UNIVERSITY OF EAST SARAJEVO
FACULTY OF MECHANICAL ENGINEERING

6th INTERNATIONAL SCIENTIFIC CONFERENCE

COMETa 2022

„Conference on Mechanical Engineering Technologies and Applications“

PROCEEDINGS

17th-19th November
East Sarajevo, RS, B&H

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University of East Sarajevo
Faculty of Mechanical Engineering
Conference on Mechanical Engineering Technologies and Applications

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PREFACE

The economic power of a society can be expressed by different indicators. However, it is certain that the competitiveness of the economy is one of the most important. In this sense, it is necessary for industrial production to follow modern development trends, which are based on current scientific achievements. Only a holistic approach in the application of knowledge in various engineering fields, and especially in the field of mechanical engineering, is a guarantee of economic progress, which enables long-term stability and prosperity of each country. Precisely for these reasons, the Faculty of Mechanical Engineering of the University of East Sarajevo organized the 1st International Scientific Conference COMETa in 2012, and this year is its 6th edition.

The main goal of the conference is to strengthen cooperation with the academic community, scientific-research institutions and, above all, with business entities. Conference COMETa 2022 is an opportunity for all participants to offer guidelines and create a better environment for more intensive industrial development through the exchange of knowledge and experience. That is going to have impact to increasing the competitiveness of national economic entities on the foreign market. The participation of a significant number of domestic and foreign scientists and researchers strengthens our conviction that in the near future we will be able to overcome challenges that are present in the technical-technological development of an advanced society in the 21st century, mainly through the generation of new ideas and the introducing of modern approaches to solving complex tasks in the field of mechanical engineering. In this sense, all your proposals and suggestions are more than welcome and will be carefully considered by the Scientific and Organizing Committee in order to improve the organization of the next conferences. Acknowledging the importance of the wide field of mechanical engineering for the overall industrial development of society, the work of the conference will take place through 7 sections, including the Student section. The program is focused on the following thematic areas:

- Manufacturing technologies and advanced materials,
- Applied mechanics and mechatronics,
- Machine design, simulation and modeling,
- Product development and mechanical systems,
- Energy and thermotechnic,
- Renewable energy and environmental protection,
- Maintenance and technical diagnostics,
- Quality, management and organization.

At this year's conference COMETa 2022, 105 papers including 4 plenary lectures will be published in the Proceedings.

We are specially looking forward that conference registered a record number of participants from abroad. Namely, 300 authors come from 25 countries. This is certainly the result of strenuous activities that were aimed at raising the international reputation and visibility of the conference in the regional, but also in the wider academic and scientific research space, which will be one of the primary goals in the future.

On behalf of the Organizing Committee of the conference COMETa 2022, we express our great gratitude to all the authors of the papers, reviewers, universities, faculties, business entities, national and international institutions and organizations that supported the conference. Without their help the organization and work of the conference would certainly not be at the level that its status deserves.

East Sarajevo, November 14th, 2022.

President of the Scientific Committee

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Full Professor

President of the Organizing Committee

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DETERMINING THE NUMERICAL VALUES OF THE POTENTIAL AT THE MEASURING POINTS

Snežana Vulović[1], Miroslav Živković[2], Rodoljub Vujanac[3], Ana Pavlović[4], Marko Topalović[5]

Abstract: In order to calibrate material parameters, a comparison of experimentally measured values and numerically predicted values is necessary. With complex FEM models, it is often impossible to create a mesh where the node of the element coincides with the position of the measuring point. The paper presents the procedure for determining the numerical values of the potential at the location of the measuring point. The first step is determining the element in which the measuring point is located and calculating the local coordinates of the measuring point in the element. The program (PAK-V) loads a file that contains the label of the measuring point, the element in which the measuring point is located and the local coordinates. Numerical values at the measuring points are calculated based on the nodal values of the element and the local coordinates. These extrapolated values are afterwards printed in a separate file for easier comparison with the measured values. After several iterations material parameters are obtained that accurately simulate geomechanical properties of a dam surroundings, facilitating structural analysis.

Key words: extrapolation, local coordinates, finite element, measuring point

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1 INTRODUCTION

Dams are objects that by their nature carry a certain risk, a potential danger to the surrounding area. That risk can never be completely removed, so the question of the safety of the dam, as well as the associated facilities, is of utmost importance. The safety of the dam implies that the dam is always in a condition in which it can fulfil all its designed functions without adverse consequences for people, the environment or property. Over time the exploitation conditions of the dam vary as the characteristics of the materials from which the objects were built, and the properties of the geotechnical environments upon which the objects were built change. Also, over time, the professional and social views of the safety criteria and risks (hydrological, seismotectonic, and others), evolve with occasional changes in standards and legal norms. Practically, the safety of dams is managed during their entire operational life.

During the construction, in order to monitor the condition of the dam and facilities, a system of technical observation is designed with a significant number of various instruments and measuring tools. The aim of data collection is to ensure relevant information on which identification and timely detection of the process that can cause threaten the security of the dam and facilities. The technical observation system changes and evolves during time. In recent years, the safety management system (SMS) for dams was formed with the aim of combining and standardizing collection, acquisition, archiving, processing and using data on technical observation.

The concept of dam safety management implies a series of procedures that are physically based, and supported with software system, which will ensure collection of the all data that are important for determination of current dam condition. Analysis of these data means engineering interpretation through mathematical models of relevant processes that are crucial for dam safety. Based on the conclusions of these predictive models, appropriate measures are adopted and implemented [1].

As a part of the dam SMS for filtration analysis and the stress-strain analysis, a finite element (FE) model of the dam and the surrounding rock mass is created. Inclusion of the position of measuring points in the FE model (i.e. matching the position of the measuring point with the FE node), results in the significant increase of the number of elements and nodes of the model. Also, the position of all of the measuring points at the time of the model creation is unknown, or the new measuring points are subsequently added. For these reasons, the process of calculating the numerical value (potential, strain...) when the position of the measuring point does not match the node in the model was developed, which is presented in this paper.

2 NATURAL COORDINATE SYSTEM IN TETRAHEDRAL ELEMENTS

The Finite Element Method (FEM) is the most common numerical methods that has implementation in almost all areas of science and technology. The basic idea of the FEM analysis is a spatial domain discretization to the subdomains on which the general laws of the continuum mechanics and numerical mathematics apply. These subdomains are terminologicaly denoted as the Finite Elements. The analysis of the system of coupled Finite Elements, obtained by discretizing continuum, enables the numerical simulation of the continuum response to the given excitation. The physical values included in the model are obtained in discreet form, i.e. in points derived from the discretization. These points are called nodes. Based on discretization of physical problem, and implementation of interpolation functions, with the introduction of natural (local) coordinate system, equilibrium equations are established for elements, and stored in the matrix form. Subsequently, element matrices are combined into a system matrix for the entire structure. Characteristic of the natural coordinate system is that it is tied to the element, that the coordinate origin is in the center of the element and that the coordinates of the nodes are 1 or -1 [2-3].

A three-dimensional (3D) finite element is used for modelling three-dimensional bodies of general shape. A three-dimensional finite element can have a different number of nodes - the usual number is from 8 to 21. For modelling complex, irregular shapes, tetrahedral finite elements with midside nodes are used. Figure 1 shows a tetrahedral element with midside nodes with an arbitrary position of the Measuring Point (M_P).

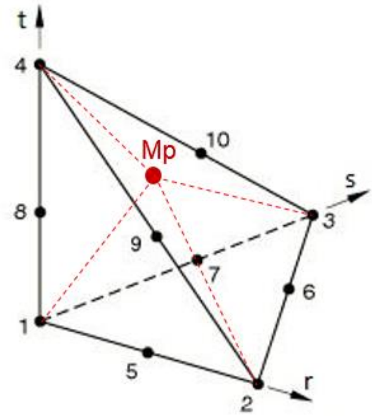

Figure 1. *Tetrahedral element with 10 nodes and the arbitrary position of the measuring point*

Geometry interpolation functions have the following form:

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \mathbf{HX}\,, \tag{1}$$

where: $\mathbf{x}$ represents the coordinate vector of the material point in the element; while $\mathbf{H}$ is the interpolation matrix. Vector of node coordinates $\mathbf{X}$ is given with:

$$\mathbf{X}^T = \lfloor X_1\ Y_1\ Z_1\ X_2\ Y_2\ Z_2 .....X_N\ Y_N\ Z_N \rfloor. \tag{2}$$

For the interpolation matrix:

$$\mathbf{H} = \begin{bmatrix} h_1 & 0 & 0 & h_2 & 0 & 0 & ...... & h_N & 0 & 0 \\ 0 & h_1 & 0 & 0 & h_2 & 0 & ...... & 0 & h_N & 0 \\ 0 & 0 & h_1 & 0 & 0 & h_2 & ...... & 0 & 0 & h_N \end{bmatrix}, \tag{3}$$

the interpolation functions for the tetrahedral finite element are given in the Table 1.

Table 1. *Interpolation functions for the tetrahedral finite element*

		$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$
h_1	$1-r-s-t$	$-\frac{1}{2}h_5$		$-\frac{1}{2}h_7$	$-\frac{1}{2}h_8$		
h_2	r	$-\frac{1}{2}h_5$	$-\frac{1}{2}h_6$			$-\frac{1}{2}h_9$	
h_3	s		$-\frac{1}{2}h_6$	$-\frac{1}{2}h_7$			$-\frac{1}{2}h_{10}$
h_4	t				$-\frac{1}{2}h_8$	$-\frac{1}{2}h_9$	$-\frac{1}{2}h_{10}$
h_5	$4r(1-r-s-t)$						
h_6	$4rs$						
h_7	$4s(1-r-s-t)$						
h_8	$4t(1-r-s-t)$						
h_9	$4rt$						
h_{10}	$4st$						

The coordinates of the measuring point MP (x, y, z) can be expressed using the coordinates of the element in which it is located, according to the equation (1), as:

$$\begin{aligned} x &= (1-r-s-t)X_1 + rX_2 + sX_3 + tX_4 \\ y &= (1-r-s-t)Y_1 + rY_2 + sY_3 + tY_4 \\ z &= (1-r-s-t)Z_1 + rZ_2 + sZ_3 + tZ_4 \end{aligned} \quad , \tag{4}$$

where X_1, Y_1, Z_1; X_2, Y_2, Z_2; X_3, Y_3, Z_3; X_4, Y_4, Z_4 are the coordinates of the nodes of the tetrahedral element (Figure 1.), while r, s, t are the local coordinates of the measuring point MP given by the equations 5-7:

$$r = \frac{\left\{ \begin{array}{l} x\lfloor Z_1(Y_3-Y_4)+Z_3(Y_4-Y_1)+Z_4(Y_1-Y_3)\rfloor + X_1\lfloor Y_3(Z_4-z)+y(Z_3-Z_4)+Y_4(z-Z_3)\rfloor + \\ X_3\left[Y_4(Z_1-z)+y(Z_4-Z_1)+Y_1(z-Z_4)\right] + X_4\left[Y_3(z-Z_1)+y(Z_1-Z_3)+Y_1(Z_3-z)\right] \end{array} \right\}}{\left\{ \begin{array}{l} X_2\left[Z_1(Y_3-Y_4)+Z_3(Y_4-Y_1)+Z_4(Y_1-Y_3)\right] + X_1\left[Y_3(Z_4-Z_2)+Y_2(Z_3-Z_4)+Y_4(Z_2-Z_3)\right] + \\ X_3\left[Y_4(Z_1-Z_2)+Y_1(Z_2-Z_4)+Y_2(Z_4-Z_1)\right] + X_4\left[Y_3(Z_2-Z_1)+Y_2(Z_1-Z_3)+Y_1(Z_3-Z_2)\right] \end{array} \right\}} \tag{5}$$

$$s = \frac{\left\{ \begin{array}{l} x\lfloor Z_1(Y_4-Y_2)+Z_2(Y_1-Y_4)+Z_4(Y_2-Y_1)\rfloor + X_1\lfloor Y_2(z-Z_4)+y(Z_4-Z_2)+Y_4(Z_2-z)\rfloor + \\ X_2\left[Y_4(z-Z_1)+y(Z_1-Z_4)+Y_1(Z_4-z)\right] + X_4\left[Y_2(Z_1-z)+y(Z_2-Z_1)+Y_1(z-Z_2)\right] \end{array} \right\}}{\left\{ \begin{array}{l} X_2\left[Z_1(Y_3-Y_4)+Z_3(Y_4-Y_1)+Z_4(Y_1-Y_3)\right] + X_1\left[Y_3(Z_4-Z_2)+Y_2(Z_3-Z_4)+Y_4(Z_2-Z_3)\right] + \\ X_3\left[Y_4(Z_1-Z_2)+Y_1(Z_2-Z_4)+Y_2(Z_4-Z_1)\right] + X_4\left[Y_3(Z_2-Z_1)+Y_2(Z_1-Z_3)+Y_1(Z_3-Z_2)\right] \end{array} \right\}} \tag{6}$$

$$t = \frac{-\left\{ \begin{array}{l} \left[X_3 (Y_1 - Y_2) + X_1 (Y_2 - Y_3) + X_2 (Y_3 - Y_1) \right] \left[X_2 (Z_1 - z) + X_1 (z - Z_2) + x (Z_2 - Z_1) \right] - \\ \left[X_2 (Y_1 - y) + X_1 (y - Y_2) + x (Y_2 - Y_1) \right] \left[X_3 (Z_1 - Z_2) + X_1 (Z_2 - Z_3) + X_2 (Z_3 - Z_1) \right] \end{array} \right\}}{\left\{ \begin{array}{l} -\left[X_4 (Y_1 - Y_2) + X_1 (Y_2 - Y_4) + X_2 (Y_4 - Y_1) \right] \left[X_3 (Z_1 - Z_2) + X_1 (Z_2 - Z_3) + X_2 (Z_3 - Z_1) \right] \\ +\left[X_3 (Y_1 - Y_2) + X_1 (Y_2 - Y_3) + X_2 (Y_3 - Y_1) \right] \left[X_4 (Z_1 - Z_2) + X_1 (Z_2 - Z_4) + X_2 (Z_4 - Z_1) \right] \end{array} \right\}} \quad (7)$$

2.1 Algorithm for determination of local coordinates

In order to determine the exact element in the whole model in which a measuring point is located, and to determine local coordinates of that measuring point, Application Programming Interface (API) for program Femap [4] was developed. The algorithm for determining local coordinates is shown in the Figure 2.

When the model of the dam and the surrounding rock mass is completed, another file is loaded that contains the positions of the measuring points. When the API is started, the user must first select the measuring points and the appropriate elements in the model. API then determines in which element in the model the measuring point is located. Based on the coordinate of nodes, the volume of the element is calculated, and the volume of »virtual« elements formed with the measuring point and the nodes of the observed element (nodes 1,2, 3,4), Figure 1. are calculated.

»Virtual« elements are: 1,2,3,M_P; 1,3,4,M_P; 1,2,4,M_P and 2,3,4,M_P. If the measuring point is located in the observed element, local coordinates are calculated according to Eqs. (5-7). API creates ASCII file PIJEZ.DAT that has an ID of measuring point, ID of element, and local coordinates of the measuring point in the element.

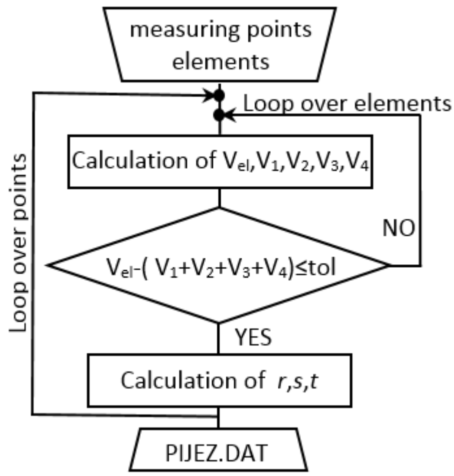

Figure 2. *Algorithm for calculation of the local coordinates*

3 CALCULATION OF POTENTIAL

The PAKV program [5] is used for the analysis of groundwater flow through porous materials. The PAKV program has been modified to calculate and print potential values at the measuring points, Figure 3. PAKV loads the input file pakp.dat from which it reads the identifier for (not) reading the measuring points, i.e. PIJEZ.DAT file. In the PAKV program, in the phase of printing the results, the potential at the measuring point is calculated by extrapolation based on the nodal values of the potential in the element and the local coordinates of the measuring point according to equation (1). The potential values at the measuring points are printed in the file pijeznivoPAK.csv. Numerically calculated values are compared with measured values and the model is calibrated, i.e. correction of the filtration coefficients in the model with the aim of obtaining the best match between measured and numerically obtained values.

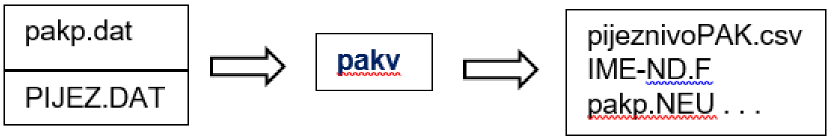

Figure 3. *Flow chart of the program PAKV*

4 CONCLUSIONS

The presented procedure was developed for the Đerdap dam model. Dam and surrounding rock mass are modeled with 3D tetrahedral elements. During the creation of the finite element model, the position of the measuring points was not considered. When the model was completed, the API was used to determine the final element in which the measuring point is located and to determine the natural (local) coordinates of the measuring point in the element; an ASCII file is created, which is one of the input files for the PAKV program. Using the extrapolation, the PAKV program calculates the potentials at the measuring points.

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