Abstract: We have shown that Noether's theorem [16] has a corollary which proves non-conservation of angular momentum and spin in "mixed" theories (the ones that combine classical and quantum approach). As laser field is treated classically and atoms with which it interacts must be treated as quantum objects (see, for instance [1], where the ADK-theory is described, which we take as a represent of "mixed" theories), it is shown that in such theories the law of conservation of the 4-angular momentum tensor is broken. As there are many other theories that can be defined as "mixed" [see, for instance, 18], we think the result being interesting enough, though that evident shortcoming does not affect the results of ADK-theory, because it never operates with spin, and the most probable ejected electrons have orbital quantum number $l = 0$ [8, 17]. However it should be stressed that the corollary we have proved **does not** imply that the law of conservation of angular momentum and spin is not valid in general, but only that one should be cautious when treating angular momentum and spin in "mixed" theories.

Corollary to Noether's theorem about the conservation of angular momentum and spin in theories that are dealing with strong laser fields

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1. Introduction

In the last twenty or so years it has been shown that the calculations and its results, called Ammosov-Delone-Krainov (ADK) theory [1], can be used to describe the process of tunneling ionization [2–5], for which Keldysh's parameter [6] $\gamma \ll 1$. In fact, it was shown in [7, 8] that the set of prescriptions given in [1] is a theory, based on Landau-Dykhne adiabatic approximation [9] and Keldysh's theory [6].

It is a well known fact [8, 10, and 11] that ADK-theory treats an atom as quantized object and laser field classically. But in both cases there are some deviations of the aforementioned. Namely in the case of the atom one is working with the states with large quantum numbers n, that is the so called quasi-classical approximation is used and in the case of the laser field one is using concepts and notions of a *quantum* and a *photon*, not only merely as a convenient terminology but also as physical quantities which are used for real estimates. There is no contradiction in that, however, because if quantized the strength of electro-magnetic field becomes the quan-

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tum operator, which for absorption of a photon by the atomic system gives factor $(n_{k\alpha})^{1/2}$ in transition amplitude [12], while for emission of a photon factor in the transition amplitude is equal to $(n_{k\alpha}+1)^{1/2}$. Here $n_{k\alpha}$ is number of photons with a given wave vector k and polarization α. In the approximation of a strong external field one can consider these matrix elements equal, so electromagnetic field could be considered classical providing that $n_{k\alpha} \gg 1$.

Yet the theories which combine classical and quantum approach might be considered ambiguous concerning angular momentum [13], i.e. it is not at first sight certain that they obey the law of conservation of angular momentum. That is why we suggest here that the problem of conservation of angular momentum in ADK-theory should be examined. Especially, as ADK-theory has shown its vitality not only in the case of strong fields but also for super strong fields [14]. There is now even a calculation for the $20-30\,\mathrm{fs}$ laser pulses with peak intensities of $10^{21}\,\mathrm{W/cm^2}$(atomic units: $e=\hbar=m_e=1$), in which non-relativistic tunnel ionization theories break down, and one needs Dirac theory of tunnel ionization [15], which is based on some assumptions of ADK-theory, but cannot be called the ADK-theory.

2. Corollary to Noether's theorem

In order to check the reliability of ADK-theory (and other similar theories, for that matter) concerning conservation laws, we shall use Noether's theorem.

Following [16] we could give next formulation of Noether's theorem: *To any* s*-parametric continuous transformation of field functions and coordinates, which keeps variation of action zero, there correspond* s*-dynamic invariants (i.e. constant in time combinations of field functions and their derivatives).*

In what follows we shall use the results and notation of book [16], except that we shall use Greek indices to denote 4 space-time coordinates (0,1,2,3) and Latin for spatial 3-coordinates (1, 2, 3). So, we shall start with expression (2.16) in [16]

$$\theta^{\nu}_{(k)} = \frac{\partial L}{\partial u_{\alpha;\nu}}(u_{\alpha;\mu}\, X^{\mu}_{k} - \Psi_{\alpha;k}) - L(x)\, X^{\nu}_{k}, \tag{1}$$

which appears in the action integral obtained for s-parametric transformations (here $L(x)$ is the Lagrangian of the system). Standard procedure is to introduce infinitesimal transformation of 4-coordinates, for which we chose infinitesimal four-dimensional Lorentz rotations

$$x_\nu \;\rightarrow\; x'_\nu = x_\nu + x^\mu\, \delta L_{\nu\mu} \tag{2}$$

where $\delta L_{\nu\mu}$ are infinitesimal parameters of the rotations; due to their antisymmetry, we may choose as independent parameters six of them

$$\delta\omega_{(\kappa\mu)} = \delta L_{\mu\nu} \quad \text{at} \;\; \mu < n, \tag{3}$$

which represent infinitesimal angles of rotation in the $x_\mu x_\nu$ plane.

After some calculations, and obtaining expressions

$$X^{(\rho\sigma)}_{\nu} = x^\sigma \delta^\rho_\nu - x^\rho \delta^\sigma_\nu, \tag{4}$$

and

$$\Psi^{\nu\,(\rho\sigma)}_{\mu} = A^{\nu\,(\rho\sigma)}_{\mu} u_\mu(x), \tag{5}$$

where, for vector fields,

$$A^{\nu\,(\rho\sigma)}_{\mu} = \delta^\rho_\mu\, g^{\nu\sigma} - \delta^\sigma_\mu\, g^{\nu\rho}\,, \quad \rho < \sigma, \tag{6}$$

one can get the 4-angular momentum tensor

$$M^{\tau\,(\rho\sigma)} = \frac{\partial L}{\partial u_{\nu;\tau}}(u^{;\rho}_{\nu} x^\sigma - u^{;\sigma}_{\nu} x^\rho) + \tag{7}$$

$$+L(x^\rho g^{\rho\tau} - x^\sigma g^{\rho\tau}) - \frac{\partial L}{\partial u_{\nu;\tau}} A^{\nu\,(\rho\sigma)}_{\mu} u_\mu(x') =$$

$$= (x^\sigma\, T^{\rho\tau} - x^\rho\, T^{\sigma\tau}) - \frac{\partial L}{\partial u_{\nu;\tau}} A^{\nu\,(\rho\sigma)}_{\mu} u_\mu(x').$$

The first two terms can be represented as

$$M^{\tau\,(\rho\sigma)}_{0} = x^\sigma\, T^{\rho\tau} - x^\rho\, T^{\sigma\tau}, \tag{8}$$

which we can identify as *orbital angular momentum* of the wave field.

It is easily seen that the first term in the last part of expression (7) corresponds to an orbital angular momentum of the wave field, and the second part, which we shall denote in the following manner

$$S^{\tau\,(\rho\sigma)} = -\frac{\partial L}{\partial u_{a;\tau}} A^{b\,(\rho\sigma)}_{a} u_b(x), \tag{9}$$

characterizes the polarization properties of the field, and in the quantized case corresponds to the spin of the particle described by the quantized field.

Thus, in the case of the multicomponent fields the last term in the (7) is different from zero, and it characterizes the polarization properties of the fields, and corresponds to the *spin angular momentum* of the field

$$M^{\tau\,(\rho\sigma)} = M^{\tau\,(\rho\sigma)}_{0} + S^{\tau\,(\rho\sigma)},$$

where

$$S^{\tau\,(\rho\sigma)} = -\frac{\partial L}{\partial u_{a;\tau}} A^{b\,(\rho\sigma)}_{a} u_b(x).$$

$$S^n = \varepsilon^{nmp}\, S^{(mp)} \quad (n,m,p = 1,2,3).$$

So, if we chose 4-rotations of space-time, it would result in conservation of 4-angular momentum (i.e. 3-angular momentum and spin), but this would not be applicable to mixed theories, because the rotations which are continuous in the classical theory, have to be quantized in the quantum theory, so there is no smooth connection between the classical and quantum part of the theory, hence for such theories the conservation of angular momentum and spin is not working.

So we have proven the following

Corollary to Noether's theorem

Physical theories which combine ("mix") the classical and quantum approach ***break*** *the law of angular momentum-spin conservation.*

This is the shortcoming of such theories, but one that does not affect the results of ADK-theory, because that theory never operates with spin, and also because the most probable ejected electrons have orbital quantum number $l = 0$ [8, 17].

Here should be stressed, though it is pretty obvious, that our corollary ***does not*** *imply that the law of conservation of angular momentum and spin is not valid in general, but only that one should be cautious when treating angular momentum and spin in "mixed" theories.*

For instance, in calculations like those given in [15], one should be extremely careful as there, due to using the Dirac equation, the spin is involved, and the laser field is treated classically, while atoms are, of course, quantized.

This proof, being all-inclusive, also holds for other mixed theories [see, for instance, 18] and is not restricted to ADK-theory solely. Yet we are applying it to that theory not only because it is most familiar to us but also because we feel that there is no need for further illustrations of this rather general principle.

3. Conclusion

Finally we can say that our discovery of breaking the law of conservation of angular-momentum and spin in mixed theories was made already in paper [13], but was not recognized, by us and no other consumer of that work, as relevant. There we proved the law of conservation of energy and momentum for such theories, and thought of breaking the law of conservation of 4-momentum, as mere illustration of our previous result.

But after reconsidering our work we discovered the negative result to be more interesting and tried to give it the dignity of a corollary, as it is, to our opinion, very general. Thus, on the basis of our result we are advising physicist that are working with mixed theories, to be extremely cautius when including angular momentum and spin into their calculations.

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