# MATHEMATICAL MEETING OF SERBIA AND MONTENEGRO 2019

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# Welcome Address

The idea of organizing "Mathematical meeting of Serbia and Montenegro" came from group of mathematicians of younger generation from Podgorica and Belgrade. After the breakup of the state union of Serbia and Montenegro, the mathematical collaboration between the two successor states has diminished to a point of near-vanishing, reduced only to few individual contacts. Moreover, Serbia and Montenegro are rapidly drifting apart in almost all respects, especially in the domain of culture and science. Needless to say, cultural and scientific isolation is often harmful, and indeed, the present rift between Serbia and Montenegro presents a great hindrance for both countries.

The response to the "Mathematical meeting of Serbia and Montenegro" was surprisingly good - over 60 mathematicians participated, not only from Serbia and Montenegro, but also from other countries in Europe and the world. The success reflected the pressing need of mathematical communities in the region to strengthen and deepen their collaboration and promises to be a good foundation for establishing "Mathematical meeting of Serbia and Montenegro" as one of the central annual event of the region's mathematical calendar.

The organization of the "Mathematical meeting of Serbia and Montenegro" was supported by

Faculty of Natural Sciences and Mathematics in Podgorica
Mathematical Institute of the Serbian Academy of Sciences and Arts
Faculty of Mathematics in Belgrade
Matica srpska - Association of Members in Montenegro
Mathematical Forum of Montenegro

# Abstracts

## SOME APPLICATIONS OF MATRIX DOMAINS OF TRIANGLES

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Sometimes, introduced sequence spaces are nothing more than "hidden" matrix domains of triangles in classical sequence spaces. Here, we will consider some recent results related to the mentioned "situation." In such way, applying the matrix domains of triangles some results can be improved and furthermore extended.

**Keywords**: sequence spaces, matrix transformations, matrix domains, bounded linear operators

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## ON SINGULAR MATRIX EQUATIONS AND CONNECTIONS WITH SINGULAR PDES

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This talk concerns theory of singular operator and matrix equations such as $AX = C$, $XB = C$, $AXB = C$, $X - AXB = C$ and $AX - XB = C$ and their implementation in discretization techniques for singular partial differential equations. Special attention will be dedicated to signular Sylvester equation and its application in reaction-diffusion pdes.

**Keywords**: operator equations, singular pdes, Sylvester equation

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# FRÉCHET DERIVATIVE AND APPLICATIONS

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We connect the Fréchet derivative with formulas related to the analytic functional calculus. Results are published in (Djordjević, D.S. Bull. Malays. Math. Sci. Soc. (2019). https://doi.org/10.1007/s40840-019-00736-6)

**Keywords**: Fréchet derivateive, Banach algebra, analytic functional calculus

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# AN APPROXIMATION OF SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS VIA TAYLOR SERIES

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An analytic approximate method for a class of stochastic differential equations with coefficients that do not necessarily satisfy the Lipschitz and linear growth conditions will be presented. Equations from the observed class have coefficients which satisfy polynomial condition and have unique solutions with bounded moments. Approximate equations are defined on partitions of the time interval, and their coefficients are Taylor approximations of the coefficients of the initial equation.

**Keywords**: Distributed stochastic differential equations, Taylor approximation, polynomial condition, $L^p$ convergence, almost sure convergence

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# EFFECTS OF PERTURBATIONS ON THE APPLICATIONS OF REFLECTED BACKWARD STOCHASTIC DIFFERENTIAL EQUATIONS

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Perturbed stochastic differential equations, in general, are the topic of permanent interest of many authors, both theoretically and in applications. Stochastic models of complex phenomena under perturbations in analytical mechanics, control theory and population dynamics, for example, can be sometimes compared and approximated by appropriate unperturbed models of a simpler structure. In this way, the problems can be translated on more simple and familiar cases which are easier to solve and investigate. Problems of perturbed backward stochastic differential equations (BSDEs) are very interesting because of their applications in economy and finance. Even in simpler cases, problems of additive and linear perturbation of BSDEs and BSDEs of Volterra type, estimates are completely different than the ones

for the forward type of stochastic equations. The most interesting problem in this field deals with a large class of reflected backward stochastic differential equations whose generators arbitrarily depend on a small parameter. The solutions of these equations, named the perturbed equations, are compared in the $L^p$-sense, $p \in ]1, 2[$, with the solutions of the appropriate equations of the equal type, independent of a small parameter and named the unperturbed equations. Conditions under which the solution of the unperturbed equation is $L^p$-stable are given. It is shown that for an arbitrary $\eta > 0$ there exists an interval $[t(\eta), T] \subset [0, T]$ on which the $L^p$-difference between the solutions of both the perturbed and unperturbed equations is less than $\eta$

**Keywords**: perturbations, reflected backward, $L^p$ estimate

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## PREDICTING THE RESULT OF A FOOTBALL MATCH - ONE STEP FURTHER

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In this paper we give an approach in predicting values of an INAR time series with Skellam marginals and identification of its Poisson components. We also give an application of this model on the prediction of the final score of a football match.

**Keywords**: INAR, thinning, Poisson marginals, Skellam marginals

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## GEOMETRY OF CR SUBMANIFOLDS

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Let $(\overline{M}, \overline{g}, J)$ be an almost complex Hermitian manifold. Following the Erlangen program of Klein: geometry is the investigation of properties which remain invariant under the action of a group, it is a natural question to investigate its submanifolds which have a special behavior with respect to the almost complex structure $J$. For that reason, the first classes of submanifolds investigated were the almost complex submanifolds (in which case $J$ maps the tangent space into the tangent space) and totally real submanifolds (in which case $J$ maps the tangent space in the normal space).

A natural generalization of the above classes of submanifolds are the so-called $CR$ submanifolds. A submanifold $M$ of $\overline{M}$ is named a $CR$ submanifold if there exist distributions $\mathcal{D}$ and $\mathcal{D}^\perp$ such that $\mathcal{D} \oplus \mathcal{D}^\perp = TM$, $J\mathcal{D} = \mathcal{D}$, $J\mathcal{D}^\perp \subset T^\perp M$. Note that totally real submanifolds ($\mathcal{D} = \{0\}$) and almost complex submanifolds ($\mathcal{D} = TM$) are trivial examples of $CR$ submanifolds. Moreover, real hypersurfaces of almost Hermitian manifolds are typical examples of CR submanifolds of maximal CR dimension.

Furthermore, the odd-dimensional analogue of CR submanifolds in Kählerian manifolds is the concept of contact CR submanifolds in Sasakian manifolds. Namely, a submanifold $M$ in the Sasakian manifold $(\widetilde{M}, \varphi, \xi, \eta, \tilde{g})$ carrying a $\varphi$-invariant distribution $\mathcal{D}$, such that the orthogonal complement of $\mathcal{D}$ in $TM$ is $\varphi$-anti-invariant, is called a contact CR submanifold.

One of the aims of submanifold geometry is to understand geometric invariants of submanifolds and to classify submanifolds according to given geometric data, for example, studying the relations and the interplay between intrinsic invariants, which only depend on the submanifold as a manifold itself, and extrinsic invariants, which depend on the immersion. As in Riemannian geometry the structure of a submanifold is encoded in the second fundamental form, we explore totally geodesic submanifolds and submanifolds close to them, we characterize several important classes of submanifolds considering certain conditions on the submanifold structure (represented by the second fundamental form) and structure naturally induced from the almost complex structure of the ambient space and we study Chen's inequalities.

On this occasion we present our research on four and five-dimensional contact CR submanifolds in five and seven-dimensional unit sphere and we show several examples of these submanifolds of product and warped product type. Moreover, we recall some of our results on CR submanifolds of complex space forms and nearly Kähler six-dimensional unit sphere.

**Keywords**: Kähler manifold, nearly Kähler manifold, Sasakian manifold, $CR$ submanifold, contact $CR$ submanifold

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## SYMMETRIC AND QUASISYMMETRIC FUNCTIONS IN COMBINATORICS

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Symmetric functions are limits of symmetric polynomials as number of variables go to infinity. They are important objects in algebraic combinatorics and representation theory. Quasisymmetric functions are relatively new generalization with a prominent application in algebraic combinatorics. Symmetric and quasisymmetric functions are universal objects in the category of combinatorial Hopf algebras, which explains their ubiquity in enumerative combinatorics. We present some recent developments in the application of symmetric and quasisymmetric functions as enumerative invariants in algebraic combinatorics. We show how these enumerative invariants appear as integer points enumerators associated to a class of convex polytopes called generalized permutohedra.

**Keywords**: counting, (quasi)symmetric function, combinatorial Hopf algebra, generalized permutohedron

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## SOLVING SOME NEW CLASSES OF DIFFERENCE EQUATIONS AND THEIR SYSTEMS IN A CLOSED FORM

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In this talk we will describe the achievement of solutions of some new significant difference equations and their systems of higher order in a closed form. There are some equations the solutions of which in closed form have not appeared in the literature hitherto.

**Keywords**: difference equations, solvable difference equations, systems of difference equations

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## FACET COLOURING OF NESTOHEDRA

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A proper colouring of a polytope is a surjective function from the set of facets to a set of $m$ colours such that every two facets associated with the same colour are separated, i.e. have no vertex in common. The chromatic number of a polytope is the minimal $m$ such that there exists a proper colouring of its facets in $m$ colours. This talk presents the chromatic numbers of associahedra and some others interesting members of the family of nestohedra.

**Keywords**: hromatic number, facet colouring, associahedron, cyclohedron

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## MINIMISERS AND KELLOGG'S THEOREM

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We extend the celebrated theorem of Kellogg for conformal mappings to the minimizers of Dirichlet energy. Namely we prove that a diffeomorphic minimiser of Dirichlet energy of Sobolev mappings between double connected domains having $C^{1,\alpha}$ boundary is $C^{1,\alpha}$ up to the boundary. It is crucial that, every diffeomorphic minimizer of Dirichlet energy has a very special Hopf differential and this fact is used to prove that every diffeomorphic minimizer of Dirichlet energy can be locally lifted to a certain minimal surface near an arbitrary point inside and at the boundary.

**Keywords**: minimizers, Kellogg's theorem, Dirichlet energy

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## MATRIX AND POLYNOMIAL RINGS OF McCOY TYPE

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Through this article $R$ denotes an associative ring with unity, $\sigma$ denotes an endomorphism of $R$ and $R[x,\sigma]$ denotes skew polynomial ring. As it is known, $R$ is a right (left) McCoy ring if the equation $f(x)g(x)=0$ holds, where $f(x), g(x) \in R[x]\backslash\{0\}$ implies that there exists a nonzero $c \in R$ such that $f(x)c=0$ ($cf(x)=0$). We say that $R$ is a weak right (left) McCoy ring whenever the polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x]\backslash\{0\}$ for which the equation $f(x)g(x)=0$ holds, implies that there exists a nonzero $s \in R\backslash\{0\}$ such that $a_i s \in nil(R)$ ($sa_i \in nil(R)$), for all $0 \leq i \leq n$, where $nil(R)$ is the set of all nilpotent elements of $R$. For a ring $R$ consider a following set of triangular matrices

$$T_n(R)=\left\{\left[\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ 0 & 0 & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_{nn} \end{array}\right] \mid a_{ij} \in R\right\}.$$

We also consider the following set of triangular matrices over ring $R$

$$T(R,n)=\left\{\left[\begin{array}{ccccc} a_0 & a_1 & a_2 & \dots & a_{n-1} \\ 0 & a_0 & a_1 & \dots & a_{n-2} \\ 0 & 0 & a_0 & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & a_0 \end{array}\right] \mid a_{ij} \in R\right\}.$$

We investigate possibility of extending McCoy property from ring to matrix extension. We also show how McCoy property can be transferred under ring isomorphism. Armendariz rings will be also considered.

**Keywords**: McCoy rings, Armendariz rings, matrix rings, skew polynomial rings

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## $wt$-DISTANCE AND ĆIRIĆ'S MAPS

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In this talk we generalize the fixed point theorem of Ćirić in the terms of $wt$-distance on complete $b$-metric space.

**Keywords**: $wt$-distance, $b$-metric, quasi-contraction, fixed point

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# q-REGULAR VARIATION AND ASYMPTOTIC ANALYSIS OF EMDEN-FOWLER TYPE SECOND ORDER q-DIFFERENCE EQUATIONS

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The theory of $q$-Karamata functions is applied to the second-order quasilinear $q$-difference equation

$$D_q(a(t)\Phi_\alpha(D_q(x(t)))) = b(t)\Phi_\beta(x(qt)),$$

where $\alpha > \beta > 0$ and $a\,, b$ are $q$-regularly varying functions, to establish necessary and sufficient conditions for the existence of strongly decaying solutions of this equation and to get information about asymptotic behavior of strongly decaying $q$-regularly varying solutions.

**Keywords**: $q$-difference equation, nonoscillatory solution, strongly decaying solutions, asymptotic behavior, regular variation, $q$-regular variation

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# ANALYSIS OF THE BEHAVIOR OF STOCHASTIC PREDATOR-PREY MODEL INFLUENCED BY ALLEE EFFECT ON PREY

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This talk presents the analysis of conditions under which the stochastic predator-prey model with Allee effect on prey population is going to be exterminated. With that purpose, we first prove the existence and uniqueness of global positive solution of considered model using the comparison theorem for stochastic differential equations. Then, we establish the conditions under which extinction of predator and prey populations occur. We also find the conditions for parameters of the model under which the solution of the system is globally attractive in mean. Finally, the numerical illustration with real life example is carried out to confirm our theoretical results.

**Keywords**: Allee effect, extinction, global attractivity in mean, predator, prey

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# TESTING UNIFORMITY - CHARACTERIZATION BASED APPROACH

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The uniform distribution is one of the most used distribution in statistical modeling and computer science. Therefore ensuring that the data come from a uniform distribution is of huge importance. Here we present a review of classical and several recent characterization based uniformity tests and their adaptations for testing the composite null hypothesis of rectangular distributions on arbitrary support. Also, some applications in time series will be presented. The presented tests will be compared with respect to several criteria.

**Keywords**: Bahadur efficiency, U-statistics, goodness-of-fit test

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# ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTIONS OF SECOND ORDER QUASILINEAR ORDINARY DIFFERENTIAL EQUATIONS IN THE FRAMEWORK OF REGULAR VARIATION

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The existence and the asymptotic behavior at infinity of positive solutions of second order quasilinear ordinary differential equations

$$(p(t)\varphi(x'(t)))' + q(t)\psi(x(t)) = 0,$$

are studied in the framework of regular variation

**Keywords**: Regularly varying solutions, slowly varying solutions, asymptotic behavior of solutions, positive solutions, second order quasilinear differential equations

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# NUMERICAL METHOD FOR A CLASS OF STOCHASTIC DIFFERENTIAL EQUATIONS WITH PIECEWISE CONSTANT ARGUMENTS

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Stochastic differential equations with piecewise constant arguments describe hybrid dynamical systems, that is, combinations of continuous and discrete systems. The mean square convergence of the sequence of the Euler-Maruyama approximate solutions is established under the global Lipschitz condition and linear growth condition, which guarantee the existence and uniqueness of the exact solution. Then we show that the initial equation is exponentially stable in mean square if and only if, for some sufficiently small step-size $\Delta$, the Euler-Maruyama method is exponentially stable in mean square. It should be pointed out that stochastic differential equations with piecewise constant arguments can be regarded as a class of stochastic differential equations with multiple time-dependent delays. In the convergence analysis of the Euler-Maruyama method of these equations in the existing literature, it is required that the delay functions satisfy Lipschitz continuity condition. In the present consideration, the delay functions do not satisfy that condition.

**Keywords**: Stochastic differential equations, Euler-Maruyama solution, mean square convergence, exponential mean square stability

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# SOLUTION OF THE INVERSE BOUNDARY PROBLEM FOR THE STURM-LIOUVILLE OPERATOR WITH DELAY

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The topic of this paper is spectral boundary problem generated by:

$$-y''(x) + q(x)y(x-\tau) = \lambda y(x) = z^2 y(x), \;\; q \in L_2[0,\pi] \tag{1}$$

$$y(x-\tau) \equiv 0, \;\; x \in [0,\tau), \;\; \tau \in [\frac{\pi}{3}, \frac{2\pi}{5}) \tag{2}$$

$$y'(0) - hy(0) = 0, \;\; h \in R, \tag{3}$$

$$y'(\pi) + Hy(\pi) = 0, \;\; H \in R, \tag{4}$$

We will present the problem in a shorter way further in the text

$$D^2 y = \lambda y = z^2 y \tag{5}$$

If potencial $q$ is absolutely continius function, then inverse problem (5) is solved and

for $q \in L_2[0,\pi]$ and $\tau \in [\frac{2\pi}{5}, \pi)$ inverse problem is also solved.

We emphasise that the problem of construction of the potential with the method of characteristic functions is reduced to solving a linear integral equation of Wolterr's type with removable argument.

The standard method of solving this equation without delay is based on the fact that Wolterrs operator is with a certain degree of contraction.

In this paper we used a new method of "small steps" and in that way we proved the fundamental result:
The two sequences of eigenvalues of the operator $D^2$ obtained by varying the boundary condition at the right end uniquely generate the operator.

**Keywords**: potencial,inverse problem,characteristic functions

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## HOMOTHETY CURVATURE HOMOGENEITY AND HOMOTHETY HOMOGENEITY

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We examine the difference between several notions of curvature homogeneity and show that the notions introduced by Kowalski and Vanzurova are genuine generalizations of the ordinary notion of $k$-curvature homogeneity. The homothety group plays an essential role in the analysis. We give a complete classification of homothety homogeneous manifolds which are not homogeneous and which are not VSI (which have some scalar curvature invariant which is non-zero) and show that such manifolds are cohomogeneity one. We also give a complete description of the local geometry, if the homothety character defines a split extension. This is a joint work with Eduardo Garcia-Rio and Peter Gilkey.

**Keywords**: bijections, magic squares, counting

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## AN OBSTRUCTION AND A CONSTRUCTION IN AN AMBIENT OF THE COTANGENT BUNDLE

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In this lecture we will give at least two good reasons why one should consider Floer-homological spectral invariants in the cotangent bundle. The first reason is that they give us an obstruction for a submanifold to be a boundary. Secondly, by homogenization of spectral invariants we can construct a family of partial quasi-morphisms on the group of Hamiltonian diffeomorphisms of the cotangent bundle.

**Keywords**: symplectic topology, Floer homology, spectral invariants, cotangent bundle

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# MERTON PORTFOLIOS IN PORTFOLIO OPTIMIZATION WITH RISK MEASURES

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Given a risk measure and a financial market with a numeraire and a finite number of risky assets, we consider three different optimal portfolio problems with respect to the risk measure: loss minimization, loss minimization with expected profit constraint, and expected profit maximization with risk exposure constraint. We demonstrate that, under reasonable assumptions, all three problems lead to a mixture of Merton portfolio and the numeraire. Examples are provided and the connection with the well known mutual fund theorems is explored.

**Keywords**: optimal portfolio, risk measure, Merton portfolio

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# NORMAL FAMILY OF FUNCTIONS IN THE SENSE OF MONTEL, BLOCH PRINCIPLE AND MÖEBIUS TRANSFORMATIONS AND THEIR APPLICATION

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On overview of Montel normality of a family of functions, the Bloch principle and Möebius transformations is given, as well as their applications in proving Lindelöf and Fatou theorems on the behavior of holomorphic and meromorphic functions at the boundary, and proving the Liouville theorem and the little Picard theorem for holomorphic and harmonic functions.

**Keywords**: normal families of functions, Bloch principle, Möebius mappings, holomorphic and harmonic functions

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# FIRST REGULARIZED TRACE OF THE LIMIT ASSIGNMENT OF STURM-LIOUVILLE TYPE WITH ROBIN BOUNDARY CONDITIONS

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We observe spectral assignment $D^2y = \lambda y$ defined by

$$-y''(x) + q_1(x)y(x-\tau) + q_2(x)y(x-2\tau) = \lambda y(x),\ \lambda = z^2 \tag{1}$$

$$q_1(x),\ q_2(x) \in L_2[0,\pi],\ \tau \in [\tfrac{\pi}{3}, \tfrac{\pi}{2})$$

$$y(x-2\tau) \equiv y(0)\varphi(x-2\tau), x \in [0,2\tau],\ y(0) = \varphi(0) = 1 \tag{2}$$

$$y'(0,z) - hy(0,z) = 0 \tag{3}$$
$$y'(\pi,z) + Hy(\pi,z) = 0 \tag{4}$$

In this paper, we construct a solution $y(x,z)$ which satisfies (1,2,3), and then (4) is used to construct the characteristic function $F(z)$, $z \in C$. Then the asymptotics of eigenvalues of the operator $D^2$ is constructed. Finally, the first regularized trace is calculated.

**Keywords**: Sturm-Liouville operator, regularized trace, differential equations with delay

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## COHERENCE IN ACTION

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In the early sixties of the 20th century a type of results in category theory appeared under the common name "Coherence theorems". The slogan "all diagrams commute" is usually tied to coherence or sometimes this notion means just a possibility of strictification in categories (almost associative operations become associative, and similar for commutativity). Although the notion of coherence is still not precisely defined one can say that it means faithfulness of some functors with geometrical categories as targets.

The aim of this talk is not to clarify the notion of coherence. We will provide some examples from working mathematics where the results of this type may be useful.

**Keywords**: category theory, commutative diagram, strictification

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## ANTI-GAUSSIAN QUADRATURE RULE FOR TRIGONOMETRIC POLYNOMIALS

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We investigate an anti-Gaussian quadrature rule with maximal trigonometric degree of exactness with respect to an even weight function on $[\pi, \pi)$. Its error is equal in magnitude but of opposite sign to corresponding Gaussian formula. We give the method for its construc- tion based on relations between nodes and weights of the quadrature rule for trigonometric polynomials and those of the quadrature rule for algebraic polynomials. Also, we introduce averaged Gaussian quadrature formula for trigonometric polynomials and, at the end, we give some numerical examples.

**Keywords**: Gauss quadrature, anti-Gaussian quadrature, trigonometric degree of exactness, nodes, weights

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# ON THE EXTREMES OF SOME GAUSSIAN RANDOM PROCESSES IN RANDOM ENVIRONMENTS

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We give a summary of a few exact asymptotic results for probability

$$\mathbf{P}\left(\sup_{t\in[0,T]} X(t) > u\right), \text{ as } u \to \infty,$$

where $X(t)$ is Gaussian random process in a random environment, $T > 0$ real constant.

**Keywords**: Gaussian random process, random environment, extremes

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# ON NON-LOCAL MODIFIED GRAVITY - A NEW COSMOLOGICAL SOLUTIONS

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Despite to all significant gravitational phenomena discovered and predicted by general theory of relativity, it is not a complete theory. One of actual approaches towards more complete theory of gravity is its non-local modification. We consider non-local modification of the Einstein theory of gravity in framework of the pseudo-Riemannian geometry, with the non-local term of the form $\mathcal{H}(R)\mathcal{F}()\mathcal{G}(R)$, where $\mathcal{H}$ and $\mathcal{G}$ are differentiable functions of the scalar curvature $R$, and $\mathcal{F}() = \sum_{n=0}^{\infty} f_n^n$ where $f_n$ are is an analytic function of the dAlambert operator . Using calculus of variations of the action induced by the metric tensor $g_{\mu\nu}$, we derived the corresponding equations of motion. Firstly, we consider several models of the above mentioned type, as well as the case when the scalar curvature is constant. Specially, we are paid our attention to the case where $\mathcal{H}(R) = \mathcal{G}(R) = \sqrt{R - 2\,\Lambda}$, and find some new cosmological solutions and we test validity of obtained solutions with experimental data. Moreover, we consider space-time perturbations of the de Sitter space. It was shown that gravitational waves are described in the class of nonlocal models $\mathcal{H}(R)\mathcal{F}()\mathcal{G}(R)$, with respect to Minkowski metric by the same equations as in general relativity.

This is joint work with I. Dimitrijević, B. Dragović, A. Koshelev and Jelena Stanković.

**Keywords**: bijections, magic squares, counting

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# FUZZY METRIC SPACES AND APPLICATIONS IN IMAGE PROCESSING

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This paper gives notions the fuzzy $T-$metric and fuzzy $S-$metric spaces, considers their properties. Numerous examples of fuzzy metrics used in the applications are mentioned. There is also a procedure for constructing new fuzzy metrics. A possible application of the fuzzy metrics in image filtering and segmentation is given.

**Keywords**: aggregation function, distance function, fuzzy metric space, image filtering, image segmentation, triangular conorm, triangular norm

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# WEAKLY CHAIN SEPARATED SETS IN A TOPOLOGICAL SPACE

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In this talk we introduce the notion of pair of weakly chain separated sets in a topological space. The notion is similar to the notion of pair of chain separated sets in a topological space given in [1].

The nonempty sets $A$ and $B$ in a topological space $X$ are weakly chain separated in $X$, if for every point $a \in A$ and every $b \in B$, there exists an open covering $\mathcal{U}$ of $X$ in $X$, such that there is no chain in $\mathcal{U}$ that connects $x$ and $y$. And by a chain in $\mathcal{U}$ that connects $x$ and $y$ we understand a finite sequence of elements of $\mathcal{U}$ such that $x$ belongs to first element, $y$ to the last, and any two neighbor elements from the chain have nonempty intersection.

If two sets are chain separated in the topological space, then they are weakly chain separated in the same space. We give an example of weakly chain separated sets in a topological space which are not chain separated in the same space. Then we study the properties of that kind of sets.

Moreover we give a criterion for chain connected set in a topological space by using the notion of weakly chain separatedness. A set $C$ is chain connected in a

topological space $X$ [1] if and only if it can not be represented as a union of two weakly chain separated sets in $X$. Then we prove the properties of chain connected sets in a topological space by using the notion of weakly chain separatedness.

At the end we will mention the criteria for two kind of two topological spaces by using the notion of chain. The topological space is totally separated if any two sigleton subsets are weakly chain separated in the space, and is discrete if they are chain separated.

[1] Z. Misajleski, N. Shekutkovski, A. Velkoska, Chain Connected Sets In A Topological Space, Kragujevac Journal of Mathematics, Vol. 43 No. 4, 2019, Pages 575-586;

**Keywords**: general topology, coverings, chain, chain connecteness, weakly chain separatedness

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# GENERALIZED AVERAGED GAUSSIAN QUADRATURE RULE AND APPLICATIONS

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It is well known that an $(\ell+1)$-node Gauss quadrature rule associated with a measure $d\sigma$ can be represented by an $(\ell+1)\times(\ell+1)$ real symmetric tridiagonal matrix $J^G_{\ell+1}(d\sigma)$ determined by the recursion coefficients of the first $\ell+1$ orthogonal polynomials associated with the measure $d\sigma$. The author, following Peherstorfer, proposed that the leading $\ell\times\ell$ tridiagonal submatrix of $J^G_{\ell}(d\sigma)$ be flipped right-left and upside-down, and appended to $J^G_{\ell+1}(d\sigma)$ to obtain a new symmetric tridiagonal matrix $J_{2\ell+1,\ell+1}$ of order $2\ell+1$. The latter matrix defines a $(2\ell+1)$-node quadrature formula referred to as a *generalized averaged Gaussian quadrature formula*. The author showed that these quadrature rules may yield a smaller quadrature error than what can be explained by just considering their algebraic degree of precision. This makes the generalized averaged Gaussian quadrature formulas attractive to use when it is inexpensive to evaluate the integrand at the nodes, it is expensive or cumbersome to compute the moment information needed to determine the Gauss rule. Applications of generalized averaged Gaussian quadrature rules to problems of this kind are described, where the quadrature rules are used to estimate quantities of interest in network analysis. A survey of the mentioned results will be presented.

**Keywords**: Generalized averaged Gaussian quadrature rules, Gauss-Kronrod quadrature rules

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## PARTIAL ORDERS IN IIPS

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We define some partial orders for matrices in indefinite inner product spaces, comparing degenerate and nondegenerate case. Several well-known results are generalized.

**Keywords**: indefinite inner product, general inverses, linear relations, partial order

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## AK GROWTH MODEL IN THE GENERALIZED LOGISTIC FORM

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In this paper we give simply modification of the Solow model, a model of the increasing growth in economics. A model is described by difference equation of the first order which is further analyzed to explain this complex dynamical system and its behavior. In our previous papers we explained importance of that mathematical tool in economics applications. In this paper we suggest modification of the AK model in three directions: modified AK model, AK model and government and simple chaotic AK model with increasing returns. We use general logistic equation, which is mathematical generalization of the logistic equation. At the end, we give data analysis with real data, estimate appropriate parameters of the models and we compare simple AK model with its generalization.

**Keywords**: AK model, logistic map, generalized logistic map

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## ON TORIC ORBIT SAPCES OF SOME GRASSMANN MANIFOLDS

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The set of all $k$-dimensional complex vector subspaces in the complex vector space $C^n$ has a manifold structure, which is known as the Grassmann manifold $G_{n,k}$. There is a natural action of the algebraic torus $(C^*)^n$ on $G_{n,k}$, which is induced by the canonical action of this torus on $C^n$. The action of the algebraic torus on $G_{n,k}$ induces a natural action of the compact torus $T^n \subset (C^*)^n$ na $G_{n,k}$. The problem of studying of the orbit space of the Grassmannian $G_{n,k}$ under the considered actions is motivated by the classical and modern problems of algebraic geometry, algebraic and equivariant topology, symplectic geometry and enumerative combinatorics. The well known papers by Gel'fand, Serganova, Goresky, MacPherson, Kapranov etc., are devoted to the action of the algebraic torus on $G_{n,k}$ and the description of the corresponding equivariant topology.

We study the action of the compact torus $T^n$ on $G_{n,k}$ by developing the methods of toric geometry, toric topology, Morse theory and we also propose the new method for the description of the topology of the orbit space $G_{n,k}/T^n$. The aim of this talk is to present some aspects of the proposed method and illustrate its application for an explicit topological description of the orbit spaces of some Grassmann manifolds and complex projective spaces.

The talk is based on joint results with V. M. Buchstaber.

**Keywords**: orbit space, torus action, Grassmann manifold

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## ON SINGULAR MATRIX EQUATIONS AND CONNECTIONS WITH SINGULAR PDES

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In this talk we introduce the notation of quasi-asymptotically almost periodic distributions and quasi-asymptotically almost periodic ultradistributions with values in a Banach space, as well as some other generalizations of these concepts. Furthermore, some applications of the introduced concepts in the analysis of systems of ordinary differential equations are provided.

**Keywords**: quasi-asymptotically almost periodic functions, quasi-asymptotically almost periodic (ultra)distributions, Stepanov $p$-quasi-asymptotically almost periodic (ultra)distributions, Banach spaces

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# INVERSE SPECTRAL PROBLEMS FOR DIFFERENTIAL OPERATORS WITH CONSTANT DELAY UNDER DIRICHLET/POLYNOMIAL BOUNDARY CONDITIONS

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This paper deals with non-self-adjoint differential operators with constant retarded argument, generated by $-y'' + q(x)y(x-\tau)$ where potential $q$ is complex-valued function, $q \in L^2[0,\pi]$ and constant $\tau \in (\frac{\pi}{2}, \pi)$ is delay. We establish properties of the spectral characteristics and research the inverse problem of recovering operators from their spectra. We prove that potential is uniquely determined from two spectra, firstly under Dirichlet boundary condition $y(0) = y(\pi) = 0$ and secondly under polynomial boundary condition $y(0) = y'(\pi) + P(\lambda)y(\pi) = 0$ of those operators.

**Keywords**: Differential operators with delays,Inverse spectral problems, Fourier trigonometric coefficients

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# INVERSE PROBLEMS FOR STURM-LIOUVILLE OPERATOR WITH POTENTIAL FUNCTIONS FROM $L_2[0,\pi]$

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This paper deals with non-self-adjoint second-order differential operators with two constant delays. We consider four boundary value problems $D_{i,k}$

$$-y''(x) + q_1(x)y(x-\tau_1) + (-1)^i q_2(x)y(x-\tau_2) = \lambda y(x),\ x \in [0,\pi]$$

$$y'(0) - hy(0) = 0,\ y'(\pi) + H_k y(\pi) = 0, k = 1,2$$

where $\frac{\pi}{3} \leq \tau_2 < \frac{\pi}{2} \leq 2\tau_2 \leq \tau_1 < \pi$, $h, H_1, H_2 \in R \setminus \{0\}$ and $\lambda$ is a spectral parameter. We assume that $q_1, q_2$ are real-valued potential functions from $L_2[0,\pi]$ such that $q_1(x) = 0$,
$x \in [0,\tau_1)$ and $q_2(x) = 0, x \in [0,\tau_2)$. The inverse spectral problem of recovering operators from their spectra has been studied. We prove that delays $\tau_2, \tau_1$ and parameters $h, H_1, H_2$ are uniquely determined from the spectra. Then we prove that potentials are uniquely determined by Volterra linear integral equations.

**Keywords**: Differential operators with delays, Inverse spectral problems, Fourier coefficients

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## ATOMIC DECOMPOSITION FOR THE HARMONIC FOCK SPACE IN THE PLANE

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The purpose of this paper refers to the development of certain functional-analytic properties for the harmonic Fock spaces in correspondence to the appropriate properties of analytic Fock spaces. Firstly, we found the pointwise estimates for the functions in the harmonic Fock spaces $\mathcal{H}^p_\alpha$, $1 \leq p \leq \infty$. Then, we describe the atomic decomposition for the harmonic Fock space $\mathcal{H}^p_\alpha$.

**Keywords**: Harmonic Fock space, orthogonal projection

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## MATHEMATICS AND INTUITION

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The philosophers Kant, Huserl, and Gödel believe that we acquire mathematical knowledge through a special kind of perception – intuition. Each of them, however, interprets that intuition in a different way. Kant takes into account the intuition of mathematical objects, Huserl distinguishes between the intuition of physical and abstract objects, and Gödel believes that the objects of mathematical intuition are mathematical statements. This difference is due to the difference of metaphysical assumptions about mathematical entities in their philosophies.

**Keywords**: intuition, objects, statement

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## INFLUENCE OF STOCHASTIC PERTURBATIONS ON STABILITY OF HEROIN MODEL WITH TWO DISTRIBUTED TIME DELAYS

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In this paper deterministic heroin model with two distributed time delays is enriched with random perturbation, that briefly describe how the environmental factors lead an individual to become a heroin user. Firstly, existence, positivity and boundedness of solution of obtained stochastic system is studied. Secondly, stability features of stochastic heroin model are investigated. Finally, the number of heroin users in USA is modeled and adequate numerical simulation is given. This simulation clear shows that theoretical results in this paper coincide with real data, that are used in the aforementioned modeling.

Research is supported by Grant No 174007 of MNTRS.

**Keywords**: Distributed delay, heroin spread, heroin spread equilibrium, Lyapunov functional, mean square stability

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## SOME ENUMERATIONS IN CLASS OF UNICYCLIC GRAPHS

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We enumerate labeled unicyclic graphs with fixed degree sequence and give recursive formula for it. Besides this, we investigate the number of non-isomorphic unicyclic graphs and give a formula of its number.

**Keywords**: unicyclic graphs, labeled graphs, degree sequence

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## BIPRODUCTS IN MONOIDAL CATEGORIES

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Let $(\mathcal{M}, \otimes, I)$ be a monoidal category with finite coproducts (including the initial object 0) preserved by each functor $a \otimes \_$ for $a \in \mathrm{ob}(\mathcal{M})$. In 2016, R. Garner and D. Schäppi characterised existence of finite *biproducts* (both coproducts and products) in $\mathcal{M}$ by existence of right duals of 0 and the coproduct $I + I$. We present a shorter proof of this theorem, and additionally, we prove that in this characterisation the assumption concerning right duals can be weakened. Precisely, for existence of finite biproducts in category $\mathcal{M}$ it is enough that object 0 and coproduct $I + I$ admit only right *semiduals*.

**Keywords**: monoidal categories, products, coproducts, biproducts, duals, semiduals

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