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Transition Rate Dependence on the Non-Zero Initial Momentum in the ADK-Theory

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Tunneling regime, introduced by Keldysh, in the interaction of strong lasers with atoms has been now accepted as the reliable method for describing processes when low frequency lasers are involved. Yet it was always assumed that the ionized electrons are leaving the atom with zero initial momentum. Because we are interested in how non-zero momentum influences the transition probability of tunnel ionization, we obtained the exact expression for the momentum. Here the estimation of the transition probability with nonzero momentum included was conducted. Potassium atoms in the laser field whose intensity varied from 10^{13} W/cm^2 to 10^{14} W/cm^2 were studied. It seems that all energy of laser field is used for tunneling ionization process at the beginning of laser pulse — ionization probability is large. After that, with further action of laser pulse, ionization probability decreases, probably because part of laser pulse energy is used for increasing momentum of ejected electrons, leaving smaller amounts of light quanta available for ionization of remaining electrons. If laser pulse lasts long enough, then the amounts of light quanta available for ionization become larger, resulting in increase in ionization probability, now with greater starting energy of ejected electrons.

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1. Introduction

In the interaction of strong lasers with atoms, tunneling regime reliably describes processes that involve low frequency lasers. Theoretical framework was given by Keldysh [1]; it was further developed, resulting in emerging of the ADK--theory (Ammosov, Delone, Krainov) [2]. Some recent papers [3–5] improved it further, but in them the authors always assumed that the ionized electrons are leaving the atom with zero initial momentum (except say in [6, 7]). Except for

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aforementioned papers, in [8] it was given thorough calculation of dependence of the initial momentum on the parabolic coordinate η. So we shall use their results in our paper.

In this paper we shall use atomic units system $e = m_{\mathrm{e}} = \hbar = 1$.

Tunneling ionization occurs when the Keldysh parameter is $\gamma \ll 1$, which was defined as $\gamma = \omega\sqrt{2E_{\mathrm{i}}}/F$, where ω is frequency of applied laser field, F is its strength, and E_{i} is ionization potential of an atom. Procedure was based on assumption that the external potential does not affect the energy of the initial state of ejected electron, and the atomic potential does not affect the final state of ejected electron (when it leaves the atom) as the electron is far enough from the nucleus. The external (Coulomb) potential was then treated as perturbation of final state energy, resulting in ADK-theory [2]. In [9] in constructing the ADK--theory the Coulomb interaction was included into calculations for the turning point τ, which was not done before. Also, it was always assumed that the initial momentum of the electron leaving the atom is zero. Besides slight tries as in [6, 7], until now there were no thorough estimations of what influence can have non-zero initial momentum on the processes thereof. Thus we are offering in this paper the calculation of transition rates for changeable initial momentum.

2. Calculating non-zero initial momentum

Because we are interested how non-zero momentum influences the transition probability of tunnel ionization, we shall try to obtain the exact expression for aforementioned momentum. In order to do this, we shall begin with stationary Schrödinger equation for charged particle in the Coulomb field

$$\left(-\frac{1}{2}\nabla^2 + V\right)\psi = E\psi. \tag{1}$$

The easiest way [8] to solve this is to introduce parabolic coordinates in the following form:

$$\xi = r + z, \quad \eta = r - z, \quad \phi = \arctan(y/x), \tag{2}$$

with $\xi, \eta \in [0, \infty]$, $\phi \in [0, 2\pi]$. In this coordinates, the Laplace operator is given by the following expression [10]:

$$\nabla^2 = \frac{4}{\xi+\eta}\left[\frac{\partial}{\partial\xi}\left(\xi\frac{\partial}{\partial\xi}\right) + \frac{\partial}{\partial\eta}\left(\eta\frac{\partial}{\partial\eta}\right)\right] + \frac{1}{\xi\eta}\frac{\partial^2}{\partial\phi^2}, \tag{3}$$

while the potential that electron feels $V = -Z/r + Fz$ gets the following form in parabolic coordinates

$$V = -\frac{2Z}{\xi+\eta} + \frac{1}{2}F(\xi - \eta). \tag{4}$$

After separating variables and introducing $f_1(\xi) = \chi_1(\xi)/\sqrt{\xi}$ and $f_2(\eta) = \chi_2(\eta)/\sqrt{\eta}$ we obtain the following equations:

$$-\frac{1}{2}\frac{\mathrm{d}^2\chi_1}{\mathrm{d}\xi^2} + \left(-\frac{C_1}{2\xi} + \frac{m^2-1}{8\xi^2} + \frac{F\xi}{8}\right)\chi_1 = \frac{E}{4}\chi_1, \tag{5a}$$

$$-\frac{1}{2}\frac{\mathrm{d}^2\chi_2}{\mathrm{d}\eta^2}+\left(-\frac{C_2}{2\eta}+\frac{m^2-1}{8\eta^2}-\frac{F\eta}{8}\right)\chi_2=\frac{E}{4}\chi_2, \tag{5b}$$

taking into account the condition for separating constants $C_1 + C_2 = Z$.

As our combined potential is given by Eq. (4), ionization occurs in the $-z$ direction, i.e. along the $\eta = r - z$ coordinate. That is the reason why we are interested in the second of two above equations. In Eq. (5b) the second term represents potential which we will denote as

$$U_2(\eta)=-\frac{C_2}{2\eta}+\frac{m^2-1}{8\eta^2}-\frac{F\eta}{8}, \tag{6}$$

and the momentum that corresponds to it is given by

$$p=\sqrt{-\frac{2E_\mathrm{i}}{4}+\frac{C_2}{\eta}-\frac{m^2-1}{4\eta^2}+\frac{1}{4}F\eta}. \tag{7}$$

The explicit form of the separation constant C_2 was obtained in paper [8]:

$$C_2=Z-\frac{\sqrt{2E_\mathrm{i}}}{2}(|m|+1). \tag{8}$$

We assume that value for m is zero.

3. Estimating the transition probability with non-zero momentum included

Determining of transition probability in ADK-theory is based on the Landau–Dykhne adiabatic approximation [1, 10], and it consists in taking transition amplitude between initial and final states to be

$$A_\mathrm{if}=\exp\left(\mathrm{i}\int_{t_1}^{\tau}\omega_\mathrm{if}(t)\mathrm{d}t\right), \tag{9}$$

where ω_if is the transition frequency from initial to final states, while τ is complex turning point in the time plane. From this follows the expression for a transition probability

$$W_\mathrm{if}=|A_\mathrm{if}|^2=\exp(-2\mathrm{Im}\delta S(\tau)). \tag{10}$$

The complex turning point is given by equation $\omega_\mathrm{if}(\tau) = 0$ that defines classically forbidden state; it can be written in the following form $E_\mathrm{f}(\tau) = E_\mathrm{i}(\tau)$, where $E_\mathrm{f}(\tau)$ and $E_\mathrm{i}(\tau)$ are the final and initial energies in the external laser field, and τ is the complex time.

As external fields are much smaller than the atomic field $F \ll F_\mathrm{at}$, we assumed that laser field influences only the final state, while the initial state remains unperturbed, so $E_\mathrm{f}(\tau) = E_\mathrm{i}(\tau)$ becomes

$$\frac{1}{2}\left(p-\frac{F}{\omega}\sin\omega\tau\right)^2-\frac{Z}{\eta(\tau)}=-E_\mathrm{i}. \tag{11}$$

From this equation, in the paper [9] there was obtained the following expression for turning point (in the zero-order approximation)

$$\tau_0 = \frac{p + \mathrm{i}\sqrt{2E_\mathrm{i}}}{F}, \tag{12}$$

that was subsequently used in calculating ADK ionization probability

$$W_\mathrm{ADK} = \left(\frac{4Z^3 e}{Fn^{*4}}\right)^{2n^*-1} \exp\left(-\frac{2Z^3}{3Fn^{*3}}\right). \tag{13}$$

In above expression, the Coulomb interaction was neglected during calculation of the turning point (12), but was included in the expression (11).

This negligence was corrected in [9], which lead to the corrected expression for a turning point

$$\tau = \frac{p + \mathrm{i}\sqrt{2E_\mathrm{i}}}{F} - \frac{\mathrm{i}Z}{(p^2 + 2E_\mathrm{i})\sqrt{2E_\mathrm{i}}}. \tag{14}$$

What follows is the corrected form for ionization probability

$$W_\mathrm{cADK} = \left(\frac{4Z^3 e}{Fn^{*4}} \frac{1}{1 + \frac{2ZF}{(p^2+2E_\mathrm{i})^2} + \frac{Z^2F^2}{2E_\mathrm{i}(p^2+2E_\mathrm{i})^3}}\right)^{2n^*-1} \exp\left(-\frac{2Z^3}{3Fn^{*3}}\right). \tag{15}$$

Expression (15) represents the transition probability without including the non--zero initial momentum into the exponent part. But if, as indicated in Sect. 2, the electron has a non-zero initial momentum (7), we obtain a final transition probability in the following form:

$$W_\mathrm{cADK} = \left(\frac{4Z^3 e}{Fn^{*4}} \frac{1}{1 + \frac{2ZF}{(p^2+2E_\mathrm{i})^2} + \frac{Z^2F^2}{2E_\mathrm{i}(p^2+2E_\mathrm{i})^3}}\right)^{2n^*-1} \exp\left(-\frac{2Z^3}{3Fn^{*3}} - \frac{p^2\gamma^3}{3\omega}\right),$$

where cADK stands for the corrected ADK. Now we shall examine how the expression for non-zero momentum (7) affects the transition probability. We plotted dependence of transition probability on intensity of field I and on η coordinate (momentum) (Fig. 1). We studied potassium atom in laser field whose intensity

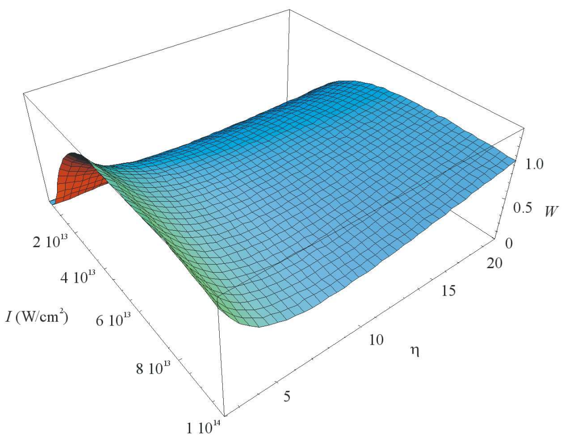

Fig. 1. Transition probability on intensity of field I and on η coordinate.

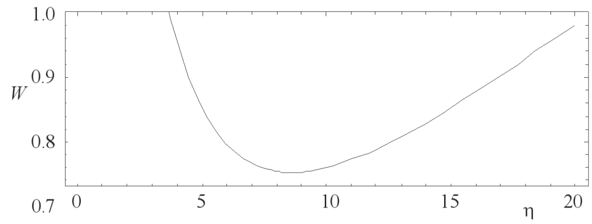

Fig. 2. Dependence of ionization probability on η coordinate.

varied from 10^{13} W/cm^2 to 10^{14} W/cm^2. It seems that all energy of the laser field is used for tunneling ionization process at the beginning of laser pulse — ionization probability is large, and the initial momentum of electrons are very small, almost zero. After that, with further effects of laser pulse, ionization probability decreases, because part of laser pulse energy is used for increasing momentum of ejected electrons, leaving smaller amounts of light quanta available for ionization of remaining electrons. If laser pulse lasts long enough, then the amounts of light quanta available for ionization become larger, resulting in increase in ionization probability with greater energy of ejected electrons now with greater starting energy of ejected electrons.

The aforementioned situation is more obvious in Fig. 2. It shows dependence of ionization probability on η coordinate (momentum).

4. Final remarks

Starting with the idea that the tunneling regime, introduced by Keldysh, in the interaction of strong lasers with atoms has been now accepted as the reliable method for describing processes when low frequency lasers are involved, we have calculated a few probabilities. Yet as it was always assumed that the ionized electrons are leaving the atom with zero initial momentum, we tried to correct this. Because we are interested how non-zero momentum influences the transition probability of tunnel ionization, we have obtained the exact expression for aforementioned momentum.

It seems that all energy of the laser field is used for tunneling ionization process at the beginning of laser pulse — ionization probability is large, and the initial momentum of electrons are very small, almost zero. Now, if laser pulse lasts yet, ionization probability decreases, probably because part of laser pulse energy is used for increasing momentum of ejected electrons, leaving smaller amounts of light quanta available for ionization of remaining electrons. If laser pulse lasts long enough, then the amounts of light quanta available for ionization become larger, resulting in increase in ionization probability with greater energy of ejected electrons now with greater starting energy of ejected electrons. This situation is most obvious in Fig. 2, which represents dependence of ionization probability on η coordinate (momentum).

Acknowledgements

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