PROCEEDINGS

4th International Congress of Serbian Society of Mechanics

Editors:
Stevan Maksimović
Tomislav Igić
Nataša Trišović

June 4-7, 2013, Vrnjačka Banja, Serbia

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Prof. Dr. Stevan Maksimović
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Computer editing
Ivana Ilić, Marija Blažić, Marko Bojanić, Bojan Međo

Press
"Beotele Prom", Beograd

Circulation
200 copies

CIP

CIP - Каталогизација у публикацији
Народна библиотека Србије, Београд

531/534(082)

SERBIAN Society of Mechanics (Beograd). International Congress (4 ; 2013 ; Vrnjačka Banja)

Proceedings / 4th International Congress of Serbian Society of Mechanics, 4-7th June, 2013, Vrnjačka Banja ; editors Stevan Maksimović, Tomislav Igić and Nataša Trišović. - Belgrade : Serbian Society of Mechanics, 2013 (Beograd : Beotele Prom). - XXIV, 1010 str. : ilustr. ; 25 cm

Tiraž 200. - Str. III: Preface / S. [Stevan] Maksimović & T. [Tomislav] Igić. - Registar. - Bibliografija uz svaki rad.

ISBN 978-86-909973-5-0

a) Механика – Зборници

COBISS.SR-ID 198308876

Published by Serbian Society of Mechanics, Belgrade
http://www.ssm.org.rs/

PREFACE

These proceedings contains the papers presented at the Forth (29th Yu) International Congress of Serbian Society of Mechanics held in Vrnjačka Banja during the period 4th – 7th June, 2013.

Theoretical and Applied Mechanics is a subject of great importance in the developing of science and technology. The aim of the Congress is to provide a forum to exhibit the progress in this field during the past years and a place to further the interaction between of modern theoretical and applied mechanics, as well as modern engineering sciences.

The papers, contributed by authors from all around the globe, have been separated into 7 sections which cover the main areas of interest: `Plenary Lectures`, Section A, Section B, Section C, Section D and two Mini-symposia.

We would like to express our gratitude to all members of the Scientific Committee and also to the participants for their engagement in organizing of the Congress, including the preparation of manuscripts to be published in the Journal Theoretical and Applied Mechanics, Scientific Technical Review and Journal of Serbian Society for Computational Mechanics.

It gives us great pleasure to express our deep appreciation for the great long-standing support that Prof. Dr. Nikola Hajdin, President of the Serbian Academy of Sciences and Arts, has given to the promotion of all aspects of theoretical and applied mechanics in Serbia.

Last, the Congress organizing committee wishes to acknowledge the collaboration of the Ministry of Education, Science and Technological Development of the Republic of Serbia, Serbian Academy of Sciences and Arts, Municipality Vrnjačka Banja and many supporting members of the Serbian Society of Mechanics.

S. MAKSIMOVIĆ & T. IGIĆ
June, 2013

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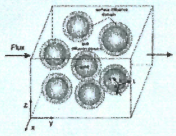
Table of Contents



Plenary lectures

Miloš Kojić
Numerical Modeling of Convective and Diffusive Mass Transport in Biological Media 3

П.С. Красильников, А.Г. Сараева
Периодические орбиты пуанкаре первого рода в плоской круговой ограниченной задаче трех тел с малым ускорением 19

Josif Vuković, Aleksandar Obradović
Constraint Reactions in Optimal Control of Mechanical Systems 25

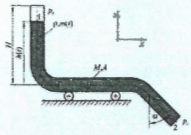
Katica (Stevanović) Hedrih
Linear and Nonlinear Dynamics of Hybrid System 43

Dragan Jovanović
Isodyne Stress Analysis of Stress State in Contact Regions 59

Philippe Vignal, Lisandro Dalcin, Nathan Collier, V. M. Calo
Petiga: Solution of Higher-order Partial Differential Equations 71

Mohsen Razzaghi
Orthogonal Functions and Hybrid Approximations for Variational Problems 81

Ardeshir Guran
Adaptive Materials and Structures: An Overview 93



Section A: General Mechanics

M. Ćurčin
Accelerometer Instrumented Physical Pendulum 105

M. Živanović
PD Control of Motion of a Scleronomic Mechanical System 111

N. Zorić, A. Simonović, Z. Mitrović, S. Stupar
Optimal Vibration Control of Smart Composite Beams using Self-Tuning Fuzzy Logic Controller 117

O. Jeremić, M. Milinović, A. Kari
Comparative Analyses of Linear and Nonlinear Serial and Redundant Links of Wire Rope Absorbers 123

P. Mandić, M. Lazarević, S. Stojanović, M. Ristanović
Real Time Fractional Order Control of Rotary Inverted Pendulum 129

O. Lazarević, V. Batinić
Analysis of Hydraulic Excavator Dynamic Behaviour 135

B. Sarić
One Common Solution to the Singularity and Perihelion Problems 141

D. Đurić
On Brachistochronic Motion of a Multibody System with Real Constraints 147

V. Vujičić
Four Dimensional Spaces with Geometric and Kinematic Constraints 153

R. Radulović
Shooting Method in Determining Global Minimum Time of Brachistochronic Motion 159

N. Vuković, Z. Miljković, M. Mitić, M. Petrović
Learning Motion Trajectories of Differential Drive Mobile Robot Using Gaussian Mixtures and Hidden Markov Model 165

S. Rusov, N. Mladenović, Z. Mitrović
Possibility of Contact Force Optimization and Aerodynamic Noise Reduction on Overhead Equipment 171

S. Šalinić, A. Nikolić
On the Free Vibration of a Multiple-Stepped Cantilever Beam 177

B. Jovanović, V. Jovanović
Maupertuis Principle and Isoenergetic Integrability 183

S. Mastilović
Dynamic Response of Brittle Systems in Confined Dimensions 187

V. Dragović, K. Kukić
From Kowalevski Top to Jurdjevic Elasticae 193

Lj. Veljović
About Kinematical Vector Rotators Defined for Rigid Body Dynamics with Coupled Rotations around Axes without Intersection 199

D. Perišić
Optimal Control under Incomplete Information and Kolmogorov Equation 205

J. Otto, L. Cvetićanin, M. Zuković
Application of the Theory of Meshchersky on a Practical Example 211

M. Živanović
Comparative Simulation Results of Sliding Mode and PD Control of a 3-DOF Manipulator 217

Section B: Fluid Mechanics

B. Zindović, Lj. Savić, R. Kapor, N. Mladenović
Comparison of Numerical and Scale Models of Stepped Spillway Flow 225

S. Ožvat, B. Mašić, G. Jeftenić, S. Kolaković, S. Vujović
Friction Factor Determination in Turbulent Pipe Flow Regime 231

D. Damljanović, B. Rašuo, A. Vitić, Đ. Vuković, J. Isaković
Measurement and Analysis of Flow Angularity in the Supersonic Test Section of the T-38 Blowdown Wind Tunnel in VTI 237

R. Petrović, M. Živković, N. Todić
Investigations of Flow Rate Ripple and Pressure Pulsation of Radial Piston Pump 243

D. Komarov, J. Svorcan, S. Stupar, A. Simonović, M. Stanojević Baltić
Numerical Investigation of S809 Airfoil Aerodynamic Characteristics 249

G. Ocokoljić, M. Samardžić, D. Marinkovski, J. Isaković, Z. Anastasijević
One-Component Transducer for Measurement of the Hinge Momoment 255

M. Samardžić, Z. Anastasijević, J. Isaković, D. Marinkovski, D. Ćurčić, B. Rašuo
Usage of Semoconductor Strain Gauges in Dynamic Experiments in the T-38 Wind Tunnel 261

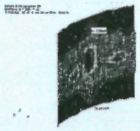
M. Kolarević, Lj. Savić, R. Kapor, N. Mladenović
Suprecritical Flow in Circular Closed-Conduit Bends 267

Đ. Čantrak, N. Janković
Reynolds Number Influence on the Statistical Characteristics of Turbulent Swirl Flow 275

M. Kozić, S. Ristić, S. Linić
Analysis of Pulverized Coal Granulation and Restitution Coefficients Impact on Coal Powder Distribution at Burners 279

D. Jerković, D. Regodić, Ž. Reljić, N. Hristov
The Prediction of Axial Aerodynamic Coefficient Reduction using Base Bleed 285

S. Mandić, S. Stojković, M. Milošević
Aerodynamic Configurations of the Modified Anti-Tank Missile Guided and Controled by Existing SCLOS System 291



Section C: Mechanics of Solid Bodies

P. Smiljanić, A. Sedmak, E. Džindo
Analysis of Composite Beams Materials and Adhesives 299

M. Bojanić, S. Boljanović, K. Maksimović
Buckling and Postbuckling Behaviour of Layered Composite Structures by Finite Elements 305

B. Deretić Stojanović, S. Stošić, S. Kostić
The Stiffness Matrix of the Fixed-End Composite Frame Element 311

J. Jarić, D. Kuzmanović, D. Šumarac
On Anisotropic Elasticity Damage Mechanics 317

D. B. Jovanović
Reconstruction of Strain Energy Surfaces at the Crack Tip Vicinity 323

Z. Đorđević, I. Atanasovska, M. Blagojević, D. Momčilović, M. Miletić
The Numerical Analysis of Strain and Stress State of Composite Shaft 329

S. Maksimović, I. Vasović, M. Maksimović, M. Djurić
Improved Computation Method to Fatigue and Fracture Mechanics Analysis of Aircraft Structures 335

N. Petrašinović, D. Petrašinović, S. Trivković, A. Simonović, S. Stupar
Fatigue Crack Growth in 2024-T3 Aluminium Alloy 341

M. Nefovska Danilović, M. Petronijević, M. Radišić
Transverse Vibration of Plates with Edge Beams using Spectral Element Method 347

B. N. Novaković
On the Post-Critical Behaviour of an Optimaly Shaped Elastic Column with Clamped-Simply Supported Ends Positioned on Elastic Foundation 353

O. Peković, S. Stupar, A. Simonović, Z. Posteljnik
Bending and Free Vibration Finite Element Analysis of Thin Composite Plates Based on Isogeometric Paradigm 359

O. Ognjanović, K. Maksimović, D. Stamenković, Z. Vasić
The Effects of Thermal Gradients on Stress Distributions 365

M. Mićunović, Lj. Kudrjavčeva, M. Topalović,
Thermomechanics of Soft Inelastics Bodies - An Application to Asphalt Behavior 371

S. Perković, M. Blažić, S. Boljanović, I. Vasović, V. Stefanović
Determination of Crack Growth Trajectory: Experimental and Numerical Comparisons 377

M. Hadžalić, S. Posavljak, N. Vukojević
Analysis of Significant Parameters on Dynamic Behaviour of the Deep Mine Hoist Installations 383

A. P. Seyranian, V. B. Glavardanov
On the Problem of Nicolai with Variable Cross-Section and Pre-Twisting Effect 389

M. Blažić, I. Ilić, Y. Assoul
Fatigue Life Analysis for Three-Dimensional Cracked Lugs 395

M. Topalović, M. Ivanović, M. Živković, A. Dišić
Comparison of FEM and SPH Methods Used for Analysis of Solid Bodies 401

B. Rakić, L. Ivanović, D. Josifović, A. Ilić
Stress-Strain Analysis at Hydro Accumulator Cylinder 407

S. Ćorić, S. Brčić
Investigation of the Frame Structure Stability in Elasto-Plastic Domain 413

M. Jovanović, A. Simonović, S. Stupar, N. Zorić, N. Lukić
Experimental Determination of Basic Parameters for Active Vibration Control System Development 419

S. Boljanović, S. Maksimović, A. Carpinteri
Crack Growth Analysis of Edge Notched Components under Cyclic Loading 425

I. Ilić, Lj. Ilić, S. Perković, M. Blažić
Strength Analysis of Composite Structural Elements for Unmanned Aerial Vehicle 431

T. Igić, D. Turnić
Limit Analysis, Optimum Conditions and Optimum Structural Design of Metal Shells 437

M. Marjanović, Đ. Vuksanović
Linear Analysis of Single Delamination in Laminated Composite Plate using Layerwise Plate Theory 443

Đ. Novković, N. Maričić, Z. Glavčić
CFD Simulations in Small Bulb Turbine with Adjustable Runner Blades 449

R. Mandić, R. Salatić, Z. Perović, M. Marinković
Experiences in the Numerical Modelling of Masonry Infilled Frames 455

S. Vasin, M. Ristić, M. Ognjanović
Gear Transmission Unit Vibration Related to Design Parameters and Resonances 461

A. Grbović, G. Kastratović, N. Vidanović , B. Rašuo
Review of Modern Numerical Methods for Stress Intensity Factor Determination 467

K. Maksimović , D. Stamenković, Lj. Milović
Influence of Surface Crack Shape on Crack Growth Rate in Plate 473

D. Ristić, D. Milosavljević
The Stress Analysis of the Spur Gears Tooth Root and its Computation Model 579

B. Škorić, M. Arsenović, M. Kutin, I. Vasović, M. Ristić, Z. Milutinović
Thermography and Numerical Simulations with Respects to Stress State and Fracture of Continuous Cast Specimens Made of Bronze Alloy 485

A. Dišić, M. Živković, V. Milovanović
Numerical Determination of Parameters of Johnson-Cook Material Model 491

J. Živković, G. Jovicić, S. Vulović, Z. Stepanović, M. Živković
The Numerical Assessment of the Structural Integrity of the Tibia-Implant using Failure Criteria 497

D. Rakić, M. Živković, V. Milovanović, N. Milivojević, D.Divac
Stress Integration of Matsuoka-Nakai Constitutive Model Using Incremental Plasticity Method 503

V. Milovanović, G. Jovičić, M. Živković, D. Rakić, A. Dišić
Analysis of Influence Choice Fatigue Failure Criteria to Assess the Integrity of Wagon Structure Parts 509

V. Dunić, N. Busarac, V. Slavković, N. Grujović, M. Živković, R. Slavković
Partitioned Thermo-Mechanical Coupling Procedure of FEM Components 517

A. Radaković, D. Milosavljević, G. Bogdanović, Lj. Veljović
Failure Analysis of a Composite Laminate Modeled Using the Higher Order Deformation Theory 523

M. Katinić, D. Kozak, N. Gubeljak, M. Rakin, B. Medjo, A. Sedmak
Numerical Determination of Creep Fracture Mechanics Parameter C^* For Single Edge Crack in a Plate under Tension 529

J. M. Đoković, R. R. Nikolić
Crack Deflection into an Interface between the Two Orthotropic Materials 535

G. Janevski, N. Nešić, P. Kozić, I. Pavlović
Transverse Vibration of a Damped Beam with One Step Change Subjected to Axial Force 541

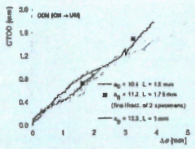
M. Vasko, A. Guran, L. Jakubovicova, P. Kopas
Effects of the Loading Rate on Contact Stresses of a Roller Bearing: A Computational Study 547

D. Karličić, R. Pavlović
Effect of Pasternak Foundation on Flexural Vibration and Buckling of Symmetric Cross-Ply Laminates 553

I. Pavlović, R. Pavlović, P. Kozić, G. Janevski, I. Ćirić
Stochastic Stability of a Viscoelastic Double-Beam System Under Wideband Noises 559

S. V. Đurković, V. Gobeljić, M. Ostojić, T. Nikolić Milkovski, T. Mihovilović, M. Grbić
Modification of Wing Structure- Extension of Wing Walk Compound 565

N. Anđelić, V. Milošević-Mitić, T. Maneski, A. Petrović
Stress Analysis of Thin-Walled Structural Elements of Turbine Blade Shape 571



Section D: Interdisciplinary and Multidisciplinary Problems

A. Hedrih, M. Lazarević, A. Mitrović-Jovanović
Fertilisation as a Biomechanical Oscillatory Phenomenon in Mamals 579

S. B. Stojanović, D. Lj. Debeljković, G. Simeunović, N. Dimitrijević
Further Results on Finite-Time Stability of Singular Time-Delay Systems: Delay-Dependent Conditions 585

V. Veljić, A. Debeljković, Đ. Koruga
Mechanical Properties Investigation of Commercial and Nanophotonics Soft Contact Lenses 591

V. Jazarević, B. Rašuo
Computation of Acoustic Sources for the Landing Gear during the Take-Off and Landing 597

S. Đ. Mesarović, J. M. Padbidri, B. Muhunthan
Micromechanics of Dilatancy, Critical State and Shear Bands in Granular Materials 603

V. Kuzmanović, Lj. Savić, N. Mladenović
Thermal Behaviour and Transversal Joint Distance Computation for Gravity RCC Dams 609

M. Petronijević, M. Radišić, M. Nefovska Danilović
Wave Propagation Due to a Moving Load 615

K. Velimirović, N. Velimirović
Flight Performance Determinaton of The Turboprop Aircraft 621

J. Vidaković, V. Kvrgić, G. Ferenc, Z. Dančuo, M. Lazarević
Kinematic and Dynamic Model of the Human Centrifuge 627

M. Milošević, V. Đurković, Z. Gajić
The Influence Terrain Slope on Impact Point of Rockets Burst Launching from MLRS 633

D. Momčilović, I. Atanasovska, Lj. Milović, Z. Đorđević
Assessment of the Fatigue Life Reduction of Machine Parts with Corrosion Pit 639

P. Elek, S. Jaramaz, D. Micković
Fragment Shape Distribution in Explosively Driven Fragmentation 645

R. Petrović, R. Slavković, N. Todić
Influence of Improved Stator Curve on the Characteristic of Vane Pump 651

I. B. Grozdanović, N. I. Burić, K. D. Todorović, N. T. Vasović
Weak and Strong Coupled Stochastic Oscillators with Delays in Coupling 657

S. Jovančić, D. Živanić, M. Milošević
Random Soil Parameters Effect to the Accuracy on Rocket Impact Points of Multiple Launcher Rocket Systems 663

S. Kostić, I. Franović, K. Todorović, N. Vasović
Nonlinear Dynamics of Spring-Block Earthquake Models 669

M. Mitić, Z. Miljković, N. Vuković, I. B. Lazarević
Visual Control of a Mobile Robot using Homography and Learning fom Demonstration Methodology 675

Z. Dančuo, V. Kvrgić, B. Rašuo, J. Vidaković
On Dynamics of a Spatial Disorientation Trainer for Pilot Training 681

K. Jovičić, R. Jovičić, M. Prokolab, B. Jegdić
Integrity Assessment for Tank with Cracks in Welded Joints 687

S. Mastilović
Molecular Dynamics Simulation of Nanoscale Taylor Test 693

D. Blagojević, M. Todić, O. Miletić, V. Golubović Bugarski
Definition and Investigation of Operation and Nonlinear Stiffness of Pulling and Buffer Devices for Load and Relaxation Process 699

Z. Perović, D. Šumarac
Elastoplastic Analysis of Trusses Subjected to Cyclic Loading 705

B. Međo, M. Rakin, N. Gubeljak, D. Kozak, I. Cvijović Alagić, A. Sedmak
Influence of Welded Joint Geometry on Fracture Behaviour – Micromechanical Assessment 711

Š. M. Bajmak
Influence Form Function Network Construction Pipeline in the Optimal Parameters in District Heating and Cooling 717

D. Jevtić, A. Savić
Mechanical Properties of Fiber Reinforced Concrete Made with Polypropylene Fibers 723

M. Blagojević, M. Živković
Development of Software Pak-M for Calculation of Magnetostatic Field 729

Z. Vasić, V. Stefanović, P. Dragičević
Practical Approach to Integrated Design of Aeronautical Composite Constructions Using Catia Software 735

N. Marković, D. Turnić, T. Igić
Elasto-Plastic Behaviour and Ultimate Load of Plate Girders Subjected to Patch Loading 741

Minisymposium 1: Computational Bioengineering

B. Ćirković, V.Isailović, Ž. Milošević, A. Sofla, M. Radišić, N. Filipović
Computer Simulation of Motion of Magnetic Particles in External Magnetic Field 751

D. Nikolić, S. Aleksandrić, M. Tomašević, M. Radović, V. Ranković, N. Filipović
Prediction of Coronary Plaque Position on Arterys with Myocardial Bridge 757

M. Živanović, G. Bijelić, A. Savić, N. Filipović
Numerical Simulation of Iontoforesis in the Drug Delivery System 763

M. Radović, M. Đoković, A. Peulić, N. Filipović
Application of Data Mining Techniques for Mammogram Classification 769

T. Đukić, S. Savić, N. Filipović
Computer Simulation of Motion of Solid Particles in Laminar Flow Using Strong Solid-Fluid Coupling Computational Scheme 775

L. Velicki, N. Čemerlić-Adjić, R. Jung, N. Tomić, O. Adjic, D. Nikolić, I. Saveljić, D. Milašinović, N. Filipović
Evaluation of Borderline Coronary Lesions Using Noninvasive Computed Fractional Flow Reserve 781

M. Milošević, M. Kojić, N. Kojić, M. Ferrari, A. Ziemys
Hierachical Model for Diffusion within Biological Media 787

R. Radaković, A. Peulić, S. Kovač, N. Filipović
Electromyography Detection of Muscle Response in Musculus Quadriceps Femoris of Elite Volleyball Players on Different Training Stimuli 793

Ž. Milošević, D. Nikolić, I. Saveljić, M. Radović, T. Exarchos, O. Parodi, N. Filipović
Three-Dimensional Computer Modeling of Plaque Formation and LDL Transport within Artery and through the Vessel Wall 797

S. Jeremić, Z. Marković, V. Stepanić
Investigation of Antioxidative Activity Mechanisms of Alisarine Molecule 803

N. Mijailović, S. Petrović, D. Nikolić, A. Peulić, N. Zdraković, B. Ristić, N. Filipović
Non-Invasively Assessment of Knee Cartilage Stress Distribution Using Motion Capture System and Finite Element Method 809

V. Potsika, V. Protopappas, M. Vavva, K. Raum, D. Rohrbach, D. Polyzos, D.I Fotiadis
Estimation of Wave Dispersion and Attenuation for the Assessment of Healing Bones 815

M. Blagojević, A. Nikolić, M. Živković, M. Živković, G. Stanković
Role of Oscillatory Shear Index in Predicting the Occurrence and Development of Plaque 821

A. Vukičević, G. Jovičić, N. Filipović
Finite Element Analysis of Generic Expandable Stent Deployment: A Worst Case Scenario Prediction 825

I. Saveljić, A. Janović, D. Nikolić, Z. Rakočević, M. Đurić, N. Filipović
Finite Element Analysis of the Facial Skeleton on Simulated Occlusal Loading 831

M. Obradović, N. Filipović
Modeling of Axonal Elongation by Stem Cell Using Finite Element Method 837

N. Todić, N. Filipović, R. Petrović
Modeling of Torque With Friction Effect for Water Hydraulic Axial Piston Pump/Motor 843

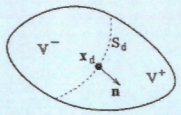
J. Đorović, Z. Marković, D. Milenković, D. Amić, S. Marković
Antioxidant Activity of Quercetin: Hat versus Set-Pt Mechanism 849

D. Petrović, M. Obradović, M. Radović , A. Jovanović, S. Jovanović, D. Baloš , M. Kojić, N. Filipović
Computer Modeling of Inhibition Process in Nanocoating of Surfaces wth Nanocontainers 855

D. Milenković, Z. Marković, J Dimitrić Marković, B. Lučić
DFT Investigation of the Reaction of Baicalein with Hydroxy Radical 861

F. D. Meo, E. H. Anouar, P. Podloucka, G. Fabre, I. Bayach, T. Desmier, N. Martín, V. Lemaur, K. Berka, J. Cornil, M. Otyepka, R. Lazzaroni, P. Trouillas
Understanding Antioxidant Properties of Natural Compounds at the Atomic Scale 867

D. Amić, V. Stepanić, B. Lučić, Z. Marković, J. M. Dimitrić Marković
Why Does O–H Bond Dissociation Enthalpy Effectively Represent Free Radical Scavenging Activity of Flavonoids? 873



Minisymposium 2: Nonlinear Dynamics

J. Simonović, M. Cajić, D. Karličić
The Forced Vibrations of Complex Circular Membrane System with Visco-Elastic Coupling 883

V. Marković, A. Stanojević, S. Maćešić, Ž. Čupić, V. Vukojević, Lj. Kolar-Anić
Dynamic States of Cortisol as a Function of Cholesterol Concentration in a Model of HPA Axis Dynamics 889

B. Stanković, Ž. Čupić, N. Pejić, Lj. Kolar-Anić
One Scenario for Transition from Supercritical to Subcritical Andronov-Hopf Bifurcation Point 895

I. Nuša Bubanja, S. Maćešić, J. Maksimović, M. Milenković, E. Greco, R. Cervellati, S. D. Furrow, Ž. Čupić, S. Anić, Lj. Kolar-Anić
Intermittences or Bursting Oscillations in Briggs-Rauscher Oscillating System 899

R. Radulović, A. Obradović, B. Jeremić
Brachistochronic Motion of a Nonholonomic Mechanical System with Limited Reaction of Constraints 903

Z. Rakarić, I. Kovačić
Bursting Oscillations in a Single Degree of Freedom Nonlinear Oscillator 909

A. Đurić, M. Filipović, W. Chen
Vizualiaztion of the Three Critical Spaces Related to the 6-DOF Machinery 915

D. Milosavljević, G. Bogdanović, Lj. Veljović, A. Radaković
Bulk Waves in Fibre Reinforced Materials 921

G. Bogdanović, D. Milosavljević, Lj. Veljović, A. Radaković
Wave Propagation in Orthotropic Materials 927

Lj. Veljović, D. Milosavljević, G. Bogdanović, A. Radaković
About Rigid Body Oscillations around Two Inclined Axes without Intersection 933

M. Ugrčić
Finite Element Modeling of Wing-Bird Strikes 939

N. Potkonjak, D. Minić, Lj. Kolar-Anić, Z. Nikolić, S. Anić
Nonlinear Phenomena of Electrochemical System Pertubated by Current Interrupt Technique 945

M. Cajić, D. Karličić, M. Lazarević
The State Space Model of a Single-Link Flexible Robot with a Fractional Order Viscoelastic Element in the Joint 949

Lj. Kevac, A. Đurić, M. Filipović
Relation between Cable-Suspended Parallel Robot and Classic Robotic Structure 955

V. Veljić, A. Debeljković, Đ.Koruga
Study of Mechanical Properties of Commercial and Nanophotonics Soft Contact Lenses by Optomagnetic Spectroscopy 961

J. Simonović
Stability Analysis of Stationary Regimes in Transverse Oscillations of Coupled Plate System 967

M. Marjanov
Unstable Orbits in the Solar System 973

M. Stamenković, M. Mikić
Testing of Singularity and Position of Non-Linear Dynamics Relative Equilibrium of Heavy Material Particle on Eccentrically Rotating Rough Circle Line, with Constant Angular Velocity 983

M. Vukčević
Dynamic of Different Structures of Galaxies - Necessary Conditions for Stable Nonlinear Structures 989

S. N. Blagojević, Ž. Čupić, S. M. Blagojević, Lj. Kolar Anić
Dynamical System Analysis for the Bray–Liebhafsky Reaction under CSTR Conditions 995

B. Adnađević, T. Đajić, J. Jovanović
The Dispersive Isothermal Kinetics of Nicotinamide Release from Poly (Acrylic-Co-Methacrylic Acid) Xerogel 999

N. Trišović
Wavelet Families - A Primer 1005

The Congress is organized by:

Serbian Society of Mechanics (SSM)

and is supported by:

Ministry of Education, Science and Technological Development of the Republic of Serbia

Ministry of Education, Science and Technological Development of the Republic of Serbia

ISBN 978-86-909973-5-0

STRESS INTEGRATION OF MATSUOKA-NAKAI CONSTITUTIVE MODEL USING INCREMENTAL PLASTICITY METHOD

Dragan Rakić[1], Miroslav Živković[1], Vladimir Milovanović[1], Nikola Milivojević[2], Dejan Divac[2]

[1] Faculty of Engineering,
The University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac
e-mail: drakic@kg.ac.rs
[2] Jaroslav Cerni Institute for the Development of Water Resources,
Pinosava, 11226 Belgrade, Serbia
e-mail: nikola.milivojevic@gmail.com

Abstract. This paper presents the integration of constitutive relations for Matsuoka-Nakai non-associative constitutive model using incremental plasticity method. The basic idea of this method is the calculation of plastic matrix thus, the calculation of the stress increment needs only total strain increment. Presented constitutive model consists of a yield surface similar to Mohr-Coulomb yield surface, but completely smooth, avoiding corners which can be problematic in numerical calculations. The yield surface divides stress 3D space into the pure elastic region and pure plastic region. In addition to the basic equations for stress integration, the paper presents the development of elastic-plastic constitutive matrix and algorithm for the implementation in the program for finite elements. Presented procedure is implemented in the PAK software. Verification is performed using examples from the literature and compared with the results obtained using Mohr-Coulomb constitutive model.

1. Introduction

Stress integration represents calculation of stress change during an incremental step, corresponding to strain increments in the step. It is in essence the incremental integration of inelastic constitutive relations to trace the history of material deformation. The stress integration is an important ingredient in the overall finite element inelastic analysis of structures. It is important that the integration algorithm accurately reproduces the material behavior since the mechanical response of the entire structure is directly dependent on this accuracy. The algorithm should be also computationally efficient because the stress integration is performed at all integration points. For general applications, this computational procedure should be robust, providing reliable results under all possible loading conditions. In this paper we present a formulation of the computational algorithm for the Matsuoka-Nakai (MN) constitutive model [1] using an incremental plasticity approach (IPM). The solutions were compared with the results obtained using Mohr-Coulomb (MC) model [2], as well as with the results obtained using concrete constitutive model.
In the next section we present formulation the MN constitutive model, followed by the derivation of the elastic-plastic constitutive matrix in general associated plasticity. Then, the general relations are implemented in MN constitutive model.

2. Matsuoka-Nakai constitutive model formulation

Matsuoka-Nakai constitutive model is based on experimental results of soil material research. Failure surface of this constitutive model is described using stress invariants in the following form:

$$f = I_3 + \frac{\cos^2 \phi}{9 - \sin^2 \phi} I_1 I_2 = 0 \tag{1}$$

where I_1, I_2 and I_3 are first, second and third stress invariants, respectively [3]. Parameter ϕ represents the angle of material internal friction. Failure surface of this constitutive model in deviatoric plane is presented in Figure 1.

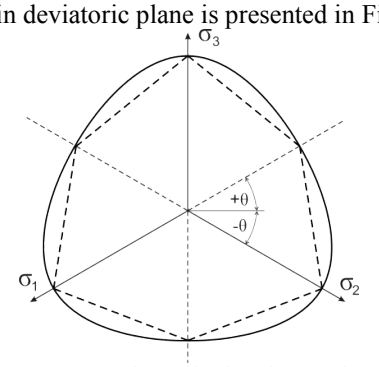


Figure 1 Mohr-Coulomb and Matsuoka-Nakai failure surface in deviatoric plane

As seen in Figure 1, for axisymmetric stress state, failure surface of Matsuoka-Nakai model matches the failure surface of Mohr-Coulomb model. However, unlike the failure surface of Mohr-Coulomb model, failure surface of Matsuoka-Nakai model is completely smooth, which is more suitable from the aspect of numerical solving.
In the case of non-associative yield condition, failure surface and the plastic potential surface of this model are different. In that case, plastic potential surface is presented by the following equation:

$$g = I_3 + \frac{\cos^2 \psi}{9 - \sin^2 \psi} I_1 I_2 = 0 \tag{2}$$

where ψ represents dilatation angle of material.
In the case of cohesive materials, failure surface (1) and plastic potential surface (2) are formulated using modified stress tensor [4] as:

$$\sigma_{ij}^{*} = \sigma_{ij} - \sigma_0 \delta_{ij} \tag{3}$$

where σ_{ij} represents effective stress tensor, δ_{ij} is Kronecker delta symbol, whereas σ_0 is defined as:

$$\sigma_0 = c \; ctg \; \phi \tag{4}$$

where c is material cohesion and ϕ is previously mentioned internal fraction angle.

3. Elastic-plastic constitutive matrix using IPM

Elastic-plastic constititutive models are described through elastic-plastic constitutive relations. In incremental plasticity theory, stress is directly proportional to strain up to reaching yield stress. After reaching yield stress, strain increment can be divided into elastic and plastic part [5]:

$$\{de\} = \{de^E\} + \{de^P\} \tag{5}$$

Only elastic part of strain causes the stress change thus the stress increment can be formulated as:

$$\{d\sigma\} = \left[C^E\right]\{de^E\} \tag{6}$$

where $\left[C^E\right]$ is elastic constitutive matrix. Substituting (5) in (6) the following is obtained:

$$\{d\sigma\} = \left[C^E\right]\left(\{de\} - \{de^P\}\right) \tag{7}$$

In the case of using elastic-plastic constitutive models, yield function is the stress state function, therefore the increment of its change can be formulated as:

$$df = \left\{\frac{\partial f}{\partial \sigma}\right\}^T \{d\sigma\} = 0 \tag{8}$$

In incremental plasticity theory it is necessary that the failure function is in every time step less or equal to zero (neutral loading condition).
Implicit stress integration implies the increment of plastic strain in the normal direction on the plastic potential surface, which can be formulated as:

$$\{de^P\} = d\lambda \left\{\frac{\partial g}{\partial \sigma}\right\} \tag{9}$$

where $d\lambda$ is positive scalar which is to be calculated and plastic potential function g is the stress state function. Substituting the plastic strain increment (9) in(7) and using (8), it is obtained:

$$df = \left\{\frac{\partial f}{\partial \sigma}\right\}^T \left(\left[C^E\right]\{de\} - d\lambda\left[C^E\right]\left\{\frac{\partial g}{\partial \sigma}\right\}\right) = 0 \tag{10}$$

Plastic parameter $d\lambda$ can be calculated from equation (10) as:

$$d\lambda = \frac{\left\{\frac{\partial f}{\partial \sigma}\right\}^T \left[C^E\right]\{de\}}{\left\{\frac{\partial f}{\partial \sigma}\right\}^T \left[C^E\right]\left\{\frac{\partial g}{\partial \sigma}\right\}} \tag{11}$$

Finally, using parameter $d\lambda$ from (11) stress increment $\{d\sigma\}$ is obtained using (7) and (9) in the function of total strain increment:

$$\{d\sigma\} = \left[C^{EP}\right]\{de\} \tag{12}$$

where term $\left[C^{EP}\right]$ represents elastic-plastic constitutive matrix.

4. Stress integration of Matsuoka-Nakai model

The derivative of the MN failure function (1) with respect to stresses can be calculated using the chain rule:

$$\left\{\frac{\partial f}{\partial \sigma}\right\}^T = \frac{\partial f}{\partial I_1}\left\{\frac{\partial I_1}{\partial \sigma}\right\}^T + \frac{\partial f}{\partial I_2}\left\{\frac{\partial I_2}{\partial \sigma}\right\}^T + \frac{\partial f}{\partial I_3}\left\{\frac{\partial I_3}{\partial \sigma}\right\}^T \tag{13}$$

The derivative of plastic potential function (2) can be calculated likewise:

$$\left\{\frac{\partial g}{\partial \sigma}\right\}^T = \frac{\partial g}{\partial I_1}\left\{\frac{\partial I_1}{\partial \sigma}\right\}^T + \frac{\partial g}{\partial I_2}\left\{\frac{\partial I_2}{\partial \sigma}\right\}^T + \frac{\partial g}{\partial I_3}\left\{\frac{\partial I_3}{\partial \sigma}\right\}^T \tag{14}$$

Certain derivatives of the failure function from (13) are:

$$\frac{\partial f}{\partial I_1} = \frac{\cos^2\phi}{9-\sin^2\phi} I_2, \quad \frac{\partial f}{\partial I_2} = \frac{\cos^2\phi}{9-\sin^2\phi} I_1, \quad \frac{\partial f}{\partial I_3} = 1 \tag{15}$$

whereas derivatives of plastic potential function from (14) are:

$$\frac{\partial g}{\partial I_1} = \frac{\cos^2\psi}{9-\sin^2\psi} I_2, \quad \frac{\partial g}{\partial I_2} = \frac{\cos^2\psi}{9-\sin^2\psi} I_1, \quad \frac{\partial g}{\partial I_3} = 1 \tag{16}$$

Using equations (13) to (16) elastic-plastic constitutive matrix can be calculated and then the stress increment (12) as well. According to this theory, algorithm for stress integration can be formed using incremental plasticity theory as given in Table 1.

Table 1 Stress integration algorithm using IPM

A. Known: $\left\{{}^{t+\Delta t}e\right\}$, $\left\{{}^{t}e\right\}$, $\left\{{}^{t}\sigma\right\}$, $\left\{{}^{t}e^{P}\right\}$

$\{d\sigma\} = \left[C^E\right]\{de^E\} = \left[C^E\right]\left(\left\{{}^{t+\Delta t}e\right\} - \left\{{}^{t}e\right\}\right)$, $\left\{{}^{t+\Delta t}\sigma\right\} = \left\{{}^{t}\sigma\right\} + \{d\sigma\}$

B. Check the solution *IF* $(F < 0)$ *elastic solution* (*GOTO* **E**)

IF $(F \geq 0)$ *elastic – plastic solution* $(CONTINUE)$

$\left\{\frac{\partial f}{\partial \sigma}\right\}$, $\left\{\frac{\partial g}{\partial \sigma}\right\}$ using (13) and (14) and $d\lambda$ using (11)

C. Local iterations:

$d\lambda$ correction

calculation $\left\{de^P\right\} = d\lambda\left\{\frac{\partial g}{\partial \sigma}\right\}$, $\left\{de^E\right\} = \{de\} - \left\{de^P\right\}$, $\{d\sigma\} = \left[C^E\right]\left\{de^E\right\}$

calculation $f\left(\sigma_{ij}\right)$ using (1)

D. Check: *IF* $(ABS(f) \geq toll)$ *return to* **C**

$\left\{{}^{t+\Delta t}e^P\right\} = \left\{{}^{t}e^P\right\} + \left\{de^P\right\}$

E. End: $\left\{{}^{t+\Delta t}\sigma\right\}, \left\{{}^{t+\Delta t}e^p\right\}$

5. Verification examples

Verification of developed algorithm of MN constitutive model is performed on the example of one-axis model compression. Model consists of single element loaded towards (Figure 2a). Model was loaded using prescribed displacement on one element face, using the function in Figure 2b. Boundary conditions of analyzed model are presented in Figure 2a.

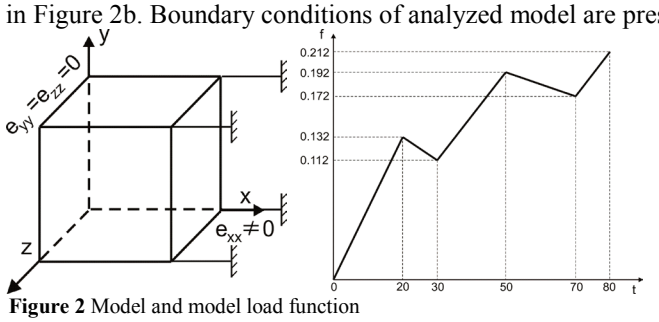


Figure 2 Model and model load function

The results are compared with the results obtained in analysis of the same example using Mohr-Coulomb constitutive model (Figure 3).

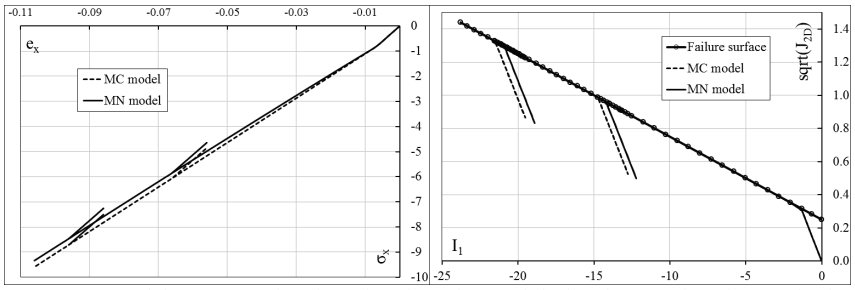


Figure 3 Axial stress as a function of strain and second deviatoric stress invariant as the function of first stress invariant

Analyzing results in Figure 3 can be noticed a relatively small deviation of results obtained using MN model compared with the results obtained using MC model. Also, the MN model is more stable in numerical terms, because the yield surface smooth, unlike the MC model.

Second example represents using of MN constitutive model in the analysis of reinforced concrete beam failure [6]. Model geometry, boundary conditions and load are presented in Figure 4. Eight-node brick finite elements for concrete beam modeling are used, whereas beam elements of appropriate dimensions are used for reinforcement. Model was loaded using prescribed displacement.

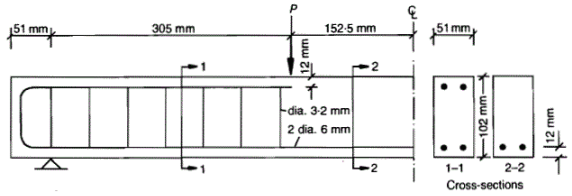


Figure 4 Load scheme, boundary condition and geometry of concrete beam

Comparison of results obtained using MN constitutive model with the results obtained using concrete constitutive model are presented in Figure 5.

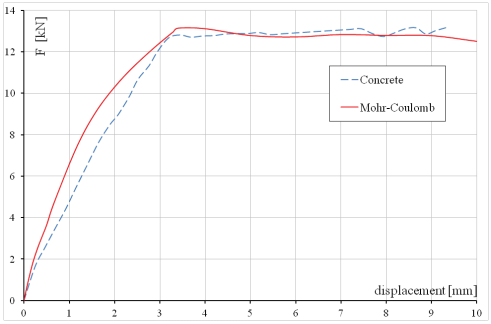


Figure 5 Force-displacement result

Analyzing results in the Figure 5, we can notice good matching of the solutions obtained using MN model with the solutions obtained using concrete constitutive model. It can be concluded that the MN model can be used to simulate the mechanical behavior of concrete structures. Mechanical characteristics of MN model for simulating the concrete behavior can be obtained by fitting the failure surfaces of these two models.

6. Conclusion

The results of the presented constitutive model are compared with the results obtained by Mohr-Coulomb constitutive model and as it can be seen, these two models provide very similar results. Results of MN model are also compared with the results obtained using concrete constitutive model. Analyzing the results a good matching is noticed. The advantage of the presented computational procedure is a general formulation, which can be applied to various yield functions, with yield function expressed in terms of stress invariants. Also, this procedure can be implemented in an explicit integration scheme (no yield condition check D in Table 1).

Acknowledgement. The authors gratefully acknowledge partial support by Ministry of Science and Technology of the Republic of Serbia, grants TR37013 and TR32036.

References

[1] Matsuoka H and Nakai T (1974), Stress-deformation and strength characteristics of soil under three different principal stresses, Proceedings of JSCE (Japan Society of Civil Engineers), no. 232, pp. 59-70.
[2] Balmer G (1952) A general analytical solution for Mohr's envelope, American Society for Testing and Materials, vol. 52, pp. 1260-1271.
[3] Kojic M and Bathe KJ (2004), Inelastic Analysis of Solids and Structures, 1 edition ed., Springer.
[4] Roddeman D (2012), TOCHNOG PROFESSIONAL User's manual.
[5] Sigma/W for finite element stress and deformation analysis - User's Guide, Geo-slope Office, Canada, 2002.
[6] Kotsovos MD and Pavlović MN (1995), Structural concrete – Finite element analysis for limit state design, Thomas Telford Services Ltd, London.