# ICSSM 2021 Proceedings

# 8th International Congress of the Serbian Society of Mechanics

# June 28-30, 2021 Kragujevac, Serbia



# ICSSM 2021 Proceedings

# 8th International Congress of the Serbian Society of Mechanics

June 28-30, 2021 Kragujevac, Serbia



# The 8th International Congress of the Serbian Society of Mechanics Kragujevac, Serbia, June 28-30, 2021

**Editors** Professor Miloš Kojić, academician Professor Nenad Filipović

**Technical Editor** Đorđe Dimitrijević

**Technical Assistant** Miloš Anić

**Proofreader** Neda Vidanović Miletić

**Publisher** Serbian Society of Mechanics, 9/1 Kneza Miloša, 11000 Belgrade, Serbia

**Press** "Grafo Ink", Kragujevac

**Impression** 100 copies

**Year** 2021

# Organizers

- Serbian Society of Mechanics (SSM)
- Faculty of Engineering, University of Kragujevac
- Faculty of Mechanical Engineering, University of Belgrade
- Faculty of Technical Science, University of Novi Sad
- Faculty of Mechanical Engineering, University of Niš
- Hellenic Society of Theoretical and Applied Mechanics
- Institute of Information Technology Kragujevac
- University of Kragujevac

with the support of

- Ministry of Education, Science and Technological Development
- Serbian Academy of Sciences and Arts

and

• Serbian Society of Computational Mechanics



# **Sponsor**

• TSI



# **Table of Contents**

Welcome Message	5
Organizing Committee	7
Keynote Speakers	
Program at a Glance	15
Technical Program	18
Camera Ready Papers	29
General Mechanics	30
Fluid Mechanics	92
Mechanics of Solid Bodies	137
Biomechanics	242
Control and Robotics	305
Interdisciplinary and Multidisciplinary Problems	354
Mini-Symposia I: 5th Serbian-Greek Symposium on Advanced Mechanics	425
Mini-Symposia II: Turbulence	508
Mini-Symposia III: Mathematical Biology and Biomechanics	578
Mini-Symposia IV: Nonlinear Dynamics	605
Plenary Lectures	662

# Welcome Message

Dear colleagues,

It is a great pleasure for us to welcome you all at *the* 8<sup>th</sup> International Congress of the Serbian Society of Mechanics in Kragujevac, Serbia Well-known for its culture, history and industrial heritage, Kragujevac was the first capital of modern Serbia and the place where the first constitution in the Balkans was proclaimed. Today, we are more than proud to say that Kragujevac is also becoming one of the scientific capitals in the region.

In this very difficult time of the COVID-19 pandemic, we decided to make this congress a hybrid event combining physical and online sessions, so that everyone interested can join us despite the obstacles we have all been facing for more than a year now.

8<sup>th</sup> International Congress of the Serbian Society of Mechanics aims to bring together leading academic scientists, researchers and research scholars to exchange and share experiences and research results on various aspects of *Theoretical and Applied Mechanics*. It will bring an interdisciplinary platform for researchers, practitioners and educators to present and discuss the most recent innovations, theories, algorithms, as well as practical challenges encountered and solutions adopted in the fields of Classical Mechanics, Solid and Fluid Mechanics, Computational Mechanics, Biomechanics, Applied Mathematics and Physics, Structural Mechanics and Engineering.

The Congress is organized by the Serbian Society of Mechanics (SSM) in partnership with: Faculty of Engineering, University of Kragujevac, Faculty of Mechanical Engineering, University of Belgrade, Faculty of Technical Science, University of Novi Sad, Faculty of Mechanical Engineering, University of Niš, Hellenic Society of Theoretical and Applied Mechanics, Institute of Information Technology Kragujevac, University of Kragujevac, with the support of the Serbian Ministry of Education, Science and Technological Development, Serbian Academy of Sciences and Arts and Serbian Society for Computational Mechanics.

Six distinguished plenary speakers will deliver lectures:

- 1. Prof. Georgios E. Stavroulakis Technical University of Crete, Greece
- 2. Prof. Themis Exarchos Ionian University, Corfu, Greece
- 3. Prof. Mihailo R. Jovanović University of Southern California, USA
- 4. Prof. Ricardo Ruiz Baier Monash University, School of Mathematics, Clayton, Australia
- 5. Dr Božidar Jovanović MISANU, Serbia
- 6. Dr Marko Janev MISANU, Serbia

The Congress encompasses six main topics: General Mechanics, Fluid Mechanics, Mechanics of Solid Bodies, Biomechanics, Control and Robotics, Interdisciplinary and Multidisciplinary Problems.

Also, there are four Mini-Symposia:

- M1: 5<sup>th</sup> Serbian-Greek Symposium on Advanced Mechanics Chairs: Prof. Georgios Stavroulakis, President of HSTAM, Greece; Prof. Nenad Filipović, President of SSM, Serbia
- M2: Turbulence Chair: Prof. Đorđe Čantrak, University of Belgrade, Serbia
- M3: Mathematical Biology and Biomechanics Chair: Dr. Anđelka Hedrih, MI SANU, Serbia
- M4: Nonlinear Dynamics Prof. Julijana Simonović, University of Niš, Serbia

Within the Congress, we are also very proud to organize the 5<sup>th</sup> Serbian-Greek Symposium on *Current and Future Trends in Mechanics*. The Symposium is organized by the Serbian Society of Mechanics (SSM) and the Hellenic Society of Theoretical and Applied Mechanics (HSTAM).

This year, 8<sup>th</sup> International Congress of the Serbian Society of Mechanics received more than 150 high-quality research papers. Each paper was reviewed and ranked by at least 2 professors and scientists in the program and the scientific review committee. As a result of the strict review process and evaluation, the committee selected 120 research papers.

We must also say that the conference would certainly not have been so successful without the efforts of many people who were actively engaged in organization of such a major nationally and internationally recognized academic event. We give our special gratitude to the members of the program and scientific review committee as well as to all chairs, organizers and committee members for their dedication and support.

On behalf of the Organizing Committee, we wish you all a pleasant stay in Kragujevac and a productive conference.

Chairs:

Prof. Nenad Filipović, *president of SSM, University of Kragujevac* Prof. Miloš Kojić, *Serbian Academy of Sciences and Arts* 

# **Organizing Committee**

# **Co-chairs:**

- Nenad Filipović, president of SSM (University of Kragujevac)
- Miloš Kojić, (Serbian Academy of Sciences and Arts)

# **Local Members:**

- Dalibor Nikolić, secretary of SSM (University of Kragujevac)
- Miroslav Živković (University of Kragujevac)
- Gordana Jovičić (University of Kragujevac)
- Dragan Rakić (University of Kragujevac)
- Vladimir Dunić (University of Kragujevac)
- Snežana Vulović (University of Kragujevac)

# **International Scientific Committee:**

Adhikari S. (UK), Atanacković T. (Serbia), Balean, D. (Turkey), Bauer S. (Russia), Borovac B. (Serbia), Bulatović R. (Montenegro), Carpinteri A. (Italy), Chernousko F. (Russia), Charalambakis N. (Greece), Challamel N. (France), Chen W. (Hohai, China), Chow Ch. (United States), Cvetičanin L. (Serbia), Djordjević V. (Serbia), Dolićanin Ć. (Serbia), Dragović V. (Serbia), Dunić V. (Serbia), Filipović N. (Serbia), Frischmuth K. (Germany), Gajić B. (Serbia), Glavardanov V. (Serbia), Golubović-Bugarski V. (R. Srpska, BiH), Grillo A. (Italy), Hedrih (Stevanović), K. (Serbia), Ibrahimović A. (France), Igić T. (Serbia), Jarić J. (Serbia), Jovanović B. (Serbia), Jovanović J. (Germany), Jovičić G. (Serbia), Katsikadelis J. (Greece), Kenjeres S. (Netherlands), Kienzler R. (Germany), Kojić M. (Serbia), Kounadis A. (Greece), Kovačić I. (Serbia), Kozak D. (Croatia), Kraseilnikov P. (Russia), Kuzmanović D. (Serbia), Lacarbonara W. (Italy), Lanusse P. (France), Lazarević M. (Serbia), Marsavina L. (Romania), Melchior P. (France), Malti R. (France), Makris N. (Greece), Maksimović S. (Serbia), Manolis G. (Greece), Manolis P. (Greece), Maretić R. (Serbia), Matthies H. (Germany), Milosavljević D. (Serbia), Mićunović M. (Serbia), Mitrović Z. (Serbia), Müller I. (Germany), Nedeljković M. (Serbia), Nigmatullin R. (Russia), Obradović A. (Serbia), Pavlović R. (Serbia), Polyzos D. (Greece), Prokopenya A. (Poland), Rakin M. (Serbia), Rakić D. (Serbia), Rega G. (Italy), Ruggeri T. (Italy), Saccomandi G. (Italy), Schrefler B. (Italy), Sedmak A. (Serbia), Seyranian A. (Russia), Simić S. (Serbia), Shitikova M. (Russia), Spanos P. (USA), Soltakhanov Sh. (Russia), Spasić D.T. (Serbia), Stevanović V. (Serbia), Sun H.G. (Hohai, China), Šumarac D. (Serbia), Tenreiro Machado J.A. (Portugal), Terze Z. (Croatia), Tikhonov A. (Russia), Tucker R. (UK), Vignjević R. (UK), Voronkova E. (Russia), Vrcelj Z. (Australia), Zarka J. (France), Zeković D. (Serbia), Živković M. (Serbia), Zorica D. (Serbia)

# **Keynote Speakers**

# "Analysis of a New Mixed Formulation for Hyperelasticity Using Kirchhoff Stress"

Monday 28 June 2021

09:15 - 09:45

**Prof. Ricardo Ruiz Baier,** Monash University, School of Mathematics, Clayton, Australia

## Abstract



Using the three-field formulation for nearly incompressible hyperelasticity introduced in [Chavan, Lamichhane, Wohlmuth, Comput. Methods Appl. Mech. Engrg. (2007), 196:4075-4086] we define a similar form valid for the fully incompressible case. We define a mixed finite element scheme and verify theoretical rates of convergence through computational tests. We also propose a new augmented Lagrangian preconditioner that improves convergence properties of iterative solvers. A few benchmark solutions are computed, and we test the formulation in models of cardiac biomechanics.

# "Coronary Atherosclerosis Assessment: A New Anatomical, Functional, Morphological and Biomechanical Approach"

Monday 28 June 2021

15:00 - 15:30

**Prof. Themis Exarchos,** Ionian University, Corfu, Greece

## Abstract



**Aims:** To investigate and compare two different endothelial shear stress (ESS) calculation techniques, compare lesion specific smartFFR and ESS values, as well as total vessel smartFFR and ESS values, and investigate the relationship between smartFFR and ESS to stress MBF (myocardial blood flow) and MFR (myocardial flow reserve).

**Methods:** A total of 10 coronary vessels of 6 patients with intermediate pre-test likelihood for coronary artery disease who have undergone both CTCA and PET-MPI with 15O-water or 13N-ammonia were included in the study. Seven (7) cases had normal stress MBF and MFR values and three (3) had abnormal ones. PET was considered abnormal when > 1 contiguous segments showed both stress MBF  $\leq 2.3 \text{ mL/g/min}$  and MFR  $\leq 2.5$  for 15O-water or <1.79 mL/g/min and  $\leq 2.0$  for 13N-ammonia, respectively. The ESS at the luminal surface of the artery was calculated as the product of viscosity and the gradient of blood velocity near the vessel wall. To calculate the smartFFR, we performed a transient simulation for each case. We used a pressure of 100 mmHg as a boundary condition at the inlet (i.e. mean human aortic pressure). At the outlet, a flow profile of 4 timesteps with a timestep duration of 0.25 sec was used. In each timestep, a volumetric flow rate of 1, 2, 3 and 4 ml/s are applied as outlet boundary conditions. The cut-off value for a pathological smartFFR is 0.83.

**Results:** There is a difference in total vessel calculated smartFFR results compared to the corresponding values of lesion specific smartFFR (0.88 vs 0.97, p=0.01). For ESS there is a negligible difference between lesion specific and total vessel values (2.22 vs 2.74, p = 0.9). There is a moderate negative correlation between both lesion specific (r = -0.543) and total vessel smartFFR and ESS (r = -0.915). ESS values were higher in vessels where vessel smartFFR was considered abnormal (1.97 vs 5.52, p = 0.01). Total vessel length smartFFR was lower in vessels with abnormal PET-MPI compared to the normal vessels (0.75 vs 0.93, p = 0.01). ESS is higher in vessels with pathological stress MBF and CFR (5.5 vs 2.0, p = 0.02)

**Conclusion:** The total vessel length smartFFR and lesion ESS appear to assess the functional significance of the vessel well, when compared to the PET-MPI measurements.

# "Auxetic and other Metamaterials in Dynamics"

Tuesday 29 June 2021

08:30 - 09:00

**Prof. Georgios E. Stavroulakis,** Technical University of Crete, Greece

## Abstract



Microstructures are in several cases reponsible for the novel mechanical behavior. Materials with negative Poisson ratio, the so-called auxetics, constitute an interesting class of mechanical metamaterials with interesting applications in statics as well as in dynamics. For example, they have enhanced damping properties. Auxetics can be produced with star-shaped microstructures and perforations. These metamaterials can serve as a first example of novel materials with nonclassical properties. Mechanical properties are mainly considered here, while in a more general setting multiphysics applications are also possible.

A short review of auxetic microstructures and optimal design of corresponding metamaterials will be presented, in view of the flexibility provided by modern additive manufacturing techniques. Applications in statics, wave propagation and dynamics will be discussed. The numerical experiments of our group will be presented together with results of current published research.

Using a classical auxetic microstructure, possible applications are studied here through numerical simulation. Finite element models are developed and dynamic analysis numerical tests are considered. First, an application related to auxetic microstructures, which are intended to be used in the core of structural panels, is presented. Then, some numerical simulation for wind turbines blades, where aluminium foam, polymeric foam and the proposed auxetic material have been considered as the core material, are taken into account. The numerical results demonstrate that the usage of auxetic microstructures results in improved dynamic response of the system in comparison to traditional, conventional materials. Further considerations of optimal design problems can be followed, either by optimizing the parameters of one given auxetic microstructure, or by using more general topology optimization tools.

Optimal design may have several goals and restrictions, depending on the application. Dynamic behaviour may be studied in frequency or time domain. In addition a complete behaviour towards the appearance of band gaps is possible. Finally, novel concepts to enhance auxeticity and control the resulting behaviour have been proposed, namely contact-activated and piezoelectric controlled microstructures. Last but not least auxetics pose challenging tasks on homogenization, especially for dynamical behaviour.

Optimal design principles, emerging research needs and technological questions will also be discussed.

# "Classical Neumann System on Stiefel Manifolds: Integrability, Geometric and Algebraic Aspects, and Linearization"

Tuesday 29 June 2021

15:00 - 15:30

**Dr Božidar Jovanović,** MISANU, Serbia

## Abstract



The Neumann system on a sphere is one of the basic classical examples of completely integrable systems. In this talk we give a review on the results concerning natural integrable generalizations of the Neumann systems to Stiefel manifolds [1,2,3,4]. Two Lax pairs for the systems are presented. A -matrix Lax representation enables us to prove non-commutative integrability of the Neumann systems, while a -matrix Lax representation implies a generalization of the Chasles theorem relating the trajectories of the systems and common linear spaces tangent to confocal quadrics. Also, by applying the second Lax representation we describe algebraic geometric properties of the systems. We show that generic complex invariant manifolds are open subsets of Prym varieties on which the flow is linear.

# "Noether's Theorem for Herglotz Type Variational Problems Involving Real and Complex Order Fractional Derivatives"

Wednesday 30 June 2021

08:30 - 09:00

# Dr Marko Janev,

MISANU, Serbia

# Abstract



In this work a variational principle of Herglotz type with a Lagrangian that depends on fractional derivatives of both real and complex orders is formulated, and the invariance of this principle under the action of a local group of symmetries is determined. By the Noether theorem the conservation law for the corresponding fractional Euler–Lagrange equation is obtained. A sequence of approximations of a fractional Euler–Lagrange equation by systems of integer order equations is used for the construction of a sequence of conservation laws. It is proved, that under certain assumptions, weakly converge to the one for the basic Herglotz variational principle. Namely, using the expansion of fractional derivative of a function into series, we analyze the approximations of already established Euler-Lagrange equation, infitezimal criteria and Noether's type theorem, in a weak sense within the dual pairing of corresponding topological spaces.

# "Color of turbulence: Stochastic Dynamical Modeling of Turbulent Flows"

Wednesday 30 June 2021

14:00 - 14:30

**Prof. Mihailo R. Jovanović,** University of Southern California, USA

## Abstract



This talk describes how to account for second-order statistics of turbulent wall-bounded shear flows using low-complexity stochastic dynamical models based on the linearized Navier-Stokes equations. The complexity is quantified by the number of degrees of freedom in the linearized evolution model that are directly influenced by stochastic excitation sources. For the case where only a subset of correlations is known, we develop a framework to complete unavailable second-order statistics in a way that is consistent with linearization around turbulent mean velocity. In general, white-in-time stochastic forcing is not sufficient to explain turbulent flow statistics. We develop models for colored-in-time forcing using a maximum entropy formulation together with a regularization that serves as a proxy for rank minimization. We show that colored-in-time excitation of the Navier-Stokes equations can also be interpreted as a low-rank modification to the generator of the linearized dynamics. Our method provides a data-driven refinement of models that originate from first principles and it captures complex dynamics of turbulent flows in a way that is tractable for analysis, optimization, and control design.

# **Program at a Glance**

Monday 28 June 2021		
08:45 - 09:15	Opening Ceremony Prof. Nenad Filipović, Presiden Prof. Miloš Kojić, full member Nikola Dašić, Major Prof. Ivica Radović, State Secretary Technological De Prof. Dobrica Milovanović, Dean o	- Welcome speech: nt of SSM, Conference Co-Chair of SASA, Conference Co-Chair r of Kragujevac City y, Ministry of Education, Science and velopment, Serbia f Faculty of Engineering, Kragujevac
09:15 - 09:45	Keynote Topic: Analysis of a New Mixed Formula Str Dr Ricardo Ruiz Baier, Monash Univer Aust	speaker: ation for Hyperelasticity Using Kirchhoff ress ersity, School of Mathematics, Clayton, ralia
09:45 - 11:00	Session M.1A Biomechanics (part I)	Session M.1B Mechanics of Solid Bodies (part I)
11:00 - 11:30	Coffee Break	
11:30 - 13:00	Session M.2A Interdisciplinary and Multidisciplinary Problems (part I)	Session M.2B Mechanics of Solid Bodies (part II)
13:00 - 14:00	Buffet Lunch	
14:00 - 15:00	Session M.3A Interdisciplinary and Multidisciplinary Problems (part II)	Session M.3B Mechanics of Solid Bodies (part III)
15:00 - 15:30	Keynote speaker: <b>Topic: Coronary Atherosclerosis Assessment: A New Anatomical, Functional,</b> <b>Morphological and Bio-mechanical Approach</b> <b>Dr Themis Exarchos,</b> <i>Ionian University, Corfu, Greece</i>	
15:30 - 16:30	Session M.4A General Mechanics (part I)	Session M.4B Mechanics of Solid Bodies (part IV)
16:30 - 17:00	Coffee	Break
17:00 - 18:30	Sessio General Mech	on M.5 banics (part II)

Tuesday 29 June 2021		
08:30 - 09:00	Keynote speaker: <b>Topic: Auxetic and other Metamaterials in Dynamics</b> <b>Dr Georgios E. Stavroulakis,</b> <i>Technical University of Crete, Greece</i>	
09:00 - 11:00	Session T.1A Mini-Symposia I: 5th Serbian-Greek Symposium on Advanced Mechanics (part I)	Session T.1B Biomechanics (part II)
11:00 - 11:30	Coffee Break	
11:30 - 13:00	Session T.2A Mini-Symposia I: 5th Serbian-Greek Symposium on Advanced Mechanics (part II)	Session T.2B Biomechanics (part III)
13:00 - 14:00	Buffet Lunch	
14:00 - 15:00	Session T.3A Mini-Symposia I: 5th Serbian-Greek Symposium on Advanced Mechanics (part III)	Session T.3B Control and Robotics
15:00 - 15:30	Keynote speaker: Topic: Classical Neumann System on Stiefel Manifolds: Integrability, Geometric and Algebraic Aspects, and Linearization Dr Božidar Jovanović, <i>MISANU</i> , Serbia	
15:30 - 16:00	Coffee	Break
16:00 - 18:00	SSM Annual Meeting	g and Assembly 2021
20:00 - 23:00	Gala Dinner - R	estoran Di Trevi

Wednesday 30 June 2021		
08:30 - 09:00	Keynote speaker: <b>Topic: Noether's Theorem for Herglotz Type Variational Problems Involving</b> <b>Real and Complex Order Fractional Derivatives</b> <b>Dr Marko Janev,</b> <i>MISANU, Serbia</i>	
09:00 - 10:15	Session W.1 Mini-Symposia III: Mathematical Biology and Biomechanics	
10:15 - 10:45	Coffee Break	
10:45 - 13:00	Session W.2A Fluid Mechanics	Session W.2B Mini-Symposia IV: Nonlinear Dynamics
13:00 - 14:00	Buffet	Lunch
14:00 - 14:30	Keynote speaker: <b>Topic: Color of turbulence: Stochastic Dynamical Modeling of Turbulent</b> <b>Flows</b> <b>Dr Mihailo R. Jovanović,</b> <i>University of Southern California, USA</i>	
14:30 - 17:30	Sessio Mini-Symposia	on W.3 n II: Turbulence
17:30 - 18:00	Closing C	Ceremony

# **Technical Program**

# Monday 28 June 2021

	Opening Ceremony - Welcome speech:
	Prof. Nenad Filipović, President of SSM, Conference Co-Chair
	Prof. Miloš Kojić, full member of SASA, Conference Co-Chair
08:45 - 09:15	Nikola Dašić, Major of Kragujevac City
	Prof. Ivica Radović, State Secretary, Ministry of Education, Science and
	Technological Development, Serbia
	Prof. Dobrica Milovanović, Dean of Faculty of Engineering, Kragujevac

09:15 - 09:45	Keynote speaker: Topic: Analysis of a New Mixed Formulation for Hyperelasticity Using Kirchhoff Stress
	<b>Prof. Ricardo Ruiz Baier</b> , Monash University, School of Mathematics, Clayton, Australia
	Chair: Hedrih A.

Session M.1A: 09:45-11:00

**Biomechanics (part I)** 

Chairs: Kojić M., Geroski V.

**M.1A.1** – Extension of our computational model for the left ventricle tissue to include hypertrophy – *Kojić M.* 

**M.1A.2** – Coupled Ohara-Rudy numerical model for heart electro-mechanics – Geroski V., Milošević M., Milićević B., Simić V., Filipović N., Kojić M.

**M.1A.3** – Electromyography detection of muscle response in musculus quadriceps femoris of elite volleyball players on different exercises – *Radaković R., Peulić A., Kovač S., Simojlović M., Filipović N.* 

Session M.1B: 09:45-11:00

Mechanics of Solid Bodies (part I)

Chairs: Mastilović S., Dunić V.

M.1B.1 – Remarks on discreteness of the nanoscale fragmentation mass distribution – Mastilović S.

M.1B.2 – Size-effect modeling of Weibull Jc cumulative distribution function based on a scaling approach – Mastilović S., Dorđević B., Sedmak A.

**M.1B.3 – Material parameters identification of concrete damage plasticity material model** – *Rakić D., Bodić A., Milivojević N., Dunić V., Živković M.* 

M.1B.4 – Using of gap element for contraction joints modeling in seismic analysis of concrete arch dams – Živković M., Jović N., Pešić M., Rakić D., Milivojević N.

**M.1B.5 – Finite element analysis of effects of multiple defects on welded joint integrity** – Aranđelović M., Sedmak S., Jovičić R., Sedmak A., Radaković Z.

Coffee Break

Session M.2A: 11:30-13:00

**Interdisciplinary and Multidisciplinary Problems (part I)** 

Chairs: Sedmak A., Nikolić D.

M.2A.1 – Noise induced dynamics of earthquake nucleation model – Kostić S., Vasović N.

M.2A.2 – Nonlinear landslide dynamics – Kostić S., Vasović N.

**M.2A.3 – Computational mechanics – welding joint as a case study** – *Jeremić L., Sedmak A., Sedmak S., Martić I.* 

**M.2A.4 – Experimental electrochemotherapy using novel design single needle device** – *Cvetković A., Cvetković D., Milasinović D., Jovičić N., Miailović N., Nikolić D., Mitrović S., Filipović N.* 

**M.2A.5** – Cavitation diagrams for merchant ships using four blade b series propellers – Veg M., Kalajdžić M.

**M.2A.6 – Microfluidic lab-on-chip system development for cell culture cultivation** – *Milivojević N., Živanović M., Nikolić D., Jovanović Ž., Šeklić D, Nikolić M., Filipović N.* 

#### Session M.2B: 11:30-13:00

Mechanics of Solid Bodies (part II)

Chairs: Rakić D., Obradović A.

**M.2B.1** – New pipe ring tensile specimen for pipeline material fracture assessment – *Trajković I., Rakin M., Milošević M., Sedmak A., Međo B.* 

M.2B.2 – Mass minimization of an AFG Timoshenko cantilever beam with a large body placed eccentrically at the beam end – *Obradović A., Mitrović Z., Zorić N.* 

**M.2B.3** – On concentrated surface loads and the flat punch contact problem in strain gradient elasticity – Zisis T., Gourgiotis P., Georgiadis H.

**M.2B.4** – Geometric optimization of shaft transition zone based on stress-strain analysis of nature inspired design – *Atanasovska I., Momčilović D.* 

M.2B.5 – A comparative analysis of fatigue behaviour between S355J2+N and Strenx 700 steel grade – Živković M., Milovanović B., Dišić A., Jovičić G., Topalović M.

M.2B.6 – Linear transient analysis of spatial curved Bernoulli – Euler beam using isogeometric approach – Jočković M., Nefovska Danilović M.

13:00 - 14:00

Buffet Lunch

Session M.3A: 14:00-15:00

**Interdisciplinary and Multidisciplinary Problems (part II)** 

Chairs: Milošević M., Šušteršič T.

**M.3A.1** – Analysis of atherosclerotic plaque in carotid arteries by using convolutional neural networks – Arsić B., Dorović S., Anić M., Gakovć B., Končar I., Filipović N.

**M.3A.2 – Structural condition assessment and rehabilitation of 'Karpos' system bridge** – *Milošević M., Živković S., Marković Branković J., Marković M.* 

**M.3A.3 – Epidemiological predictive modelling of COVID-19 spread** – Šušteršič T., Blagojević A., Cvetković D., Cvetković A., Lorencin I., Baressi Šegota S., Car Z., Filipović N.

M.3A.4 – In vitro and in silico testing of stent device – Nikolić D., Saveljić I., Filipović N.

Session M.3B: 14:00-15:00

Mechanics of Solid Bodies (part III)

Chairs: Dunić V., Milovanović V.

**M.3B.1** – Waves in composite layer reinforced with two families of inextensible fibres – *Milosavljević D., Radaković A., Čukanović D.* 

**M.3B.2 – Simulation of damage evolution in metal structures** – Živković J., Dunić V., Milovanović V., Živković M.

**M.3B.3** – Analysis of Hertz contact stresses for metallic material with carbides – Atanasovska I., Momčilović D.

M.3B.4 – Tuning a fractionally damped piezoelectric energy harvester with attached concentrated masses – *Paunović S.* 

	Keynote speaker: Topic: Coronary Atherosclerosis Assessment: A New Anatomical, Functional,
15:00 - 15:30	Morphological and Bio-mechanical Approach
	Prof. Themis Exarchos, Ionian University, Corfu, Greece
	Chair: Filipović N.

#### Session M.4A: 15:30-16:30

**General Mechanics (part I)** 

Chairs: Rakić D.

**M.4A.1 – Design and practical realization of a frame with stiffness at its ending point** – *Radomirović D., Kovačić I., Gatti G.* 

M.4A.2 – Problematizing the orbital mechanics' two first integrals – Nedić S.

**M.4A.3 – Simple algorithm for computing the stiffness matrix of composite cross-section** – *Lazović Radovanović M., Nikolić J., Radovanović J.* 

M.4A.4 – Topological interface states in acoustic metamaterials – Cajić M., Karličić D., Adhikari S.

Session M.4B: 15:30-16:30

**Mechanics of Solid Bodies (part IV)** 

Chairs: Mićunović M., Anđelić N.

**M.4B.1** – Low-cycle fatigue damage modeling with hysteretic energy loss – *Perović Z., Šumarac D.* **M.4B.2** – A model for the analysis of elasto-plastic buckling of compressed columns – *Milekić N., Bendić M., Ćorić S., Perović Z.* 

M.4B.3 – On diffuse instability of orthotropic viscoplastic plates – Kudrjavčeva Lj., Mićunović M.

**M.4B.4** – One view on the optimization of thin-walled cantilever channel-section and Z-section beams – Anđelić N., Milošević Mitić V., Petrović A., Đurđević Đ.

16:30 - 17:00	Coffee Break

Session M.5: 17:00-18:30

**General Mechanics (part II)** 

Chairs: Nikolić D.

M.5.1 – Determination of dynamic load factor for single element camshaft using various laws of follower motion – *Jovanović D., Milenković B.* 

M.5.2 – Behaviour, examination and stability of the constrained mechanical system described with nonlinear equations – *Vesović M., Radulović R.* 

M.5.3 – Kowalevski exponents for the Kirchhoff equations – Dragović V., Gajić B.

**M.5.4 – Added mass method application for dam-accumulation interaction analysis** – *Rakić D., Jelić L, Živković M., Milivojević N., Bojović M.* 

**M.5.5** – Comparative bending analysis of composite laminate and functionally graded plates based on the new shape function – *Milosavljević D., Radaković A., Čukanović D., Bogdanović G., Ivanović L.* 

**M.5.6 – Vehicle motion in the presence of nonholonomic constraints** – Žigić M., Grahovac N.

# **Tuesday 29 June 2021**

08:30 - 09:00	Keynote speaker: Topic: Auxetic and other Metamaterials in Dynamics
	<b>Prof. Georgios E. Stavroulakis,</b> <i>Technical University of Crete, Greece</i> Chair: Kojić M.

## Session T.1A: 09:00-11:00

Mini-Symposia I: 5th Serbian-Greek Symposium on Advanced Mechanics (part I)

Chairs: Filipović N., Kojić M.

T.1A.1 – Rolling without slipping a heavy homogeneous rigid ball on a torus – Hedrih Stevanović K.

**T.1A.2 – Exact augmented perpetual manifolds: a corollary for their uniquess** – *Georgiades F.* 

**T.1A.3 – Effect of chiral materials on presence of band gaps** – *Koutsianitis P., Tairidis G., Kougkolos A., Stavroulakis G.* 

**T.1A.4** – **Viscoelastic fillers in cellular shear wall panels: behavior under dynamic loads** – *Syrimi P., Papathanasiou S., Tsopelas P.* 

**T.1A.5** – Mathematical modelling of electro-elastic dislocations in piezoelectric materials and the J-integral – Agiasofitou E., Lazar M.

**T.1A.6** – Free vibration of axially functionally graded Timoshenko cantilever beam with a large rigid body attached at its free end – *Obradović A., Šalinić S., Tomović A.* 

**T.1A.7 – A study of rectangular plates under colinear point load with both approximate and exact solution** – *Tsiatas G., Pavlović M.* 

**T.1A.8 – Bullet penetration in auxetic star-shape armor plates** – *Ntintakis I., Stavroulakis G.* 

Session T.1B: 09:00-11:00

**Biomechanics (part II)** 

Chairs: Kojić M., Marković Z.

**T.1B.1** – Torsional oscillations of a tree trunk with branches through a biomechanical oscillatory model in the form of a complex cantilever – *Hedrih A., Hedrih Stevanović K.* 

**T.1B.2** – Inhibitory effect of coumarin derivate on vitamin K epoxide reductases (VKOR) responsible for anticoagulation effect – *Milanović Ž., Avdović E., Milenković D., Marković Z.* 

**T.1B.3** – Substituent effect on the binding mode and toxicity of selected 1,4-benzodiazepin-2-one – *Milanović Ž., Dimić D., Kesić A., Milenković D.* 

**T.1B.4 – Entresto (Sacubitril/Valsartan): molecular docking simulations with Neprilysin and Angiotensin II receptor** – Antonijević M., Marković Z., Filipović N., Dorović Jovanović J.

**T.1B.5** – Inhibitory effect of the 4-Hydroxy-(E)-N'-(1-(2,4-Dioxochroman-3-YL)Ethyl)-Benzohydrazide on the α-Glucosidase – Antonijević M., Simijonović D., Avdović E., Đorović Jovanović J.

**T.1B.6 – Left ventricle model generated from echocardiographic data** – *Milićević B., Milošević M., Geroski V., Simić V., Trifunović D., Filipović N., Kojić M.* 

**T.1B.7 – Fluid-electro-mechanical parametric model of the left ventricle** – *Simić V., Milošević M., Milićević B., Geroski V., Filipović N., Kojić M.* 

**T.1B.8 – Numerical analysis of blood flow through the cerebral aneurysm** – *Milovanović A., Saveljić I., Jovanikić O., Filipović N.* 

Coffee Break

#### Session T.2A: 11:30-13:00

Mini-Symposia I: 5th Serbian-Greek Symposium on Advanced Mechanics (part II)

Chairs: Kojić M., Vulović A.

**T.2A.1 – Analysis of freight wagons for transporting of bulk materials** – *Vulović S., Pavlović D., Živković M., Vujanac R., Topalović M.* 

**T.2A.2 – Physics-informed neural networks for elastic plate problems** – *Muradova A., Stavroulakis G.* 

**T.2A.3** – Crack analyses in a sandwich specimen under flexural loading – *Theotokoglou E., Tourlomousis I.* 

**T.2A.4** – Stochastic response of nonlinear energy sinks subjected to gaussian white noise base excitation – Karatzia D., Tsiatas G.

**T.2A.5 – Ageing of equilibrium contact angle in capillary flows** – Lazaridis K., Wu Y., Muniyal Krishna S., Yu C., Krivilyov M., Sekulić D., Mesarović S.

**T.2A.6** – Calculation of femoral cortical bone elasticity modulus from computed tomography scans – *Vulović A., Filipović N.* 

Session T.2B: 11:30-13:00

**Biomechanics (part III)** 

Chairs: Milošević M., Đorović S.

**T.2B.1 – Blood flow in arterial bifurcation calculated by turbulent finite element model** – *Nikolić A., Topalović M., Simić V., Blagojević M.* 

**T.2B.2** – Analytical model: calculation of fractional flow reserve – Starčević S., Savić S., Filipović N.

**T.2B.3 – Computational model for polymeric bioresorbable Poly-L-Lactic acid (PLLA) stents** – *Milošević M., Anić M., Geroski V., Nikolić D., Isailović V., Filipović N., Kojić M.* 

**T.2B.4** – Computational modeling of tumor cell circulation in capillary with platelets – Nikolić A., Simić V., Milošević M., Ziemys A., Yokoi K., Kojić M.

**T.2B.5** – Alterations of human articular cartilage due to osteoarthritis in terms of real and complexconjugate derivatives – Žigić M., Mikulić Gutman S., Grahovac N., Mitić I., Spasić D.

**T.2B.6 – Numerical analysis of knee joint at maximum power tennis serve** – Jovičić G., Vulović A., Dorović S., Vukićević A., Vulović R., Radaković R., Filipović N.

13:00 - 14:00

Buffet Lunch

Session T.3A: 14:00-15:00

Mini-Symposia I: 5th Serbian-Greek Symposium on Advanced Mechanics (part III)

Chairs: Car Z., Simić V.

**T.3A.1 – On inelasticity of damaged quasi rate independent orthotropic materials** – *Mićunović M., Kudrjavčeva Lj.* 

**T.3A.2** – Artificial intelligence approach to parameter learning in nonlinear control systems – *Nestorović T., Pal A., Oveisi A.* 

T.3A.3 – Determining inverse kinematics of a serial robotic manipulator through the use of genetic

**programming** – Car Z., Baressi Šegota S., Anđelić N., Lorencin I., Musulin J., Štifanić D., Mrzljak V. **T.3A.4** – **Biomechanics of left ventricle and in silico drug testing** – Filipović N., Milićević B., Milošević M., Simić V., Georski V., Kojić M.

Session T.3B: 14:00-15:00

## **Control and Robotics**

Chairs: Lazarević M., Obradović A.

**T.3B.1** – **H**∞ **proportional-integral control of a turbofan engine** – *Zorić N., Jazarević V., Obradović A.* 

**T.3B.2 – PI velocity control of a mechanical system** – Živanović M.

**T.3B.3 – Robust constrained state space ILC for 3DOF robot manipulator** – *Dubonjac A., Lazarević M.* 

**T.3B.4 – Adaptive iterative learning control of robotic system based on particle swarm optimization** – *Živković N., Lazarević M., Petrović M.* 

**T.3B.5 – Open closed-loop PD\mu/PD type ILC control of neuroarm robotic system** – *Cvetković B., Lazarević M., Mandić P., Šekara T., Lino P.* 

15:00 - 15:30	Keynote speaker: Topic: Classical Neumann System on Stiefel Manifolds: Integrability, Geometric and Algebraic Aspects, and Linearization Dr Božidar Jovanović, <i>MISANU, Serbia</i> Chair: Lazarević M.
---------------	--

15:30 - 16:00	Coffee Break
16:00 - 18:00	SSM Annual Meeting and Assembly 2021
20:00 - 23:00	Gala Dinner - Restoran Di Trevi

# Wednesday 30 June 2021

08:30 - 09:00	Keynote speaker: Topic: Noether's Theorem for Herglotz Type Variational Problems Involving Real and Complex Order Fractional Derivatives
	Dr Marko Janev, MISANU, Serbia
	Chair: Mađarević D.

## Session W.1: 09:00-10:15

Mini-Symposia III: Mathematical Biology and Biomechanics

Chairs: Hedrih A., Simonović J.

**W.1.1 – Volume optimization of the worm gear using biologically inspired algorithm** – *Milenković B., Jovanović D.* 

**W.1.2 – Generalized function of fractional type dissipation energy in double DNA helix chain** – *Hedrih Stevanović K.* 

W.1.3 – Bone regenerative potential driven by periodic excitation: deterministic and stochastic mathematical models – *Simonović J., Woolley T.* 

**W.1.4 – Modelling the influence of externally induced cholesterol pulses on hypothalamic-pituitaryadrenal axis perturbed with ethanol** – *Stanojević A., Anđelković M., Čupić Ž., Kolar Anić Lj.* 

W.1.5 – Rotation transformation matrix of the joint coordinate system with the application to the knee joint – Rosić N.

10:15 - 10:45	Coffee Break

Session W.2A: 10:45-13:00

**Fluid Mechanics** 

Chairs: Saveljić I., Topalović M.

**W.2A.1** – Influence of second order effects on pressure distribution in microtubes – *Guranov I., Milićev S., Stevanović N.* 

**W.2A.2** – Simulating fluid flow within coronary arteries using parallelized sparse lattice Boltzmann method – *Đukić T., Filipović N.* 

**W.2A.3** – **Saffman-Taylor instability - History and application** – *Cvetković I., Milićev S., Pihler Puzović D.* 

**W.2A.4 – Multi-component mixture of Euler fluids - continuum/kinetic closure** – *Pavić Čolić M., Mađarević D., Simić S.* 

**W.2A.5** – Multi-component mixture of Euler fluids - shock structure analysis – Mađarević D., Pavić Čolić M., Simić S.

**W.2A.6 – FSI analysis of hydrofoils using FEM and SPH methods** – *Topalović M., Nikolić A., Vulović S., Milovanović V.* 

**W.2A.7** – Natural convection due to lower plate temperature nonuniformity – Jovanović M., Milanović S., Spasić Ž.

W.2A.8 – Numerical simulation of laminar flow in periodical grooved channel – Lečić L., Stevanović

V.

**W.2A.9** – Numerical modeling the motion of otoconia particles in the semicircular canal under whole body vibration – Saveljić I., Mačužić Saveljić S., Nikolić D., Đukić T., Đorović S., Lukić J., Filipović N.

Session W.2B: 10:45-13:00

Mini-Symposia IV: Nonlinear Dynamics

Chairs: Hedrih Stevanović K., Simonović J.

W.2B.1 – Schools of asymptotic methods on nonlinear mechanics Krilov-Bogolyubov-Mitropolsky and scientific research in Serbia – *Hedrih Stevanović K*.

W.2B.2 – Tracking of periodic signal through the orthogonal lattice of nonlinear chains by synchronisation – Simonović J.

**W.2B.3** – Forced vibration of the nano-system composed from elastically connected nano-plate and nano-shell with influence of different parameters – *Stamenković Atanasov M., Pavlović I., Jovanović D.* 

**W.2B.4** – Exact augmented perpetual manifolds: corollary about linear and nonlinear perpetual mechanical systems – *Georgiades F.* 

**W.2B.5** – Nonlinear vibration of fractional viscoelastic beam on nonlinear layer – *Nešić N., Cajić M., Karličić D., Jović S., Simonović J.* 

**W.2B.6** – Analysis of evolution equations of a nonstationary axisymmetric body in a nonstationary central gravitational field – *Minglibayev M., Prokopenya A., Bizhanova S.* 

**W.2B.7** – Vibration suppression and energy harvesting application of an axially moving beam – *Karličić D., Cajić M., Adhikari S.* 

**W.2B.8** – Motion of two coupled bodies on a rough horizontal plane with variable coefficient of friction – *Prokopenya A*.

**W.2B.9** – Finite-time stability of neutral fractional-order time-varying delay systems with nonlinear parameter uncertainties and perturbations – Lazarević M., Radojević D., Maione G., Pisl S.

13:00 - 14:00	Buffet Lunch

14:00 - 14:30	Keynote speaker: Topic: Color of turbulence: Stochastic Dynamical Modeling of Turbulent Flows Prof. Mihailo R. Jovanović, University of Southern California, USA Chair: Čantrak Đ.
---------------	---

Session W.3: 14:30-17:30

Mini-Symposia II: Turbulence

Chairs: Čantrak Đ.

W.3.1 – High efficiency optimization of a multistage centrifugal pump based on combination of modified BA and CFD – *Wang W., Pei J., Osman M.* 

**W.3.2** – Challenges to accurate computation of propeller performances at low angular velocities – *Svorcan J.* 

**W.3.3 – Energy harvesting from the hot water transportation pipelines** – *Sonawat A., Kim J.* 

**W.3.4 – Experimentally determined response of impinging turbulent axisyimmetric air jet modified by low-amplitude sound modulation** – *Cvetinović D., Erić A., Škobalj P., Milutinović N., Tihon J., Nakabe K., Tatsumi K.* 

**W.3.5 – Convolutional neural networks for flow sensing in wall turbulence** – *Cuellar Martín A., Güemes A., Discetti S., Ianiro A.* 

W.3.6 – TSI Presentation – Sponsor presentation

W.3.7 – Model-based design of riblets for turbulent drag reduction – Jovanović M., Ran W., Zare A.

W.3.8 – Sizing valve regulating element – Savić Lj., Janković N., Čantrak D., Ilić D.

**W.3.9** – **Simulating transitional and turbulent flow around airfoils at medium angles-of-attack** – *Svorcan J., Hasan M., Tanović D., Popović L.* 

**W.3.10** – Five hole fast response probe for measurements of 3D velocity and pressure fields – *Bojović D., Čantrak D., Janković N., Nedeljković M.* 

**W.3.11 – Numerical investigation of reynolds number effects on rotor aerodynamic performances in hover** – *Svorcan J., Kovačević A., Ivanov T., Jovanović M.* 

**W.3.12 – Forced vortex in turbulent swirling flow** – *Čantrak Đ., Janković N., Lečić L.* 

17:30 - 18:00	Closing Ceremony
---------------	------------------

СІР - Каталогизација у публикацији - Народна библиотека Србије, Београд

531/534(082)

SRPSKO društvo za mehaniku. Međunarodni kongres (8 ; 2021 ; Крагујевац) Proceedings / The 8th International Congress of Serbian Society of Mechanics, Kragujevac, Serbia, June 28-30, 2021 ; [editors Miloš Kojić, Nenad Filipović]. - Belgrade : Serbian Society of Mechanics, 2021 (Kragujevac : Grafo Ink). - 661 str. : ilustr. ; 28 cm

Tiraž 100. - Bibliografija uz svaki rad.

ISBN 978-86-909973-8-1

а) Механика - Зборници

COBISS.SR-ID 41508105

ICSSM 2021 Proceedings



## FSI ANALYSIS OF HYDROFOILS USING FEM AND SPH METHODS

Marko D. Topalovic<sup>1</sup>, Aleksandar V. Nikolic<sup>1</sup>, Snezana D. Vulovic<sup>1</sup>, Vladimir P. Milovanovic<sup>2</sup>

 <sup>1</sup> Institute for Information Technologies, University of Kragujevac, Serbia e-mail: topalovic@kg.ac.rs, dziga@kg.ac.rs, vsneza@kg.ac.rs
<sup>2</sup> Faculty of of Engineering, University of Kragujevac, Serbia e-mail: vladicka@kg.ac.rs

#### Abstract

The purpose of this research was to investigate the advantages and disadvantages of coupled SPH-FEM analysis of a hydrofoil in regards to using FEM only approach. The both methods (SPH and FEM) are based on the continuum mechanics, however, SPH implementation uses Lagrangian material framework, while FEM uses an Eulerian formulation for the fluid analysis, and Lagrangian formulation for the solid analysis. In the case of combined SPH-FEM analysis, the hydrofoil is modeled with FEM, while the surrounding water is modeled with SPH particles. The contact between the two is done using nodes to surface algorithm, while if we use the FEM only, the weak coupling is done on the common interface nodes between Eulerian fluid domain and Lagrangian solid domain. The Lagrangian framework of the SPH means that we need to generate particles at one end, and to destroy them on the other, in order to generate a continuous fluid flow. The simplest way to do this is by using activation and deactivation planes, which is a solution implemented in the commercial LS-Dyna solver. Results show that velocity fields obtained by SPH-FEM coupling are similar to velocity fields obtained by FEM. FEM only solution has a clear advantage in regards to execution time, however, SPH-FEM coupling offers greater insight into phenomena such as cavitation, that justifies the extra computational cost.

Key words: SPH, FEM, Lagrangian formulation, Eulerian formulation, Fluid structure interaction, Cavitation

#### 1. Introduction

Smoothed Particle Hydrodynamics (SPH) is a mesh-free numerical method [1], based on the continuum mechanics, which means that analysed continuum is divided into subdomains called pseudo-particles [2] that describe a certain section of material, fluid or solid, and not the individual real particles. It was originally designed for solving astrophysical problems [3,4]. Its purpose was later extended to Computational Fluid Dynamics (CFD) problems, governed by the Navier–Stokes equations [5], and finally solid mechanics, by adding the strength of materials into SPH equilibrium equations [6]. SPH is also based on the Lagrangian material framework, meaning that the motion of the particular material section is observed [1]. On the other hand, when used for CFD analysis, FEM implements Eulerian spatial formulation which observes a fixed volume through which the fluid flows [7]. Eulerian formulation can predict pressure and fluid velocity with great accuracy [7], as well as forces exerted by fluid on the surrounding vessel, but, if we want to observe the movements and interactions of particles within the fluid flow [2], we would still need a Lagrangian formulation.

Although fluid structure interaction is commonly studied using coupled SPH-FEM with fluid modelled with SPH particles and thin walled structure (aircraft fuselage or ship hull) modelled with FEM shell elements [2], these simulations are not done within the continuous fluid flow, the issue that we will address in this paper.

In the section 2 of this paper a brief retrospective of SPH kernel and particle approximation is given, along with the calculation of stress in viscous fluid. In the section 3 we will discuss flow generation in LS-Dyna [8], and all of the problems we have encountered. Section 4 contains a practical implementation of the previously described methodology, demonstrated on hydrofoil analysis and results discussion. Conclusion summarizes outcomes shown in this paper and points out ways to improve the continuous flow analysis using SPH which will be the focus of our further research.

#### 2. SPH Approximations and fluid model

SPH uses kernel approximation and particle approximation [1] to model continuum mater for both solid and fluid, and the conservation laws of continuum mechanics [1] are expressed in the form of partial differential equations which are transformed into integral equations by interpolation function that gives "kernel estimate" of the field variables at point [1]. The exact value of the function  $f(\mathbf{x})$  in integral form is given with (1):

$$f(\mathbf{x}) = \int_{\Omega} f(\mathbf{x}') \delta(\mathbf{x} - \mathbf{x}') d\mathbf{x}', \qquad (1)$$

where  $f(\mathbf{x})$  is a function of position vector  $\mathbf{x}$  defined in the domain  $\Omega$  and  $\delta(\mathbf{x}-\mathbf{x}') = \begin{cases} 1 & \mathbf{x}=\mathbf{x}' \\ 0 & \mathbf{x}\neq\mathbf{x}' \end{cases}$  is the Dirac delta measure [1]. Replacing  $\delta(\mathbf{x}-\mathbf{x}')$  with bell-shaped kernel function  $W(|\mathbf{x}-\mathbf{x}'|,h)$  where *h* is the smoothing length (bell base radius), gives us a kernel approximation [1] of function  $f(\mathbf{x})$ :

$$\langle f(\mathbf{x}) \rangle = \int_{\Omega} f(\mathbf{x}') W(|\mathbf{x} - \mathbf{x}'|, h) d\mathbf{x}'.$$
 (2)

The integral form given in Eq. (2) is not practical for numerical implementation because analysed continuum is divided into a finite number of particles which carry individual mass and occupy individual space, so Eq. (2) is converted to discrete form of summation over all particles within the support domain [1]. The infinitesimal volume dx'is replaced by finite volume of the particle  $\Delta V_j = m_j / \rho_j$  where  $m_j$  and  $\rho_j$  are particle mass and particle density [1]. With the summation of all particles within support domain implemented in Eq. (2) we get particle approximation of a function  $f(\mathbf{x})$  for particle *i*:

$$\left\langle f\left(\mathbf{x}_{i}\right)\right\rangle \cong \sum_{j=1}^{NNP} f\left(\mathbf{x}_{j}\right) W\left(\left|\mathbf{x}_{i}-\mathbf{x}_{j}\right|,h\right) dV_{j} = \sum_{j=1}^{NNP} \frac{m_{j}}{\rho_{j}} f\left(\mathbf{x}_{j}\right) W\left(\left|\mathbf{x}_{i}-\mathbf{x}_{j}\right|,h\right),$$
(3)

where NNP is the number of nearest neighbouring particles.

Total stress tensor  $\sigma_{\alpha\beta}$  in the viscous fluid [2] consists of the hydrostatic pressure p and viscous stress  $^{visc}\tau_{\alpha\beta}$ :

$$\sigma_{\alpha\beta} = -p\delta_{\alpha\beta} + {}^{\text{visc}}\tau_{\alpha\beta} \,. \tag{4}$$

Viscous stress [1] can be calculated as:

$$^{visc}\tau_{\alpha\beta} = \mu \left(\partial_a v_\beta + \partial_a v_\beta - \frac{2}{3}\partial_\gamma v_\gamma \delta_{\alpha\beta}\right) = \mu \varepsilon_{\alpha\beta} , \qquad (5)$$

where  $\mu$  is the coefficient of dynamic viscosity and  $\varepsilon_{\alpha\beta}$  is the strain rate tensor [1]:

$$\varepsilon_{\alpha\beta}^{i} = \sum_{j=1}^{NNP} \frac{m_{j}}{\rho_{j}} v_{\beta}^{ji} \frac{\partial W^{ij}}{\partial x_{\alpha}^{i}} + \sum_{j=1}^{NNP} \frac{m_{j}}{\rho_{j}} v_{\alpha}^{ji} \frac{\partial W^{ij}}{\partial x_{\beta}^{i}} - \left(\frac{2}{3} \sum_{j=1}^{NNP} \frac{m_{j}}{\rho_{j}} \mathbf{v}^{ji} \cdot \nabla_{i} W^{ij}\right) \delta_{\alpha\beta} \,. \tag{6}$$

#### 3. SPH fluid flow in LS-DYNA

In the commercial LS-Dyna solver [8], continuous fluid flow in SPH is not the main focus of research and development, and there are no examples, or published papers using this feature. However, in the manual [8] there is a description of keywords BOUNDARY\_SPH\_FLOW and CONTROL\_SPH (BOXID) that are used for activation and deactivation of SPH particles [8], which can be used to create the continuous fluid flow shown in Fig. 1.



Fig. 1. SPH flow in LS-DYNA

This flow generation is similar to the activation and deactivation planes used by Jonsson et. al. [9], and it also has some issues that we will now point out. First and foremost, the activation plane does not create new particles, it only activates them, so all of the particles that are going to flow through model, must be created before the analysis is started. Having most particles deactivated during the analysis saves computational time, but still deactivated particles make the model cumbersome. The same goes for the deactivation box, this boundary only flags SPH particles as inactive, they are no longer part of any calculation, but they are still there as a part of the model. BOUNDARY\_SPH\_FLOW keyword requires the prescription of node velocities for all of the particles centers i.e. inlet velocity. On the outlet side, we can define box which could prescribe node velocity of the exiting particles, but that velocity would need to be the same, i.e. we cannot define a parabolic velocity profile, unless we create a large number of boxes. Furthermore, if the model is too short, as we will see in the case of hydrofoil, the particles do not have enough space to even out their velocities. Without the velocity prescription box, the outlet is a free surface, which could make the model unrealistic. Another solution is to add new SPH part with sparse fixed particles to act like a sieve and to slow exiting particles down.

Since this is a 2D problem, we also constrained translation of all particles in the direction normal to the observed plane. We used mirror SPH planes as boundary conditions [8], but since these planes provide frictionless contact with ghost particles, we had to constrain two layers of particles on top and bottom in the flow direction to get more realistic behavior.

#### 4. Results of SPH Analysis of hydrofoils and discussion

We used two hydrofoil profiles to test the continuous fluid flow performance of SPH solver built in the commercial LS-DYNA program [8]. The first hydrofoil is a part of a high-speed passenger ship and its 1267 mm long with maximum thickness of 184 mm. Velocity of the ship i.e. inlet velocity of the model is 300 mm/s. SPH-FEM model has 8383 SPH particles, a number that include 5280 initially deactivated particles, and 595 FEM elements. FEM model used for comparison has 11974 elements and it is analyzed in PAKF solver [7]. The total analysis time is 2 s, with a time step of 0.01 s. Velocity field after 0.15 s shown in Fig. 2.



Fig. 2. Ship hydrofoil velocity field a) SPH-FEM in LS-DYNA b) PAKF

The second hydrofoil is a part of a surfboard, and its 55.2 mm long with maximum thickness of 9.9 mm. SPH-FEM model has 11599 SPH particles, a number that includes 6000 initially deactivated particles, and 110 FEM elements. FEM model analyzed in PAKF [7] used for comparison has 7070 elements. The total analysis time is 1 s, with a time step of 0.01 s, and inlet velocity of 300 mm/s. In the Fig. 3. we can see the velocity field for the surfboard hydrofoil at 0.2 s.



Fig. 3. Surf board hydrofoil velocity field a) SPH-FEM in LS-DYNA b) PAKF

As our analysis shows, slow moving hydrofoil with velocity of 300 mm/s cannot break the bonds between SPH particles in the boundary layer, so they slide over the contact surface. If the prescribed velocity is increased to 400 mm/s, the gap between SPH particles appear on top of the hydrofoil, which corresponds to the formation of the cavitation bubble nucleus [10], Fig. 4.



Fig. 4. Cavitation as gaps between SPH particles

As it can be seen from Fig. 4. FEM-SPH coupling can be used to forecast location of formation of cavitation bubbles, but the full extent of the cavitation effect on the wetted surface cannot be predicted using these methods. In reality, decrease of velocity and increase of pressure of surrounding fluid causes the rapid collapse of cavitation bubbles which is followed by the great noise and occurrence of micro jets which can damage hydrofoil surface. In FEM-SPH coupled analysis, filling the gaps between SPH particles is not so sudden, and it does not create spikes in pressure on hydrofoil surface.

#### 5. Conclusions

SPH method coupled with FEM, is often used in maritime engineering to predict fluidstructure interaction between vessels and surrounding water. However, these calculations are done with static water, and vessels that impact water at high speeds. In this paper, we analysed SPH-FEM coupling for FSI analysis of a continuous fluid flow. The obtained results are similar to those obtained by Eulerian FEM analysis, but the prescription of boundaries is much more difficult in LS-DYNA with coupled SPH-FEM solvers. Execution time is also significantly longer for SPH-FEM analysis. On the other hand, SPH coupled with FEM can be used as a prediction tool for the location where the cavitation will occur, but not to model the damage it causes. SPH has a great potential to be used in the maritime industry for analysis of impact of floating bodies on vessels, but to realize its full potential, boundary conditions for generation and destruction of SPH particles need to be implemented, in order to simulate continuous fluid flow.

#### Acknowledgements

This research is supported by the Ministry of Education, Science and Technological Development, Republic of Serbia, Grant TR32036 and 451-03-9/2021-14/200378.

#### References

- [1] Liu G., Liu M., Smoothed Particle Hydrodynamics, World Scientific Publishing, 2003.
- [2] Vignjević R., Campbell J., Brief review of development of the smooth particle hydrodynamics (SPH) method, IConSSM 2011 - The 3rd International Congress of Serbian Society of Mechanics, Vlasina lake, Serbia, 5-8 July 2011, 24-43.
- [3] Gingold R. A., Monaghan J. J., *Smoothed particle hydrodynamics: theory and application to non-spherical stars*, Monthly Notices Royal Astronomical Society, Vol. 181, 375-389, 1997.
- [4] Lucy L. B., *A numerical approach to the testing of fusion process*, Astronomical Journal, Vol. 88, 1013-1024, 1997.
- [5] Monaghan, J J., Pongracic H., *Artificial viscosity for particle methods*, Applied Numerical Mathematics Vol. 1, 187-194, 1985.
- [6] Libersky L. D., Petschek A. G., Carney T. C., Hipp J. R., Allahdadi F. A., High Strain Lagrangian Hydrodynamics: A Three-Dimensional SPH Code for Dynamic Material Response, Journal of Computational Physics, Vol. 109. 67-75, 1993.
- [7] Kojic M., Filipovic N., Stojanovic B., Kojic N., Computer Modeling in Bioengineering: Theoretical Background, Examples and Software, John Wiley & Sons, 2009.
- [8] Hallquist J. O., *LS-DYNA Theory Manual*. Livermore Software Technology Corporation, 2006.
- [9] Jonsson P., Jonsén P., Andreasson P., Hellström J. G. I., Lundström T. S., Smoothed Particle Hydrodynamics Modeling of Hydraulic Jumps, Proceedings of Particle-Based Methods II— Fundamentals and Applications, Barcelona, 26-28 October 2011. 490-501.
- [10] Roohi E., Zahiri A. P., Passandideh-Fard M., Numerical simulation of cavitation around a two-dimensional hydrofoil using VOF method and LES turbulence model, Applied Mathematical Modelling, Vol 37, 6469–6488, 2013.