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Overview of the Sections

Volume 13 (2013) of PAMM "Proceedings in Applied Mathematics and Mechanics" assembles the contributions to the 84$^{\text{th}}$ Annual Meeting of the Gesellschaft für Angewandte Mathematik und Mechanik, held 18 – 22 March 2013 at the University of Novi Sad, Serbia.
The contributions are grouped according to the minisymposia and sessions of the conference.

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Stress Integration of the Hoek-Brown Material Model Using Incremental Plasticity Theory

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This paper presents the formulation of the stress integration procedure for the Hoek-Brown (HB) material model with non-associative yielding condition by using the incremental plasticity method. Main idea of this method is to reach the solution by calculating the plastic matrix according to the method of incremental plasticity (used for elastic constitutive matrix corrections), and with the use of the total strain increment. Computational procedure is implemented within the PAK program package. Results of this procedure were compared with the solutions obtained by other program that contains this material model.

1 Introduction

Stress integration represents calculation of stress change in an incremental step, corresponding to the strain increments. The integration algorithm has to reproduce accurately material behaviour since the mechanical response of the entire structure is directly dependent on this accuracy. The algorithm should be also computationally efficient because the stress integration is performed at all integration points. In this paper we present a formulation of the computational algorithm of the Hoek-Brown (HB) material model [1] using incremental plasticity method (IPM) [2]. The solutions were compared with the results obtained using other software packages and applied to some practical problems.

2 Stress integration of the Hoek-Brown model

Elastic-plastic constitutive models are described using elastic-plastic constitutive relations. In the incremental plasticity theory, stress is directly proportional to strain up to reaching yield stress. After reaching yield stress, strain increment can be divided into elastic and plastic part [3]

$$\{de\} = \left\{de^E\right\} + \left\{de^P\right\} \tag{1}$$

Only elastic part of strain causes the stress change. Implicit stress integration implies the increment of plastic strain in the normal direction on the plastic potential surface, which can be formulated as

$$\left\{de^P\right\} = d\lambda \left\{\frac{\partial g}{\partial \sigma}\right\} \tag{2}$$

where $\partial\lambda$ represents positive scalar which is to be calculated. In the incremental plasticity theory plastic parameter $d\lambda$ is calculated as

$$d\lambda = \frac{\left\{\frac{\partial f}{\partial \sigma}\right\}^T \left[C^E\right]\{de\}}{\left\{\frac{\partial f}{\partial \sigma}\right\}^T \left[C^E\right]\left\{\frac{\partial g}{\partial \sigma}\right\}} \tag{3}$$

Using parameter $d\lambda$ from (3), increment of plastic strain (2) can be obtained so we can calculate the stress increment at the end of the step. Yield function of the HB model is given in the form [3]

$$f = \frac{I_1}{3} m_b \sigma_c^{\left(\frac{1}{a}-1\right)} - s\sigma_c^{\frac{1}{a}} + 2^{\frac{1}{a}}\left(\sqrt{J_{2D}}\cos\theta\right)^{\frac{1}{a}} + m_b\sqrt{J_{2D}}\sigma_c^{\left(\frac{1}{a}-1\right)}\left(\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\right) \tag{4}$$

whereas plastic potential function has a similar form. Terms I_1 and $\sqrt{J_{2D}}$ in (4) represent first stress invariant and second deviatoric stress invariant, while term θ represents Lode angle (function of stress invariants). Algorithm for stress integration can be formed using incremental plasticity theory as given in Table 1.

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Table 1: Stress integration algorithm using IPM.

A. Known $\left\{^{t+\Delta t}e\right\}$, $\left\{^{t}e\right\}$, $\left\{^{t}\sigma\right\}$, $\left\{^{t}e^{P}\right\}$

B. Check the solution IF (f<0) elastic solution (GOTO **E**)

IF ($f \leq 0$) elastic-plastic solution (CONTINUE)

$\left\{\frac{\partial f}{\partial \sigma}\right\}^{T}, \left\{\frac{\partial g}{\partial \sigma}\right\}^{T}$

C. Local iterations: dλ correction, calculation of $\left\{de^{P}\right\}$ and $f\left(\sigma_{ij}\right)$

D. Check: IF($ABS\left(f\right) \geq toll$) RETURN to **C**

E. End $\left\{^{t+\Delta t}e^{P}\right\}$, $\left\{^{t+\Delta t}\sigma\right\}$, $\left\{^{t+\Delta t}e\right\}$

3 Verification Example

Verification of the algorithm from Table 1 is performed through the example of slope stability analysis. Geometry of the analysed model and material data are shown in Figure 1.

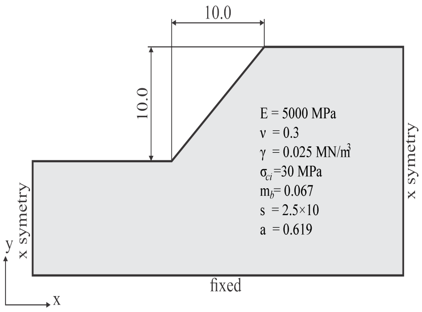

Fig. 1: Geometry of the model and material data.

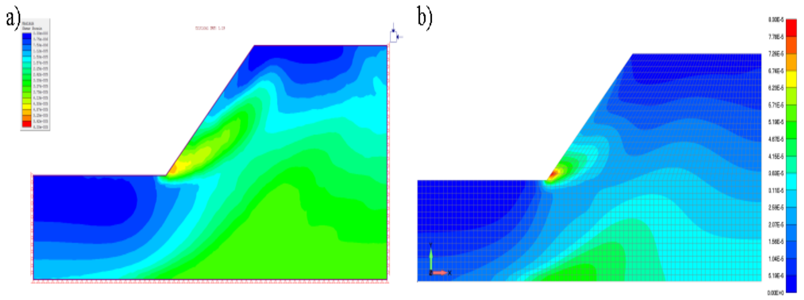

Fig. 2: Shear stress distribution a) Phase, b) PAK.

The result of the analysed problem using algorithm implemented in the program PAK [4] is compared with the solution obtained using program Phase2 [5]. Shear strength reduction method was used for determination of safety factor of the slope. Shear stress distribution obtained using programs Phase2 and PAK is shown in Figure 2 and obtained safety factors are 1.19 (Phase2) and 1.20 (PAK). Similar shear stress distribution whose exceeding leads to losing the stability of the slope can be seen. Safety factors obtained using these two programs are also very similar.

4 Conclusion

The advantage of presented computational procedure is its general formulation, which can be applied to various yield functions and be expressed in terms of stress invariants. The results of the presented HB material model implemented in the program PAK are compared with the results obtained using program Phase2. It can be seen that these two algorithms provide very similar results. The presented algorithm is applied in the analysis of a real construction in the second example. The stability of the tunnel during its excavation was analysed. To simulate the gradual excavation and lining of the tunnel, option of element death and birth was used. It was shown that the developed algorithm is robust and applicable in solving real geotechnical problems.

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