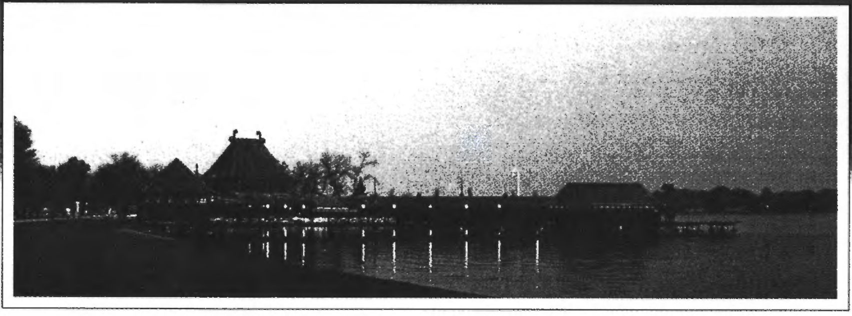
Proceedings

IConSSM 2009

2nd International Congress of Serbian Society of Mechanics

Palić (Subotica), 1-5 June 2009



Editors

Teodor Atanacković
Dragan Spasić
Srboljub Simić

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2nd International Congress of Serbian Society of Mechanics (IConSSM 2009)

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PREFACE

The present volume contains plenary lectures and papers of young authors, competing for the prize "dr Rastko Stojanović", presented at the Second International Congress of Serbian Society of Mechanics, held at Palić (Subotica) during the period 1^{st}-5^{th} June 2009. The Congress was organized by the Serbian Society of Mechanics. The aim of the Congress is presentation of original high level work, at the forefront of research, in various areas of Theoretical and Applied Mechanics.

During the Congress 123 papers were presented, grouped in four traditional sections: General Mechanics, Fluid Mechanics, Mechanics of Solid Bodies and Interdisciplinary and Multidisciplinary Problems. Moreover, three mini-symposia are held: (M1) MM-IX Fractional Calculus and Applications, (M2) Computational Methods in Structural Analysis and Fracture Mechanics and (M3) Computational Biomechanics. In addition, 8 invited plenary lectures were presented by the authors from France, Germany, Italy, Serbia, Slovakia and United States.

The Editors would like to thank all the authors of the papers for their active participation during the Congress, the reviewers of the papers, the members of the Scientific and Organizing Committee, the members of the Executive Committee of the Serbian Society of Mechanics, and the distinguished invited lecturers who kindly accepted the invitation to come to Congress and helped make it success.

Special thanks are also due to those organizations which supported financially this Congress: Serbian Society of Mechanics, Engineering Chamber of Serbia, Ministry of Science of Serbia, Provincial secretariat for Science and Technological Development of the Province of Vojvodina. and Municipality of Subotica.

Palić, June 2009

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2nd International Congress of Serbian Society of Mechanics
IConSSM 2009, Palić (Subotica), 1-5 June 2009

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COMPARATIVE ANALYSIS OF IMPLICIT AND EXPLICIT NUMERICAL METHODS IN DYNAMIC PROBLEMS SOLVING

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Abstract. Theoretical basics formulation of the implicit Newmark-method and the explicit central difference method for numerical integration of dynamic motion equations is presented in this paper. According to theoretical assumptions, algorithms for the integration of dynamic motion equation are developed. Algorithms are implemented into the software package PAK. Verification of the developed algorithms has been done for elementary examples which have analytical solutions. At the end of the comparative result analysis obtained by applying both methods of numerical integration, dynamic behavior analysis of buildings on the surface soil due to impact of the wagon wheels in motion on underground railway tracks.

1. Introduction

In practice, forces applied on constructions change during the time so displacement of some spots depends on time. Construction motion is affected by inertial characteristics which have to be considered during motion calculation. Therefore, instead of static equilibrium conditions differential motion equations have to be set, whose integration leads to displacement, velocity and acceleration of specific construction spots.

Such as in static analysis, finite element method has a significant implementation in the field of construction dynamics. Specific dynamic problems in practice are solved by using approximate explicit and implicit numerical integration method of dynamic motion equations.

This paper presents theoretical bases of implicit Newmark method and explicit central difference method, a then according to presented theoretical bases, algorithms for integration of dynamic motion equations are developed and implemented in PAK program package [1]. Section 4 of the paper shows comparable analysis of obtained results by using both numerical integration methods on the example of dynamic buildings behavior analysis on the surface due to the wagon wheels crash during motion in subway. In the end, the conclusion shows some observations obtained by comparable analysis of implicit and explicit numerical integration method while dynamic problem solving.

2. Differential motion equations

Using the virtual work principle $\delta \mathrm{A}_u = \delta \mathrm{A}_s$, differential motion equations of continuum divided into finite elements [2] are given in equation (1), Figure 1:

$$\int_V \sigma_{kj} \delta e_{kj} dV = \int_V F_j^v \delta u_j dV + \int_{S^\sigma} F_j^s \delta u_j dS + \sum_i F_k^i \delta u_k^i \tag{1}$$

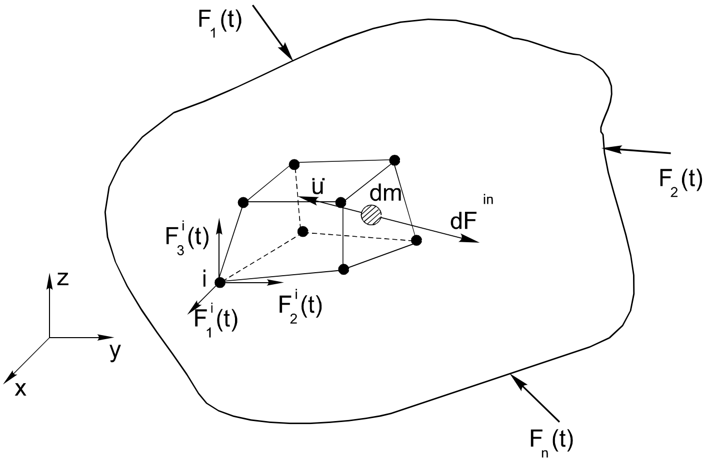

Figure 1. Forces applied on elementary finite element mass *dm*

Left side of the equation (1) represents virtual work of the inside forces:

$$\delta \mathrm{A}_u = \int_V \sigma_{kj} \delta e_{kj} dV \; . \tag{2}$$

Right side of the equation (1) represents virtual work of the outside forces:

$$\delta \mathrm{A}_s = \int_V F_k^v \delta u_k dV + \int_{S^\sigma} F_k^s \delta u_k dS + \sum_i F_k^i \delta u_k^i \, , \tag{3}$$

In dynamic analysis inertial forces are added to volume forces $\mathbf{F}^V$ in the equation of finite element equilibrium. Elementary volume inertial force $d\mathbf{F}^{in}$ which corresponds to elementary mass dm is:

$$d\mathbf{F}^{in} = -\ddot{\mathbf{u}} dm = -\ddot{\mathbf{u}} \rho dV \; . \tag{4}$$

Substituting inertial force equation in relation (1) we get

$$\int_V \delta \mathbf{e}^T \boldsymbol{\sigma} dV = \int_V \delta \mathbf{u}^T \left(\mathbf{F}^V - \rho \ddot{\mathbf{u}} \right) dV + \int_S \delta \mathbf{u}_S^T \mathbf{F}^s dS \, . \tag{5}$$

Differentiating with respect to time of relation for displacement

$$\mathbf{u} = \mathbf{HU} \, , \tag{6}$$

we get velocity and acceleration for arbitrary material point

$$\dot{\mathbf{u}} = \mathbf{H}\dot{\mathbf{U}} \tag{7}$$

$$\ddot{\mathbf{u}} = \mathbf{H}\ddot{\mathbf{U}} \tag{8}$$

Substituting equation (6) for $\delta\mathbf{u}$, in equation (5) follows

$$\delta\mathbf{U}^T\left(\int_V \mathbf{H}^T \rho \mathbf{H} dV\right)\ddot{\mathbf{U}} + \delta\mathbf{U}^T\int_V \left(\mathbf{B}^T\mathbf{C}\mathbf{B}dV\right)\mathbf{U} = \delta\mathbf{U}^T\mathbf{F} \tag{9}$$

wherefrom:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}(t)\,. \tag{10}$$

where $\mathbf{M}$ is finite element mass matrix, defined by the equation

$$\mathbf{M} = \int_V \rho \mathbf{H}^T \mathbf{H} dV\,. \tag{11}$$

Vector $\mathbf{F}(t)$ represents outside forces applied on the finite element nodes, and comprises surface, volume, and concentrated forces depending on time.

In real materials appear damping forces proportional to point velocity. Elementary damping force $d\mathbf{F}^d$ which corresponds to elementary volume dV is

$$d\mathbf{F}^d = -b\dot{\mathbf{u}}dV\,, \tag{12}$$

where b is dumping coefficient. Replacing equations for dumping force (12) in (5), as complementary volume force, by using (7), finite element equation of motion is obtained:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{B}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}(t) \tag{13}$$

where $\mathbf{B}$ is dumping matrix,

$$\mathbf{B} = \int_V b \mathbf{H}^T \mathbf{H} dV\,. \tag{14}$$

Damping matrix $\mathbf{B}$ and mass matrix $\mathbf{M}$ have the same dimensions as element stiffness matrix $\mathbf{K}$, so the same procedure for matrix members organizing is used during forming mass matrix and dumping matrix of the construction.

Equations (11) and (14) define consistent mass matrix and consistent damping matrix.

In order to save computer operating time, it is convenient to use lumped mass matrix, which contains members only on the main diagonal, instead of consistent mass matrix in the construction calculation with large number of degree of freedom. These members represent equivalent "concentrated masses". Construction mass matrix is in this case also diagonal so it is stored in the memory as vector of length equal to numbers of degrees of freedom.

3. Numerical integration methods of differential motion equations

Numerical methods of direct differential equation system integration have a wide usage in FEM and instead of analytical results for any moment of time t; they give results in discrete moments of time $0, \Delta t, 2\Delta t, \ldots, n\Delta t$. Namely, we consider that the total period of time T is divided into n intervals $\Delta t = T/n$, within which we are interested in result of differential equation system. The procedure is based on searching the result in the moment $t+\Delta t$ at the end of the step according to the known result in moment t at the beginning of the interval Δt. Thereat, assumptions for change of displacement, velocity and acceleration within the interval Δt are introduced. These methods are often called step by step methods in the literature.

3.1. The Newmark method

Implicit methods mean fulfillment of differential motion equations at the moment $t+\Delta t$, so according to the known results at the moment t, unknown results are obtained at the moment $t+\Delta t$ and thus their fulfillment is certain and accurate. Choice of time step Δt in implicit methods is related to the desirable result accuracy and less to stability, since these methods can reach unconditional stability.

Implementation of Newmark method of direct system equation integration (10) or (13) is based on the assumption that generalized accelerations are constant in the time interval Δt, and that they can be written in the following way:

$$\ddot{\mathbf{U}}(\tau) = (1-\delta)\,{}^{t}\ddot{\mathbf{U}} + \delta\,{}^{t+\Delta t}\ddot{\mathbf{U}}, \tag{15}$$

where $\ddot{\mathbf{U}}(\tau)$ is acceleration at the moment $t \le \tau \le t+\Delta t$, further denoted as $\ddot{\mathbf{U}}$, $0 \le \delta \le 1$ is parameter, ${}^{t}\ddot{\mathbf{U}}$ and ${}^{t+\Delta t}\ddot{\mathbf{U}}$ are accelerations at the moment u t and $t+\Delta t$. Here left superscript shows time moment that presented value it related to.

Integrating equation (15) in the interval Δt, we get:

$$ {}^{t+\Delta t}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \left[(1-\delta)\,{}^{t}\ddot{\mathbf{U}} + \delta\,{}^{t+\Delta t}\ddot{\mathbf{U}}\right]\Delta t \tag{16}$$

$$ {}^{t+\Delta t}\mathbf{U} = {}^{t}\mathbf{U} + {}^{t}\dot{\mathbf{U}}\Delta t + \frac{1}{2}\left[(1-\delta)\,{}^{t}\ddot{\mathbf{U}} + \delta\,{}^{t+\Delta t}\ddot{\mathbf{U}}\right]\Delta t^2 . \tag{17}$$

In order to achieve better stability and result accuracy, instead of obtained equation (17) for ${}^{t+\Delta t}\mathbf{U}$, the following equation is used:

$$ {}^{t+\Delta t}\mathbf{U} = {}^{t}\mathbf{U} + {}^{t}\dot{\mathbf{U}}\Delta t + \left[\left(\frac{1}{2}-\alpha\right){}^{t}\ddot{\mathbf{U}} + \alpha\,{}^{t+\Delta t}\ddot{\mathbf{U}}\right]\Delta t^2 , \tag{18}$$

where α is coefficient.

$$\delta \ge \frac{1}{2}; \quad \alpha \ge \frac{1}{4}\left(\frac{1}{2}+\delta\right)^2 \tag{19}$$

In calculation, the most widely used

$$\delta = \frac{1}{2} \quad \alpha = \frac{1}{4} \tag{20}$$

Figure 2 shows approximation for speed $\dot{\mathbf{U}}$ that corresponds to the assumptions above. So, acceleration is constant in the interval Δt, corresponding to (15) and it is expressed through acceleration value at the beginning and end of time step. It is noticed that for $\delta = 0$ $\ddot{\mathbf{U}} = {}^{t}\ddot{\mathbf{U}}$, for $\delta = 1$ $\ddot{\mathbf{U}} = {}^{t+\Delta t}\ddot{\mathbf{U}}$, and for $\delta = 0.5$ $\ddot{\mathbf{U}} = \left({}^{t}\ddot{\mathbf{U}} + {}^{t+\Delta t}\ddot{\mathbf{U}}\right)/2$. Speed $\dot{\mathbf{U}}$ is linear function and generalized displacement $\mathbf{U}$ is square function in the interval Δt.
Newmark method is implicit so equation (13) is written for the moment $t + \Delta t$,

$$\mathbf{M}\,{}^{t+\Delta t}\ddot{\mathbf{U}} + \mathbf{B}\,{}^{t+\Delta t}\dot{\mathbf{U}} + \mathbf{K}\,{}^{t+\Delta t}\mathbf{U} = {}^{t+\Delta t}\mathbf{F} \tag{21}$$

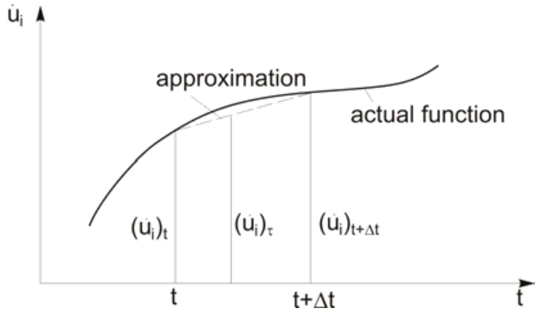

Figure 2. Approximation of generalized speed for the time step

Further procedure is based on expressing the vector ${}^{t+\Delta t}\ddot{\mathbf{U}}$ through ${}^{t}\mathbf{U}, {}^{t}\dot{\mathbf{U}}, {}^{t}\ddot{\mathbf{U}}$ and ${}^{t+\Delta t}\mathbf{U}$ according to equation (18)

$${}^{t+\Delta t}\ddot{\mathbf{U}} = \frac{1}{\alpha\left(\Delta t\right)^2}\left[{}^{t+\Delta t}\mathbf{U} - {}^{t}\mathbf{U} - {}^{t}\dot{\mathbf{U}}\Delta t - \left(\frac{1}{2} - \alpha\right)\left(\Delta t\right)^2\,{}^{t}\ddot{\mathbf{U}}\right] \tag{22}$$

Then , ${}^{t+\Delta t}\dot{\mathbf{U}}$ is expressed, substituting equation (16) in (20), in the form:

$${}^{t+\Delta t}\dot{\mathbf{U}} = \frac{\delta}{\alpha\Delta t}\left({}^{t+\Delta t}\mathbf{U} - {}^{t}\mathbf{U}\right) - \left(\frac{\delta}{\alpha} - 1\right){}^{t}\dot{\mathbf{U}} - \left(\frac{\delta}{2\alpha} - 1\right)\Delta t\,{}^{t}\ddot{\mathbf{U}} \tag{23}$$

Finally , replacing final equations for ${}^{t+\Delta t}\ddot{\mathbf{U}}$ and ${}^{t+\Delta t}\dot{\mathbf{U}}$ in equation (21), we obtain system of algebraic equations with respect to unknown generalized displacements,

$$\hat{\mathbf{K}}\,{}^{t+\Delta t}\mathbf{U} = {}^{t+\Delta t}\hat{\mathbf{F}} \tag{24}$$

which can be solved with respect to unknown displacements ${}^{t+\Delta t}\mathbf{U}$. Matrix $\hat{\mathbf{K}}$ and right side vector $\hat{\mathbf{F}}$ are:

$$\hat{\mathbf{K}} = \mathbf{K} + a_0\mathbf{M} + a_1\mathbf{B} \tag{25}$$

$${}^{t+\Delta t}\hat{\mathbf{F}} = {}^{t+\Delta t}\mathbf{F} + \mathbf{M}\left(a_0\,{}^{t}\mathbf{U} + a_2\,{}^{t}\dot{\mathbf{U}} + a_3\,{}^{t}\ddot{\mathbf{U}}\right) + \mathbf{B}\left(a_1\,{}^{t}\mathbf{U} + a_4\,{}^{t}\dot{\mathbf{U}} + a_5\,{}^{t}\ddot{\mathbf{U}}\right) \tag{26}$$

where $a_0, a_1, \ldots a_5$ are coefficients

$$
\begin{aligned}
&a_0 = \frac{1}{\alpha(\Delta t)^2}, \quad a_1 = \frac{\delta}{\alpha \Delta t}, \quad a_2 = \frac{1}{\alpha \Delta t} \\
&a_3 = \frac{1}{2\alpha} - 1, \quad a_4 = \frac{\delta}{\alpha} - 1, \quad a_5 = \left(\frac{\delta}{2\alpha} - 1\right)\Delta t
\end{aligned}
\tag{27}
$$

It is noticed that in case of linear problem (system matrixes are constant) and during the same integration step, it is possible to factorize matrix $\hat{\mathbf{K}}$ only once, and then for each time step to determine right side vector ${}^{t+\Delta t}\hat{\mathbf{F}}$ as well as to efficiently get results using backward algorithm. Since factorization of matrix $\hat{\mathbf{K}}$ takes most time, this way computer operating time is saved. Time step choice affects result accuracy and stability, especially while solving nonlinear problems. In practice, time step is most often determined to be at least 20 smaller than period that correspond to first eigen value, which means that analysis of eigen values should always be done first, and then, according to the obtained results take the time step.

3.1. The Central Difference Method

Explicit methods are based on fulfillment of differential motion equations at the moment t in order to determine values at the moment $t+\Delta t$, whereas all the values and their derivatives are used at the moment t. These methods demand considerably less integration step Δt in order to get stable results and fulfilling accuracy. Stability means that the initial condition change cannot lead to result divergention (displacement increase that does not correspond to system motion).

Central difference method is one of the most frequently used explicit methods for numerical integration. It is developed according to central difference equations for velocity and acceleration. According to Figure 3, velocity in the moment t is given in equation (28) and (29) and acceleration in the equation (30) [3],[4]

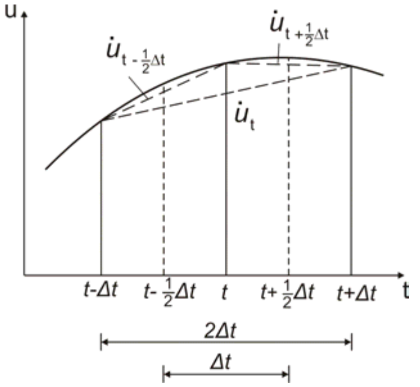

Figure 3. Discretization by central difference method

$$
{}^{t}\dot{\mathbf{U}} = \frac{1}{\Delta t}\left({}^{t+\frac{1}{2}\Delta t}\mathbf{U} - {}^{t-\frac{1}{2}\Delta t}\mathbf{U} \right) \tag{28}
$$

$$ {}^{t}\dot{\mathbf{U}} = \frac{1}{2\Delta t}\left({}^{t+\Delta t}\mathbf{U} - {}^{t-\Delta t}\mathbf{U} \right) \tag{29} $$

$$ {}^{t}\ddot{\mathbf{U}} = \frac{1}{\Delta t^2}\left({}^{t+\Delta t}\mathbf{U} - 2\,{}^{t}\mathbf{U} + {}^{t-\Delta t}\mathbf{U} \right) \tag{30} $$

Central difference method is explicit, so the equation (13) is written for moment t ,

$$ \mathbf{M}\,{}^{t}\ddot{\mathbf{U}} + \mathbf{B}\,{}^{t}\dot{\mathbf{U}} + \mathbf{K}\,{}^{t}\mathbf{U} = {}^{t}\mathbf{F} \tag{31} $$

Replacing relations for ${}^{t}\ddot{\mathbf{U}}$ and $\dot{\mathbf{U}}$ from (29) and (30) respectively, in the equation (31) we have

$$ \left(\frac{1}{\Delta t^2}\mathbf{M} + \frac{1}{2\Delta t}\mathbf{B} \right) {}^{t+\Delta t}\mathbf{U} = {}^{t}\mathbf{F} - \left(\mathbf{K} - \frac{2}{\Delta t^2}\mathbf{M}\,{}^{t} \right)\mathbf{U} - \left(\frac{1}{\Delta t^2}\mathbf{M} - \frac{1}{2\Delta t}\mathbf{B} \right) {}^{t-\Delta t}\mathbf{U}\,, \tag{32} $$

Or the algebraic equation system with respect to unknown generalized displacements,

$$ \hat{\mathbf{K}}\,{}^{t+\Delta t}\mathbf{U} = {}^{t}\hat{\mathbf{F}} \tag{33} $$

which can be solved with respect to unknown displacements ${}^{t+\Delta t}\mathbf{U}$. Matrix $\hat{\mathbf{K}}$ and right side vector $\hat{\mathbf{F}}$ are:

$$ \hat{\mathbf{K}} = a_0\mathbf{M} + a_1\mathbf{B} \tag{34} $$

$$ {}^{t}\hat{\mathbf{F}} = {}^{t}\mathbf{F} - \left(\mathbf{K} + a_2\mathbf{M} \right) {}^{t}\mathbf{U} - \left(a_0\mathbf{M} - a_1\mathbf{B} \right) {}^{t-\Delta t}\mathbf{U} \tag{35} $$

where a_0, a_1, a_2, a_3 are coefficients

$$ a_0 = \frac{1}{\Delta t^2}, \quad a_1 = \frac{1}{2\Delta t}, \quad a_2 = 2a_0, \quad a_3 = \frac{1}{a_2}\,. \tag{36} $$

At the moment $t = 0$, considering the initial conditions ${}^{0}\mathbf{U}, {}^{0}\dot{\mathbf{U}}, {}^{0}\ddot{\mathbf{U}}$ we get:

$$ {}^{-\Delta t}\mathbf{U} = {}^{0}\mathbf{U} - \Delta t\,{}^{0}\dot{\mathbf{U}} + a_3\,{}^{0}\ddot{\mathbf{U}}\,. \tag{37} $$

An important thing while using central difference method is choice of integration time step Δt , which has to be smaller than critical value $\Delta t_{critical}$. For different types of elements, time step is determined according to the equation (38),

$$ \Delta t < \Delta t_{critical} = \frac{l}{c}\,, \tag{38} $$

Where l is minimal element length, and c is sound velocity through a corresponding material

$$ c = \sqrt{\frac{E}{\rho}} \tag{39} $$

E - Young-elasticity module, ρ - material density.

4. Example

4.1. Model Description

3D model is taken from [5] and it is shown in Figure 4. There are three buildings on the surface of the ground and underground tunel. Dynamic loading arise when the train passes through the tunel.

3D isoparametric finite elements with 8 nodes are used for the model [6], and there are two materials – material of ground and material of buildings. Material data are given in Table 1.

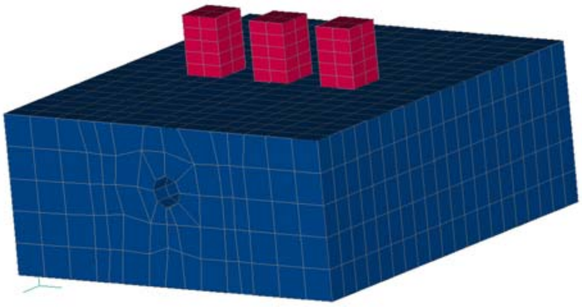

Figure 4. 3D Model

Table 1. Material data

Material ID	Young's Modulus	Poisson's ratio	Density
ground	$3.5\ 10^{7}$	0.3	1900
buildings	$3.2\ 10^{10}$	0.3	81

Bottom and side surfaces of ground are constrained in all degree of freedom (all translations are constrained).

4.2. Dynamic loading

Dynamic load is applied as combination of impulse forces which are applied in 10 pairs of nodes. Force is applied in vertical direction. There are 20 elements along the tunnel, i.e. there are 21 nodes on the length. Load is applied as impulse forces in every second pair of nodes along the tunnel. Time function of impulse force for the first pair of nodes, on the beginning of tunnel, is shown in Figure. 5. The same time function is used for defining other nine pairs of forces, which sequently delays for τ=0.4 s for every pair. Time function and delay directly depend of velocity of train.

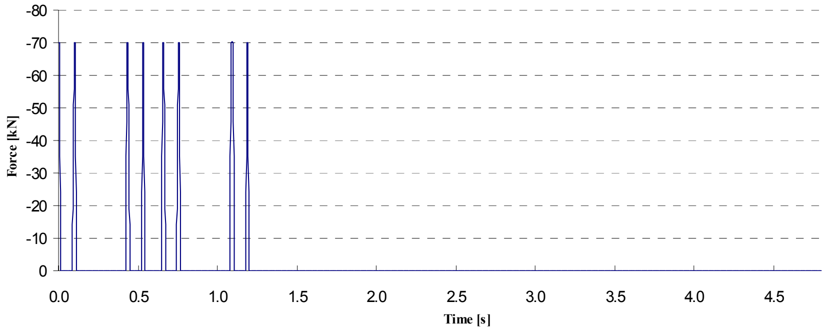

Figure 5. Time function of impulse force in the nodes 1070 i 1169.

Node number and correspond time delay τ are given in Table 2.

Table 2. Pairs of nodes and correspond delays

NODE ID		Delay τ [s]
1070	1169	0
1072	1171	0.4
1074	1173	0.8
1076	1175	1.2
1078	1177	1.6
1080	1179	2.0
1082	1181	2.4
1084	1183	2.8
1086	1185	3.2
1088	1187	3.6

4.3. Dynamic Analysis and Results

The objective is to obtain response of model under dynamic loading using implicit Newmark and explicit central difference numerical time integration method. Time for train passes trough the tunnel is 4.8 s. Dynamic analysis is performed in software package PAK.
Using implicit Newmark numerical time integration method, problem is solved using 600 equal time steps of length $\Delta t = 8 \cdot 10^{-3} s$. The first integration parameter is 0.5, and the second is 0.25. In this calculation case we use lumped mass matrix.
Using explicit central difference numerical time integration method, problem is solved using 48000 time steps of length $\Delta t = 1 \cdot 10^{-4} s$, according equations (39) and (38).

Figures 6, 7, and 8, show comparative analysis of numerical results obtained for node 576 displacement in X, Y, Z direction, respectively, by using both numerical integration methods. Node 576 is the middle node on the top of the first building, from left to right, Figure 4.

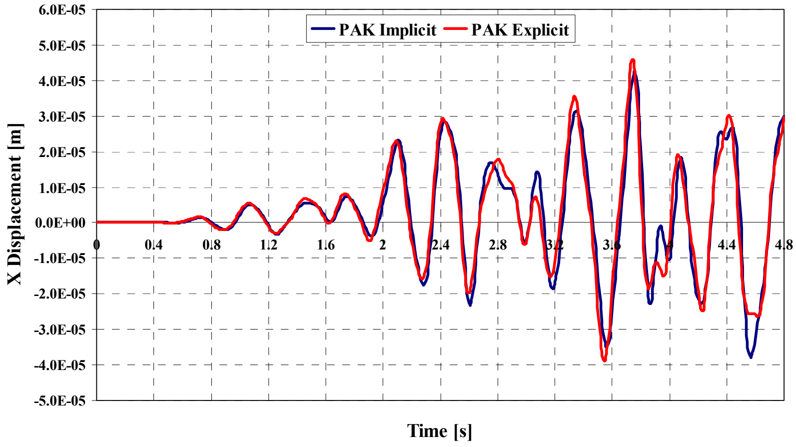

Figure 6. Node 576 displacement in X direction

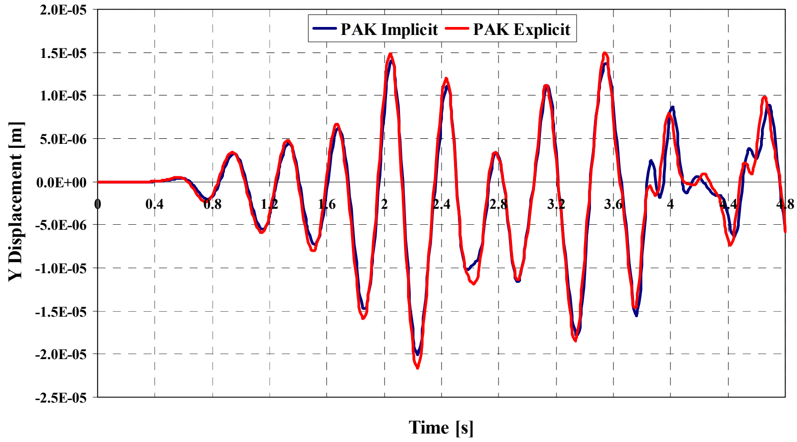

Figure 7. Node 576 displacement in Y direction

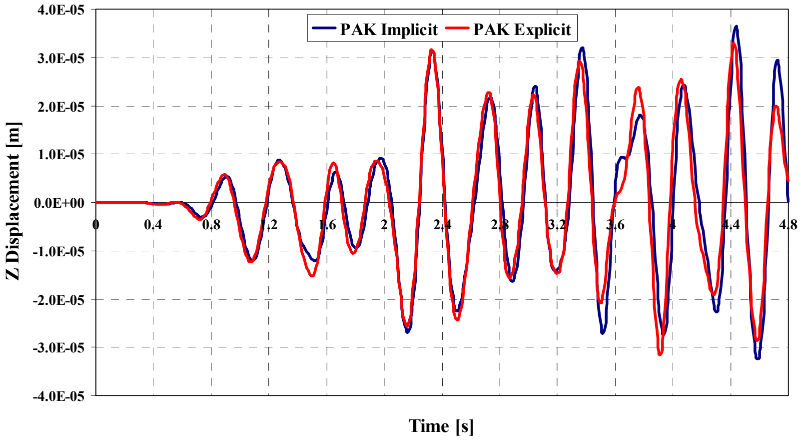

Figure 8. Node 576 displacement in Z direction

A very good matching of the results obtained by using both numerical integration methods for the specific dynamic problem is verified.

Figures 9. and 10. show comparative analysis of numerical results obtained for X displacement of nodes 1146 and 1716, respectively, by using implicit and explicit integration methods. Node 1146 is the middle node on the top of the middle building, whereas node 1716 is the middle node on the top of the third building (Figure 4).

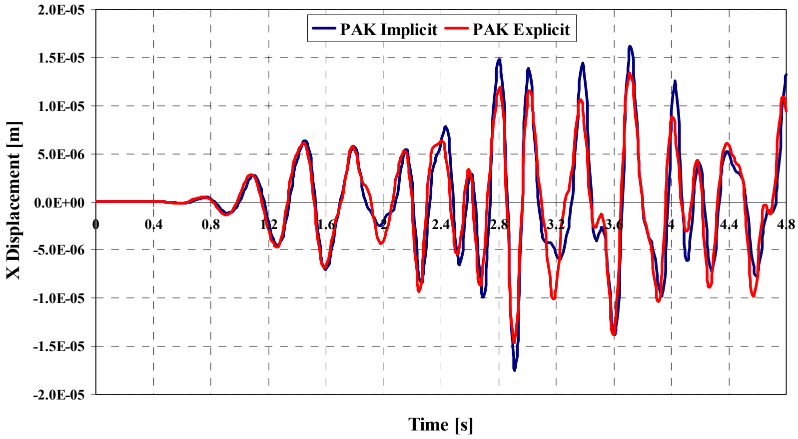

Figure 9. Node 1146 displacement in X direction

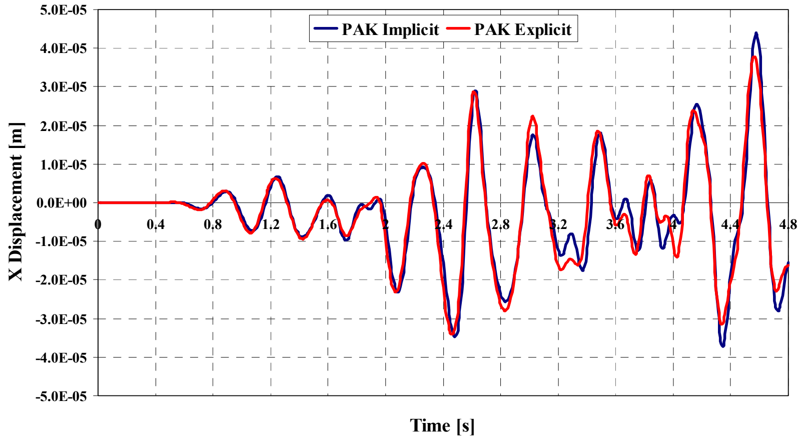

Figure 10. Node 1716 displacement in X direction

Such as from previous figure, a good matching of the results obtained by using both numerical integration methods for the specific dynamic problem is recognized.

The good matching of the results obtained by Newmark implicit method and central difference explicit method proves reliability of used numerical integration methods.

5. Conclusions

This aim of this paper was to compare two already known numerical integration methods used in solving of dynamic motion equations. The paper presents comparative analysis of Newmark implicit and central difference explicit method. According to the theory, algorithms implemented in software package PAK are developed. Dynamical analysis of 3D model in software package PAK, shown in section 4, comparative results obtained by using these two numerical methods are shown. The results and their good matching prove reliability of both methods and that they can be used for dynamic problem analysis. However, it should be noticed that calculation amount of time for the shown dynamic problem is considerably smaller 132 times when Newmark implicit method is used, because central difference explicit method requires a very small time step.

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