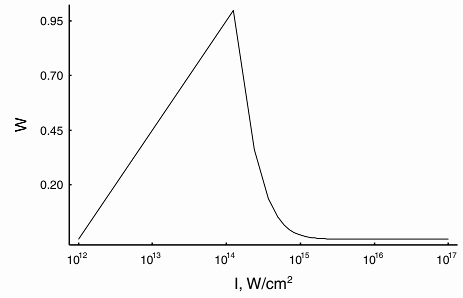
Abstract: In the case of the short-range potential [1,5,8,9], the estimation of the probability of ionization of atoms is carried along taking into account the approximation (Keldysh approximation) which states that this kind of potential does not affect the energy of the final state f of the ejected electron in the laser field, because the electron is far enough from the nucleus. When the Coulomb potential is taken into account, it can be treated as a perturbation to the energy of the final state [1,10]. Yet, originally [1,10], the Coulomb potential in this kind of estimation was not included into calculating the turning point. This was done in [7], but only for the fields below the atomic field (10^{16} W/cm^2). Now, based on the results [11,12], we are extending our calculation that included the Coulomb correction into the estimating the turning point to the fields that are much stronger (up to 10^{17} W/cm^2). That results in the shift of the position of the turning point τ. This paper is dealing with the influence of that shift on the ionization probability for atoms in the low-frequency electromagnetic field of superstrong lasers.



Ionization probability plotted vs. intensity of the field

Turning point behaviour in tunnel ionization of atoms in super-strong, low-frequency laser fields

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1. Introduction

Multiphoton processes in atoms have been investigated both in its experimental and theoretical aspects for more then fifty years. Half of this time in the part of this processes named tunnel ionization, the ADK-theory [1] was one of the most propulsive theories [2–5]. Tunnel ionization is actual when Keldysh parameter [6] $\gamma \ll 1$, and for this case in [1] was obtained the result which was experimentally confirmed many times (see, for instance, papers [2–5,7,8], to mention a few), for fields up to 10^{12} W/cm^2 (throughout this paper the atomic unit system $e = \hbar = m_e = 1$ is used). Now, as the ADK-theory is extended to the case of superstrong fields [11,12], after improving the treatment of the turning point in the case of strong fields (up to 10^{14} W/cm^2) in [7], we are investigating the behavior of the corrections to the turning point in ADK-theory due to Coulomb interaction in the case of the superstrong fields up to 10^{17} W/cm^2 (which is just over the atomic field strength – 10^{16} W/cm^2, and just bellow the field strength at which relativistic effects become predominant i.e. 10^{18} W/cm^2 [13]).

Using Landau-Dykhne adiabatic approximation [8–11] the ADK-theory [1] starts with the transition amplitude between initial and final states ($E_f > E_i$ on real axes)

$$A_{if} = \exp\left\{ i \int_{t_1}^{\tau} \omega_{if}(t)\, dt \right\} , \qquad (1)$$

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where ω_{if} is frequency of the transition from i - initial to f - final state in the presence of the external field, and τ is the complex turning point in the time plane.

The complex turning point is obtained from equation, which is classically forbidden

$$\omega_{if}(\tau) = 0\,. \tag{2}$$

Also, the transition rate $i \to f$ is given by expression [9,12]

$$W_{if} = |A_{if}|^2 = \exp\{-2\,\mathrm{Im}\,\delta S(\tau)\}\,. \tag{3}$$

In the case of the short-range potential [1,5,9,12], the estimation of the probability of ionization of atoms using (3) is carried along taking into account the approximation (Keldysh approximation) which states that this kind of potential does not affect the energy of the final state f of the ejected electron in the electromagnetic field, because the electron is far enough from the nucleus. When the Coulomb potential is taken into account, it can be treated as a perturbation to the energy of the final state [1,10]. Yet, originally [1,10], the Coulomb potential in this kind of estimation was not included into calculating the turning point from (2). This was done in [7], but only for the fields below the atomic field (up to 10^{14} W/cm^2). Now we are extending our calculation that included the Coulomb correction into the estimating the turning point to the fields that are much stronger (up to 10^{17} W/cm^2), which is justified by the results of [11,12]. That results in the shift of the position of the turning point τ. This paper is dealing with the influence of that shift on the ionization probability for atoms in the low-frequency electromagnetic field of superstrong lasers.

2. Influence of the Coulomb interaction on the complex turning point

The method for calculating the probability of tunnel ionization using the Landau-Dykhne adiabatic approximation is given in book [9]. It begins with the equation (2), given as

$$E_f(\tau) = E_i(\tau)\,, \tag{4}$$

where $E_i(\tau)$, $E_f(\tau)$ are the initial and final energy, respectively, in the external electromagnetic field, and τ is the complex time, related to the turning point. Because external field F is much smaller than the atomic field F_{at}, we will take in consideration its influence only on final state, while assuming that initial state is non-perturbed, so it follows

$$\frac{1}{2}\left(p - \frac{F}{\omega}\sin\omega\tau\right)^2 = -E_i\,. \tag{5}$$

As Coulomb term in (5) is small compared to other terms, we will be using iteration. In approximation of zero order, only the external electric field is taken into account

$$\frac{d^2 z}{dt^2} = -F\cos\omega t\,,$$

which, after integration and remembering that $z = \eta/2$ (we are using parabolic coordinates, as in [9]), gives

$$\eta(\tau) = -2i\sqrt{2E_i}\tau - \frac{2F}{\omega^2}(1-\cos\omega\tau)\,. \tag{6}$$

First term in (6) was chosen in such a way as to insure that, at the initial time $t = 0$, energy of the electron equals atomic energy - E_i. By neglecting a second term in (6), as was done earlier in ADK theory [1], one has $\eta(\tau_0) = -2i\sqrt{2E_i}\tau_0$, where τ_0 is the turning point in the zero-order approximation

$$\tau_0 = \frac{p + i\sqrt{2E_i}}{F}\,. \tag{7}$$

But, if we form a power series of cosine: $\cos\omega\tau \approx 1 - \omega^2\tau^2/2$, expression (6) becomes

$$\eta(\tau) = -2i\sqrt{2E_i}\tau + F\tau^2\,. \tag{8}$$

By following an iteration procedure, we put τ_0 instead of τ and obtain

$$\eta(\tau_0) = \frac{p^2 + 2E_i}{F}\,. \tag{9}$$

Now, let us go back to expression (5): because external field is low-frequential ($\omega \ll \omega_{at}$), we are allowed to expand sine in power series, $\sin\omega\tau \approx \omega\tau$; we get a following expression

$$p - F\tau = -i\sqrt{2E_i}\sqrt{1 - \frac{1}{\eta(\tau)}\frac{Z}{E_i}}\,. \tag{10}$$

If, in expression (10), we substitute $\eta(\tau_0)$, because of iteration, we will obtain

$$p - F\tau = -i\sqrt{2E_i}\sqrt{1 - \frac{F}{p^2 + 2E_i}\frac{Z}{E_i}}\,. \tag{11}$$

As Coulomb correction under the root of the above expression is insignificant compared with ionization potential E_i there follows another expanding: $\sqrt{1-x} \approx 1 - x/2$, which gives

$$p - F\tau = -i\sqrt{2E_i}\left(1 - \frac{1}{2}\frac{F}{p^2 + 2E_i}\frac{Z}{2E_i}\right)\,,$$

i.e.

$$\tau = \frac{p}{F} + \frac{i\sqrt{2E_i}}{F}\left(1 - \frac{Z}{2E_i}\frac{F}{p^2 + 2E_i}\right)\,,$$

and, finally, an expression for Coulomb-correction-included turning point, already obtained in [7]

$$\tau = \frac{p + i\sqrt{2E_i}}{F} - \frac{iZ}{(p^2 + 2e_i)\sqrt{2E_i}}\,. \tag{12}$$

3. Estimation of pre-exponential part of the expression for a rate of tunneling ionization using a newly obtained turning point

The time has come for us to examine the influence of turning point (12) on a pre-exponent obtained earlier in ADK-theory. We start by including a Coulomb interaction into the time-dependant part of the action, which will lead to the following expression for energy of the final state

$$E_f(t) = \frac{1}{2}(p - Ft)^2 - \frac{2n_2 - 1}{n^*\eta(t)}\,, \tag{13}$$

where $\eta(t) = -2i\sqrt{2E_i} + Ft^2$, n_2 is one of parabolic quantum numbers that define a given state, and $n^* = Z/\sqrt{2E_i}$ is an effective principal quantum number – all of those follow from our expressing a Coulomb interaction in parabolic coordinates.

And so, Coulomb interaction gives the following part of the action

$$\delta S_c = -\int\limits_0^\tau \frac{(2n_2+1)\sqrt{2E_i}}{2\eta(t)Z}dt\,. \tag{14}$$

which we shall divide into two parts [7]

$$\delta S_c = \delta S_0 + \delta S_a + \int\limits_0^{t_a} + \int\limits_{t_a}^{\tau}\,, \tag{15}$$

where t_a represents time related to arbitrary turning point $\eta_a = 2r_a$, that we invoked because, at this distance from atom, an influence of atomic residue on ejecting electron is already small, and an external field can still be neglected. Now, in a region $\eta < \eta_a$, which corresponds to time $t < t_a$, quantum effects are very strong, so it should be treated in a completely different manner than region $\eta > \eta_a$, which corresponds to time $t > t_a$.

Corresponding gain to the action by δS_0 can be obtained using semi-classical approximation, and by taking into account that, for $t < t_a$, the wave function can be treated as unperturbed atomic function

$$\mathrm{Im}\,\delta S_0 = -\frac{2n^*-1}{2}\ln\left(\frac{2Ze}{n^{*2}}\sqrt{2E_i}t_a\right)\,.$$

Analogous gain to the action by δS_a can be obtained by integration of

$$\delta S_a = \int\limits_{t_a}^{\tau}\frac{(2n_2+1)\sqrt{2E_i}}{\eta(t)}dt\,.$$

After substituting a turning point $\eta(\tau) = -2i\sqrt{2E_i}\tau + F\tau^2$ and a few elementary transformations, we have

$$\delta S_a = i\frac{2n_2+1}{2}\ln\left(\frac{\tau - \frac{2i\sqrt{2E_i}}{F_0}}{\tau}\,\frac{t_a}{t_a - \frac{2i\sqrt{2E_i}}{F_0}}\right)\,.$$

Now, we shall include here an expression for Coulomb-corrected turning point (12); after rather cumbersome but pretty much straightforward procedure, we obtain expression for imaginary part

$$\mathrm{Im}\,\delta S_a = \frac{2n^*-1}{2}\times \tag{16}$$

$$\times\left\{\left[1 + \frac{2ZF}{(p^2+2E_i)^2} + \frac{Z^2F^2}{2E_i\,(p^2+2E_i)^3}\right]\frac{Fn^{*2}}{4Ze2E_i}\right\}\,,$$

where we used a fact that $2n_{2\max} = 2n^* - 2$.

Finally, for the ionization probability one has (S_{sr} being the part of the action due to the short-range potential)

$$W = \exp\left(-2\,\mathrm{Im}\,\delta S_{sr}\right)\exp\left(-2\,\mathrm{Im}\,\delta S_c\right) = \tag{17}$$

$$= \left[\frac{4Z^3e}{Fn^{*4}}\frac{1}{1+\frac{2ZF}{(p^2+2E_i)^2}+\frac{Z^2F^2}{2E_i(p^2+2E_i)^3}}\right]^{2n^*-1}\times$$

$$\times\exp\left(-\frac{2}{3}\frac{Z^3}{Fn^{*3}}\right)\,.$$

In expression (17) for ionization probability W, the second rational term in the parentheses is a correction for the Coulomb interaction, which we obtained. For the fields up to 10^{12} W/cm^2, the correction is small and could be neglected (for instance, in the case of potassium ionization in the laser field of 10^{12} W/cm^2 [4], it is 0.10876478), but for greater fields (we used fields up to 10^{14} W/cm^2 $< I_{at} \sim 10^{16}$ W/cm^2, because at that time the ADK-theory was not extended to the fields that are greater than the atomic [11,12]) this correction gains in amount (for instance, 10^{14} W/cm^2 gives 1.340258, and 10^{17} W/cm^2 gives 314.185).

If plotted for the fields from 10^{12} W/cm^2 to 10^{17} W/cm^2 (because for 10^{18} W/cm^2 and higher fields relativistic effects become predominant [13], and, also, it is extremely difficult to obtain so strong laser pulses as continuous, and for higher intensities that is even impossible, for the moment), probability (17) gives behaviour which were predicted many times theoretically, but with rather poor experimental support yet (see [11]). One has, after rapid increase of the probability of ionization of atoms, until the field intensities of 10^{14} W/cm^2, sudden decreasing at intensities of order 10^{14} W/cm^2 (which can be explained by the ionization via tunneling effect of all available electrons in the last orbits which, in the cases of alkali metals or noble gases, gives nuclear charge $Z > 10$), then a saturation at very low level of probability for fields from 10^{15} W/cm^2 to 10^{17} W/cm^2 - see Fig. 1. Because $Z > 10$ the Keldysh approximation is still valid [11], we can use ADK-theory to describe ionization of electrons from deeper orbits.

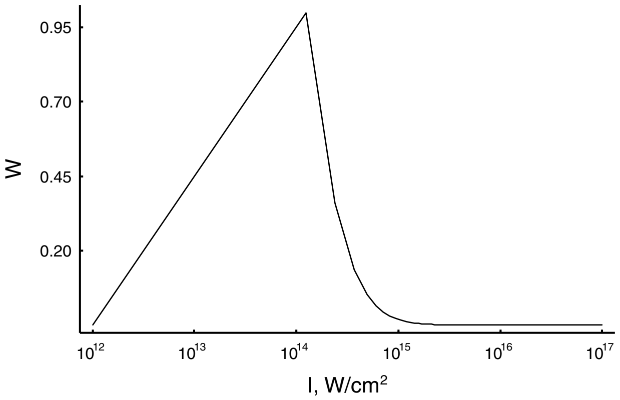

Figure 1 Ionization probability plotted vs. intensity of the field. I is in a.u. system, and W is in arbitrary units

4. Conclusion

For short-range potential, estimation of probability of ionization of atoms was made, based on assumptions of Keldysh approximation [6], that short-range potential does not affect energy of the final state of ejected electron, when it leaves the atom. Coulomb potential is then treated as perturbation of final state energy. This was done in [7], though only for fields with intensities below those of the atomic field. But as the ADK-theory was recently extended to the case of superstrong fields [11,12], our calculations now include extension of potential range up to 10^{17} W/cm^2, that leads to the shift of position of the turning point τ, which then influences ionization probability for atoms in the low-frequency electromagnetic field of superstrong lasers.

Finally, we can conclude that, for the fields whose intensities vary from 10^{12} W/cm^2 to 10^{17} W/cm^2, probability given by (17) shows behaviour which was predicted many times theoretically (but with rather poor experimental support so far (see [11]), i.e. it shows sudden decrease at laserfield intensities of order 10^{14} W/cm^2, and then a saturation at very low level of probability for fields from 10^{15} W/cm^2 to 10^{17} W/cm^2, which is, as mentioned above, all applicable only to multi-charged ions with the ion charge larger than ten and with at least one electron left in a bound state.

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