain in mid-surface of the shell are assumed using the concept of MITC method. In other words, the mid-surface of 4-node quadrilateral shell finite element is subdivided into four triangular domains, in order to construct the assumed membrane strain field by using the membrane strains obtained from those four triangular subdomains of the shell finite element [2]. The purpose is to reduce membrane locking that happen when the MITC4 shell elements are geometrically distorted in curved geometries, while the element retains good membrane behavior. Numerical results obtained with E4 shell finite element [3], which is implemented in software package PAK, are presented at the end of the paper and compared with those obtained with MITC4+ element. This numerical example shows the main problems of E4 element in the case of very thin shells and curved geometries, which should be significantly reduced by using the abovementioned modified element.

Key words: shell finite element, 4-node element, MITC method, membrane locking, convergence

1. Introduction

Although the shell structures are vastly present in the technical practice, their behavior is not easy to predict. Shell finite elements have been developed during the past decades with the purpose of analysing these structures, and their application in the finite element method is immence [4]. Great number of shell finite elements have been presented so far, based on different theoretical approaches. However, the general shell finite element that is absolutely superior over other shell elements does not exist.

The behavior of shell structures belongs to one of the three asymptotic categories, depending on the geometry of the shell, boundary conditions and applied load, which are: membrane dominated, bending dominated and mixed shell behavior [4-6]. However, the ideal element of a shell should give the correct solution that is independent of the shell geometry, asymptotic category and its thickness. Of course, such element that has an optimal uniform convergence behavior for any problem, even when it is used in distorted meshes, still has not been formulated.

The greatest obstacle in analysis of the shell structure is the dependence of the behavior of the element to the parameter of thickness, known as membrane and shear locking. The MITC method has been successfully used for the development of the finite elements with reduced locking influence, [1,7-11]. At first, this method has been developed to reduce the shear locking of the MITC4 element [1]. The MITC9 and MITC16 quadrilateral shell finite elements were later developed by Bucalem and Bathe, and both shear and membrane locking of this triangular shell finite elements have been successfully reduced by using this method [10, 11]. Beside that, the MITC method has been successfully used to develop isotropic triangular shell elements, MITC3 and MITC6, by Lee and Bathe [9].

There have been several attempts to reduce membrane locking of 4-node continuum mechanics based shell finite elements. The method of reduced integration [12-14] can greatly alleviate membrane locking as well as shear locking, but the elements do not have proper physical rigid body modes. Also, the method of assumed strain field was applied to the membrane strains in order to reduce membrane locking [15]. That modification gave as a result severely deteriorated in-plane behaviors of 4-node shell elements.

In this paper a new MITC4 element is presented, MITC4+ element, with a modification of membrane strains based upon the MITC method. The problem of shear locking in this shell element is solved with the help of a well known assumed field for transverse shear strain of the original MITC4 shell element [1], whereas a new membrane strain field was proposed for the improvement of membrane behavior. This field is obtained by determining the membrane deformation in triangular domains which divide the mid-surface of the shell element. In this way, one gets an element, which is, in the sence of membrane deformation, less sensitive to the irregularity of the shape and the opportunity to determine more accurately the membrane rigidity.

In the following chapter the formulation of the MITC4+ shell element is presented. The formulation of the originally MITC4 element is given in [1,16]. At the end of this paper, the performances of the shell element E4 are presented using the numerical example.

2. MITC4+ element

In the new 4-node quadrilateral shell finite element, the mid-surface of element is divided into four flat triangular domains, where instead of the membrane strains, which are obtained from the displacement field of the element [1], a new covariant membrane strain field is assumed. The new membrane strain field is obtained by introducing the interpolation of the mentioned triangular domains and determining covariant membrane strains into every triangle separately.

2.1 Defining the position of the center point 5

Figure 1 shows four triangular domains on the mid-surface of the shell finite element. Domains are defined so that their intersection is in point 5, which represents the center in the mid-surface of shell finite element [2].

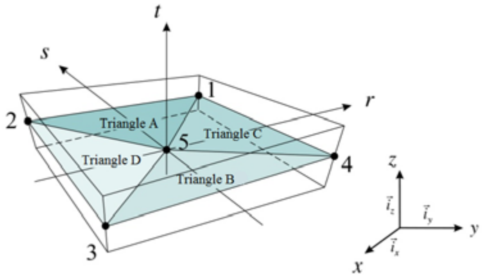

Figure 1. Triangular domains and the central point 5 of the mid-surface shell finite element

The position of central point 5 is given with the following equation

$$X_i^5 = \sum_{k=1}^{4} \gamma_k \mathrm{X}_i^k \quad i = 1,2,3 \tag{1}$$

in which X_i^5 are coordinates of the central point 5, X_i^k are coordinates of nodal point k in global Cartesian coordinate system x, y, z, where $i = 1$ coresponds to axis x, $i = 2$ to axis y and $i = 3$ to axis z.

Similar to equation (1), the displacement of point 5 can be calculated as

$$U_i^5 = \sum_{k=1}^{4} \gamma_k U_i^k \quad i = 1,2,3 \tag{2}$$

where U_i^5 is the displacement of point 5 in direction of the i axis , while U_i^k is the displacement of node k in the direction of the coresponding axis in the global Cartesian coordinate system x, y, z.

2.2 Interpolation for the geometry and triangular domain displacement

Defined center point is used to divide the mid-surface of the quadrilateral shell finite element into four flat triangular domains, while the geometry interpolation is introduced for every triangular domain in the following form

$$\widehat{x}_i(r,s,t) = \sum_{k=1}^{3} \widehat{h}_k(r,s) \widehat{X}_i^k \quad i = 1,2,3 \tag{3}$$

where $\widehat{x}_i(r,s,t)$ are the coordinates of the triangular domain's material points, and $\widehat{X}_i^k$ are coordinates of node k which defines the triangular domain A, B, C or D.

Interpolation functions $\widehat{h}_k(r,s)$ of the 3-node triangular elements, that corespond to the nodes that define them are

$$\begin{aligned} \widehat{h}_1(r,s) &= r \\ \widehat{h}_2(r,s) &= s \\ \widehat{h}_3(r,s) &= 1 - r - s \end{aligned} \tag{4}$$

Triangular domains, Figure 1, are defined so that the following is true

$$\begin{aligned} &\widehat{\mathbf{X}}^1 = \mathbf{X}^1, \widehat{\mathbf{X}}^2 = \mathbf{X}^2, \widehat{\mathbf{X}}^3 = \mathbf{X}^5 \text{ for triangle A} \\ &\widehat{\mathbf{X}}^1 = \mathbf{X}^3, \widehat{\mathbf{X}}^2 = \mathbf{X}^4, \widehat{\mathbf{X}}^3 = \mathbf{X}^5 \text{ for triangle B} \end{aligned} \tag{5}$$

$$\overset{\frown}{\mathbf{X}}^1 = \mathbf{X}^4, \overset{\frown}{\mathbf{X}}^2 = \mathbf{X}^1, \overset{\frown}{\mathbf{X}}^3 = \mathbf{X}^5 \text{ for triangle C}$$
$$\overset{\frown}{\mathbf{X}}^1 = \mathbf{X}^2, \overset{\frown}{\mathbf{X}}^2 = \mathbf{X}^3, \overset{\frown}{\mathbf{X}}^3 = \mathbf{X}^5 \text{ for triangle D}$$

Based on the interpolation function (4) we can define the position of the axes $\overset{\frown}{r}$ and $\overset{\frown}{s}$, Figure 2, of the triangular domains.

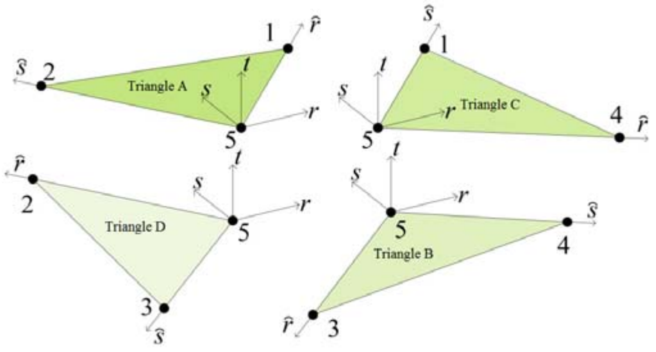

Figure 2. Positions of the triangular domain's axes $\overset{\frown}{r}$ and $\overset{\frown}{s}$

Similar to the geometry interpolation, we calculate the displacements of the triangular domains' material points as

$$\overset{\frown}{u}_i(r,s,t) = \sum_{k=1}^{3} \overset{\frown}{h}_k(r,s)\overset{\frown}{U}_i^k \quad i = 1,2,3 \tag{6}$$

where $\overset{\frown}{u}_i(r,s,t)$ are displacements of the triangular domains' material points, and $\overset{\frown}{U}_i^k$ the displacement of node k of the coresponding triangular domain in the direction of the i axis.

Displacement vectors of the triangular domains' nodes are defined analogously to (5),

$$\begin{gathered}
\overset{\frown}{\mathbf{U}}^1 = \mathbf{U}^1, \overset{\frown}{\mathbf{U}}^2 = \mathbf{U}^2, \overset{\frown}{\mathbf{U}}^3 = \mathbf{U}^5 \text{ for triangle A} \\
\overset{\frown}{\mathbf{U}}^1 = \mathbf{U}^3, \overset{\frown}{\mathbf{U}}^2 = \mathbf{U}^4, \overset{\frown}{\mathbf{U}}^3 = \mathbf{U}^5 \text{ for triangle B} \\
\overset{\frown}{\mathbf{U}}^1 = \mathbf{U}^4, \overset{\frown}{\mathbf{U}}^2 = \mathbf{U}^1, \overset{\frown}{\mathbf{U}}^3 = \mathbf{U}^5 \text{ for triangle C} \\
\overset{\frown}{\mathbf{U}}^1 = \mathbf{U}^2, \overset{\frown}{\mathbf{U}}^2 = \mathbf{U}^3, \overset{\frown}{\mathbf{U}}^3 = \mathbf{U}^5 \text{ for triangle D}
\end{gathered} \tag{7}$$

2.3 Triangular domain's covariant membrane deformations

As the displacements of the material points $\overset{\frown}{u}_i(r,s,t)$ of triangular domains are defined by equation (6), the components of covariant membrane strains $\overset{\frown}{e}_{ij}^{m}$ in triangular domains can be calculated with

$$\overset{\frown}{e}_{ij}^{m(\triangle)} = \frac{1}{2}\left(\frac{\partial \overset{\frown}{\mathbf{x}}}{\partial r_i} \cdot \frac{\partial \overset{\frown}{\mathbf{u}}}{\partial r_j} + \frac{\partial \overset{\frown}{\mathbf{x}}}{\partial r_j} \cdot \frac{\partial \overset{\frown}{\mathbf{u}}}{\partial r_i} \right) = \frac{1}{2}\left(\overset{\frown}{\mathbf{g}}_i \cdot \frac{\partial \overset{\frown}{\mathbf{u}}}{\partial r_j} + \overset{\frown}{\mathbf{g}}_j \cdot \frac{\partial \overset{\frown}{\mathbf{u}}}{\partial r_i} \right) \quad i,j = 1,2 \tag{8}$$

where $\overset{\frown}{\mathbf{g}}_1, \overset{\frown}{\mathbf{g}}_2$ and $\overset{\frown}{\mathbf{g}}_3 = \mathbf{g}_3$ are covariant base vectors tangent to the coordinate axes of the triangular domains, where the base vector $\overset{\frown}{\mathbf{g}}_3$ has the direction of the normal, so it matches the covariant base vector $\mathbf{g}_3$ of the natural coordinate system r, s, t, Figure 3.

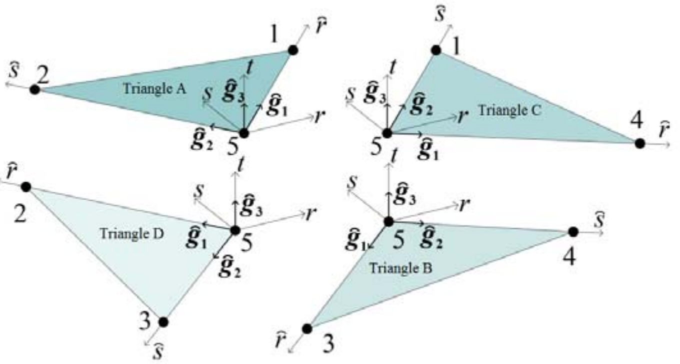

Figure 3. Covariant base vectors of the triangular domains

Deformation vector $\widehat{\mathbf{e}}^{m(\triangle)}$ of the coresponding triangular domain can be respresented with a matrix of derivatives of interpolation functions $\widehat{\mathbf{B}}^{(\triangle)}$ and vectors of node displacements $\mathbf{U}$ in the following way

$$\widehat{\mathbf{e}}^{m(\triangle)} = \begin{Bmatrix} \widehat{e}_{11}^{m(\triangle)} \\ \widehat{e}_{22}^{m(\triangle)} \\ \widehat{\gamma}_{12}^{m(\triangle)} \end{Bmatrix} = \widehat{\mathbf{B}}^{(\triangle)}\mathbf{U} = \begin{bmatrix} \widehat{\mathbf{B}}^{1(\triangle)} & \widehat{\mathbf{B}}^{2(\triangle)} & \widehat{\mathbf{B}}^{3(\triangle)} & \widehat{\mathbf{B}}^{4(\triangle)} \end{bmatrix} \begin{Bmatrix} \mathbf{U}^1 \\ \mathbf{U}^2 \\ \mathbf{U}^3 \\ \mathbf{U}^4 \end{Bmatrix} \tag{9}$$

2.4 Strain transformation

When the covariant membrane strains of the triangular domains are completely defined, strain transformations must be done $\widehat{e}_{ij}^{m(\triangle)}$, which are defined by (8), to the natural axes r, s, t of the 4-node shell finite element, of the following form

$$\bar{e}_{ij}^{m(\triangle)} = \widehat{e}_{kl}^{m(\triangle)}\left(\mathbf{g}_i \cdot \widehat{\mathbf{g}}^k\right)\left(\mathbf{g}_j \cdot \widehat{\mathbf{g}}^l\right), \quad i,j = 1,2 \; i \; k,l = 1,2 \tag{10}$$

where $\widehat{\mathbf{g}}^k$ and $\widehat{\mathbf{g}}^l$ are the covariant vectors of the coresponding triangular domain, while $\widehat{\mathbf{g}}^i \cdot \widehat{\mathbf{g}}_j = \delta_{ij}$ is the Kronecker delta symbol.

Equation (10) in expanded form can be written as

$$\begin{aligned} \bar{\mathbf{e}}^{m(\triangle)} = \begin{Bmatrix} \bar{e}_{11}^{m(\triangle)} \\ \bar{e}_{22}^{m(\triangle)} \\ \bar{\gamma}_{12}^{m(\triangle)} \end{Bmatrix} &= \begin{bmatrix} \left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^1\right)\left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^1\right) & \left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^2\right)\left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^2\right) & 2\left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^1\right)\left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^2\right) \\ \left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^1\right)\left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^1\right) & \left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^2\right)\left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^2\right) & 2\left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^1\right)\left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^2\right) \\ \left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^1\right)\left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^1\right) & \left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^2\right)\left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^2\right) & 2\left(\mathbf{g}_1 \cdot \widehat{\mathbf{g}}^1\right)\left(\mathbf{g}_2 \cdot \widehat{\mathbf{g}}^2\right) \end{bmatrix} \begin{Bmatrix} \widehat{e}_{11}^{m(\triangle)} \\ \widehat{e}_{22}^{m(\triangle)} \\ \widehat{\gamma}_{12}^{m(\triangle)} \end{Bmatrix} = \\ &= \widehat{\mathbf{T}}^{(\triangle)}\widehat{\mathbf{e}}^{m(\triangle)} \end{aligned} \tag{11}$$

where $\widehat{\mathbf{T}}^{(\triangle)}$ is the tranformation matrix of the corresponding triangular domain.

Submatrices $\bar{\mathbf{B}}^{k(\triangle)}$ of the corresponding triangular domain are equal to

$$\overline{\mathbf{B}}^{k} = \widehat{\mathbf{T}}\widehat{\mathbf{B}}^{k} \tag{12}$$

Now, the strains of the corresponding triangular domain can be presented like the following, using (9)

$$\overline{\mathbf{e}}^{m(\triangle)} = \widehat{\mathbf{T}}^{(\triangle)}\widehat{\mathbf{B}}^{(\triangle)}\mathbf{U} \tag{13}$$

2.5 Assumed membrane strain field

Using the covariant membrane strains (13) obtained from four triangular domains, a strain field was suggested that reduces the sensitivity to the irregularities of shape, and has the following form

$$\tilde{e}_{ij}^{m} = a + br + cs + drs \quad i,j = 1,2 \tag{14}$$

Where the coefficients a,b,c,d are obtained by using the covariant membrane strains from the triangular domains, in the following way

$$a = \frac{1}{4}\left(\overline{e}_{ij}^{m(A)} + \overline{e}_{ij}^{m(B)} + \overline{e}_{ij}^{m(C)} + \overline{e}_{ij}^{m(D)}\right),$$

$$b = \frac{1}{2}\left(-\overline{e}_{ij}^{m(D)} + \overline{e}_{ij}^{m(C)}\right),$$

$$c = \frac{1}{2}\left(-\overline{e}_{ij}^{m(B)} + \overline{e}_{ij}^{m(A)}\right),$$

$$d = 0$$

Assumed strain field (14) is defined so that the positions of points A, B, C and D are symmetrical regarding the element's center, as shown in Figure 4 [2].

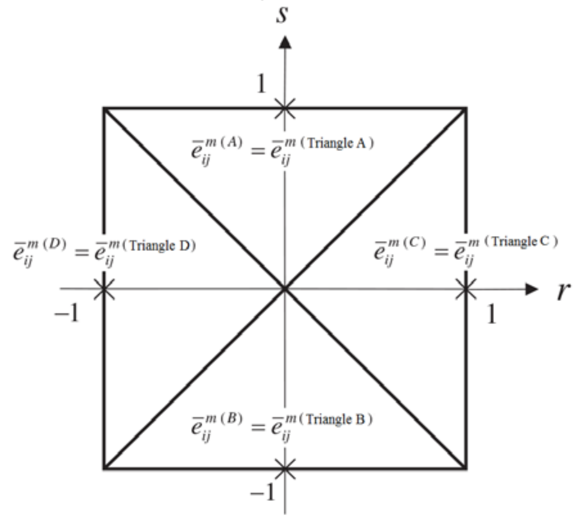

Figure 4. Tying positions of points (A), (B), (C) and (D), that belong to the assumed membrane strain field of the MITC4+ shell element

Considering the transformation (11), and in accordance with (12) and (13), strain vector $\tilde{\mathbf{e}}^{m}$ of the shell element can be written in following form

$$\tilde{\mathbf{e}}^{m} = \left\{ \frac{1}{4} \left(\widehat{\mathbf{T}}^{(A)} \widehat{\mathbf{B}}^{(A)} + \widehat{\mathbf{T}}^{(B)} \widehat{\mathbf{B}}^{(B)} + \widehat{\mathbf{T}}^{(C)} \widehat{\mathbf{B}}^{C} + \widehat{\mathbf{T}}^{(D)} \widehat{\mathbf{B}}^{(D)} \right) + \left(-\widehat{\mathbf{T}}^{(D)} \widehat{\mathbf{B}}^{(D)} + \widehat{\mathbf{T}}^{(C)} \widehat{\mathbf{B}}^{(C)} \right) r + \right. \\ \left. + \left(-\widehat{\mathbf{T}}^{(B)} \widehat{\mathbf{B}}^{(B)} + \widehat{\mathbf{T}}^{(A)} \widehat{\mathbf{B}}^{(A)} \right) s \right\} \mathbf{U} = \widetilde{\mathbf{B}} \mathbf{U} \tag{15}$$

where $\widehat{\mathbf{T}}^{(\triangle)}$ are corresponding transformation matrices of the triangular domains, and $\widehat{\mathbf{B}}^{(\triangle)}$ is a matrix of derivatives of inerpolation functions of triangular domains defined for every triangular domain. The matrix of derivatives of interpolation functions $\widetilde{\mathbf{B}}$ of the shell element in a natural coordinate system that fits the suggested strain field is obtained from this equation.

3. Numerical analysis

The convergence studies for the E4 shell finite element which is implemented into the software package PAK are presented in this chapter. The obtained results are compared with those of MITC4+ shell element. In order to display improved performances of MITC4+ shell finite element comparing to those of E4 element, the convergence curves of both elements are presented. Convergence curves of MITC4+ element are taken from [2].

For convergence studies, we use the s-norm [17] defined as

$$\left\| \mathbf{u} - \mathbf{u}_h \right\|_s^2 = \int_{\Omega} \Delta\boldsymbol{\varepsilon}^T \Delta\boldsymbol{\tau} d\Omega \tag{16}$$

where $\mathbf{u}$ denotes the exact solution and $\mathbf{u}_h$ denotes the solution obtained by finite element discretization. Here, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\tau}$ are the strain vector and the stress vector in the global Cartesian coordinate system, respectively.

In the practical use of this norm, the finite element solution using a very fine mesh $\mathbf{u}_{ref}$ is adopted instead of the exact solution $\mathbf{u}$, so the equation (16) can be expressed as

$$\left\| \mathbf{u}_{ref} - \mathbf{u}_h \right\|_s^2 = \int_{\Omega_{ref}} \Delta\boldsymbol{\varepsilon}^T \Delta\boldsymbol{\tau} d\Omega_{ref} \tag{17}$$

It is well known that the behavior of the element significantly deteriorates with decreasing of the thickness, so the relative error E_h is used for analysis of the sensitivity of the element to the thickness parameter

$$E_h = \frac{\left\| \mathbf{u}_{ref} - \mathbf{u}_h \right\|_s^2}{\left\| \mathbf{u}_{ref} \right\|_s^2} \tag{18}$$

3.1 Cilindrical shell problems

We observe a cylindrical shell of uniform thickness t, length $2L$ and radius R, which is loaded by the pressure $p(\theta)$ normal to the shell surface, as shown in Figure 5

$$p(\theta) = p_0 \cos(2\theta)$$

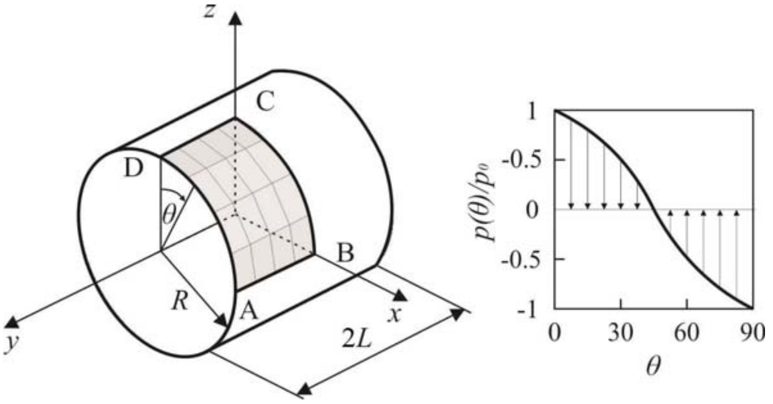

Figure 5. Cylindrical shell loaded by the pressure distribution $p(\theta)$ with free ends

Material properties and dimensions are: $\mathrm{L=R=1m}$, $\mathrm{E=2\times10^5 MPa}$, $\nu=1/3$, $\mathrm{p_0=1Pa}$. A detailed study of this shell problem is presented in [2, 18].

Depending on the boundary conditions at shell structure ends, it shows two different asymptotic behaviors: the bending dominated behavior under a free boundary condition and the membrane dominated behavior under a clamped boundary condition. For the clamped cylindrical shell problems both shell elements, MITC4 and MITC4+, give similary good convergence behavior, as shown in [2], so we consider only a bending dominated behavior.

Because of the symmetry, we can limit calculations to the region ABCD, shown in Figure 5. For the free edge case, the following boundary conditions are imposed: $v=\varphi_1=0$ along BC, $u=\varphi_2=0$ along DC and $w=\varphi_2=0$ along AB. As shown in [2], when using regular mesh all 4-node shell elements have a flat geometry, so the membrane locking does not occur and because of that cylindrical shell was modeled with distorted mesh with $N\times N$ ($N=4,\ 8,\ 16,\ 32,\ 64$).

Figure 6 present the convergence curves of the E4 and MITC4+ shell finite elements for the free cylindrical shell problem. Results are obtained for three values of thickness parameter t/L : 1/100, 1/1000 and 1/10000.

The relative error used here is based on the reference solution obtained with a regular mesh of 72×72 E4 shell elements. The element size in the convergence curves is $h=L/N$.

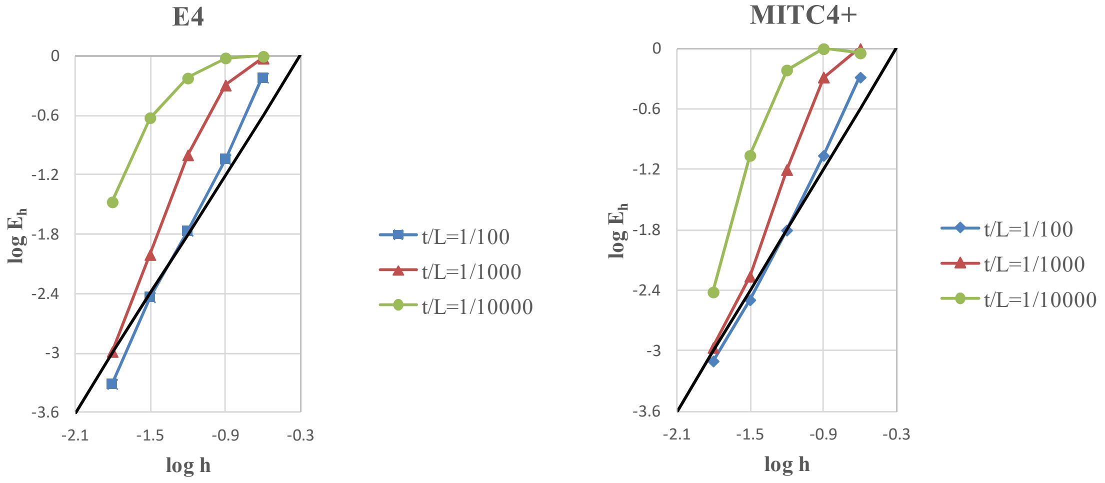

Figure 6. Convergence curves for the free cylindrical shell problem with distorted meshes

Figure 6 shows the main problem in the analysis of shell finite element, that is the influence of thickness parameter of distorted elements on the accuracy of the solution. Very thin shells extremely slowly converge to the reference solution. By increasing mesh density, numerical solution of very thin shells is approaching to the reference solution very slowly with a large deviation. It is obvious that by reducing thickness of distorted shell finite element, the speed of convergence to the reference solution is decreased, i.e. thicker shells for distortion shows better convergence.

It can be seen that MITC4+ shell finite element shows a slightly better convergence in the case of very thin shell. The obtained values are close enough to the exact values comparing to those of E4 shell element.

4. Conclusions

The main idea of this paper is the formulation of the modified 4-node quadrilateral MITC4 shell finite element. With the introduction of the center point 5, four triangular domains are defined. In that way the suggested membrane strain field is constructed using membrane strains obtained from triangular domains, thus eliminating the bilinear term in the field of strain. Theoretical proposals of this element are presented with aforementioned correction of its field with the aim of alleviation of the membrane locking which is strongly present in the distorted elements.

Major problems of the E4 shell element are also presented in this paper by numerical analysis of distribution of loaded pressure $p(\theta)$ on a cylinder with free ends, which shall be significantly reduced with the application of the modified element presented in this paper. The fact that the finite shell elements show variable sensitivity with reducing of thickness, depending on their geometry and boundary conditions, was confirmed with the same example. It has been observed that in distorsion elements the influence of the thickness parameter is especially evident, i.e., the speed of convergence of the numerical solution to the correct one is significantly slower in case of reduced thickness. The element shows great sensitivity to the irregularity of shape, which is a very important property for general application, and it must be taken into consideration because it is usually impossible to avoid the finite distorsion elements in modeling of structures.

The major problem that occurs during the analysis of finite elements is related to the reduction of the locking which appears in the shells with dominant bending, as in the case of the cylinder with free boundaries analysed in this paper. The tendency in analysis of finite elements of shells is to formulate the element that will converge uniformly, indipendent of the geometry, boundary conditions and its thickness. Of course, such element still does not exist and it is extremely difficult to formulate it. However, the application of the correction, presented in this paper, should reduce the sensitivity of the element when it is geometrically distorted. It is still necessary to implement the corrected element in the programming package PAK and test its performance on specific examples.

Acknowledgements

The authors gratefully acknowledge partial support by Ministry of Science and Technology of the Republic of Serbia, grants TR32036.

References

[1] Bathe K. J., Dvorkin E. N., *A formulation of general shell elements-the use of mixed interpolation of tensorial components,* International Journal for Numerical Methods in Engineering, Vol. 22, 697-722, 1986.

[2] Ko Y., Lee P. S., Bathe K. J., *The MITC4+ element and its performance,* Computers and Structures, Vol. 169, 57-68, 2016.

[3] Slavković R., Živković M., Kojić M., *Enhanced 8-node three-dimensional solid and 4-node shell elements with incompatibile generalized displacements,* Communications in Numerical Methods in Engineering, Vol. 10, 699-709, 1994.

[4] Chapelle D., Bathe K. J., *The finite element analysis of shells-fundamentals,* Springer, Berlin, 2003.

[5] Chapelle D., Bathe K.J., *Fundamental consideration for the finite element analysis of shell structures,* Computers and Structures, Vol. 66, 19-36, 1988.

[6] Lee P. S., Bathe K. J., *On the asymptotic behavior of shell structures and the evaluation in finite element solutions,* Computers and Structures, Vol. 80, 235-255,2002.

[7] Bathe K. J., Dvorkin E. N., *A continuum mechanics based four-node shell element for general nonlinear analysis,* Engineering with Computers, Vol. 1, 77-88, 1984.

[8] Bucalem M. L., Bathe K. J., *Higher – order MITC general shell elements,* International Journal for Numerical Methods in Engineering. 36, 3729-3754, 1993.

[9] Lee P. S. Bathe K. J., *Development of MITC isotropic triangular shell finite elements,* Computers and Structures, Vol. 82, 945-962, 2004.

[10] Bathe K. J., Lee P. S., Hiller J. F., *Towards improving the MITC9 shell element,* Computers and Structures, Vol. 81, 477-489, 2003.

[11] Beirão da Veiga L., Chapelle D., Paris Suarez I., *Towards improving the MITC triangular shell element,* Computers and Structures, Vol. 85, 1589-1610, 2007.

[12] Zienkiewicz O. C., Taylor R. L., Too J. M., *Reduced integration technique in general analysis of plates and shells,* International Journal for Numerical Methods in Engineering, Vol. 3, 275-291, 1971.

[13] Belytschko T., Tsay C. S., *A stabilization procedure for the quadrilateral plate element with one-point quadrature,* International Journal for Numerical Methods in Engineering, Vol. 19, 405-419, 1983.

[14] Belytschko T., Leviathan I., *Projection schemes for one-point quadrature shell elements,* Computer Methods in Applied Mechanics and Engineering, Vol. 115, 277-286, 1994.

[15] Vampa V., *Analysis of in-layer strains in the low order MITC shell element,* Latin American Journal of Solids and Structures, Vol. 4, 87-102, 2007.

[16] Kojic M., Slavkovic R., Zivkovic M., Grujovic N., *Metod konacnih elemenata 1,* Masinski fakultet u Kragujevcu, Kragujevac, 2010.

[17] Hiller J. F., Bathe K. J., *Measuring convergence of mixed finite element discretizations: an application to shell structures,* Computers and Structures, Vol. 81, 639-654, 2003.

[18] Bathe K. J., Iosilevich A., Chapelle D., *An evaluation of the MITC shell elements,* Computers and Structures, Vol. 75, 1-30, 2000.