298

Abstract: Estimation of transition rate of ionization of atoms for short range potential, based on assumptions of Keldysh approximation, shows that short-range potential does not affect the energy of the final state of ejected electron, when it leaves the atom. Coulomb potential is then treated as perturbation of final state energy leading to the ADK-theory. But Coulomb interaction was not originally included in calculating the turning point. This we corrected in [1], though only for fields with intensities below those of the atomic field. But as the ADK-theory was recently extended to the case of superstrong fields, our calculations now include extension of potential range up to 10^{17} W/cm²; this leads to the shift of position of the turning point τ , which then influences transition rates for atoms in the low-frequency electromagnetic field of superstrong lasers. On Figs. 1 and 2 obtained from our calculations the value of Z was switched from 1 to 10 at field intensity of 5×10^{13} W/cm² (atomic unit system), which is done for the first time ever. The second figure also indicates an enormous activity at fields 10^{13} W/cm², which is due to strong tunneling effect for this energies of laser field. After that, there is saturation at a very low level of transition rate for fields from 10^{15} W/cm² to 10^{17} W/cm².



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Transition rate dependence on the improved turning point in ADK-theory

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1. Introduction

Ever since the multiphoton ionization was discovered and the first theoretically framework for it was layed down [2], there has been works devoted to the low frequency case. Yet in the last two decades of XX century, the ADK- theory [3] was among the theories which were most often used in that area of research [1,4].

Putting it briefly, for short-range potential [5,6], estimation of transition rate of ionization of atoms was made, based on assumptions of Keldysh approximation [2] – that short-range potential does not affect energy of the final state of ejected electron, when it leaves the atom. Coulomb potential was then treated as perturbation of final state energy thus leading to the ADK-theory [3]. In [1,7] it was noticed that, when constructing ADK-theory, the Coulomb interaction was not included into calculations for the turning point τ . This was not so important at that time as the laser fields in experiments were of the order of 10^{12} W/cm² [7], and corresponding corrections (see Eq. (2) and comment under it), were of the order of 0.10876478.

Yet as the laser fields in experiments were constantly increasing, there emerged a neccessity for ADK-theory to include these cases as well. Therefore the ADK-theory was

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Figure 1 W_{ADK} plotted vs. intensity of field

recently extended to the case of superstrong fields [8,9], and our calculations now include extension of potential range up to 10^{17} W/cm²; this leads to the shift of position of the turning point τ , which then influences transition rates for atoms in the low-frequency electromagnetic field of superstrong lasers.

Throughout this paper the atomic unit system ($e = \hbar = m_e = 1$) will be used.

In paper [1] it was shown that correct calculation for the turning point in ADK- theory gives, instead of initial expression for the turning point in the zero-order approximation

$$\tau_0 = \frac{p + i\sqrt{2E_i}}{F} \,,$$

improved expression for the turning point, one that now includes the Coulomb interaction:

$$\tau = \frac{p + i\sqrt{2E_i}}{F} - \frac{iZ}{\left(p^2 + 2E_i\right)\sqrt{2E_i}}\,,$$

where Z is the nucleus charge after the tunnel ionization of the atom, which is occurring in the electromagnetic field of the strong low frequency laser (CO₂, for instance).

2. The corrected transition rate

As a consequence of this, we have shown in [1,7] that corresponding transition rates which we shall denote as W_{ADK} and W_{CADK} ("C" standing for Coulomb correction) are, respectively:

$$W_{ADK} = \left(\frac{4Z^3e}{Fn^{*4}}\right)^{2n^*-1} \exp\left(-\frac{2Z^3}{3Fn^{*3}}\right),$$
 (1)

which is already known result [3,4], and

$$W_{CADK} = \tag{2}$$



Figure 2 W_{CADK} plotted vs. intensity of field

$$= \left[\frac{1}{1 + \frac{2ZF}{(p^2 + 2E_i)^2} + \frac{Z^2F^2}{2E_i(p^2 + 2E_i)^3}}\right]^{2n^* - 1} W_{ADK} \,.$$

In expression (2) for transition rate W_{CADK} , (which is the new result, published in [1]) the second rational term in the parentheses is a correction due to the Coulomb interaction. For the fields up to 10^{12} W/cm², the correction:

$$K = K_1 + K_2 = \frac{2ZF}{(p^2 + 2E_i)^2} + \frac{Z^2F^2}{2E_i(p^2 + 2E_i)^3}$$

is small and could be neglected (for instance, in the case of potassium ionization in the laser field of 10^{12} W/cm², its value is 0.10876478, [1]). In [1], the fields that were used had intensities of 10^{14} W/cm² < $I_{at} \sim 10^{16}$ W/cm². But for stronger fields the correction gains in amount (for instance, other conditions unchanged, 10^{14} W/cm² gives 1.340258, and 10^{17} W/cm² gives 314.185).

When plotted for the fields from 10^{10} W/cm² to 10^{17} W/cm² (because for 10^{18} W/cm² and higher fields relativistic effects become predominant [10]; also, it is extremely difficult to obtain so strong laser pulses as continuous – for higher intensities that is even impossible, for the moment), transition rate (1) shows behavior that was predicted many times theoretically [4]. Results of present calculation are shown on Figs. 1 and 2. It should be noted that on Fig. 2 is given the improved result of ADK-theory, which includes the turning point calculated with the Coulomb correction (therefore transition rate was given an index CADK: W_{CADK}); the saturation can be seen at $\sim 10^{14}$ W/cm². On Fig. 1 this saturation appears only after intensities of $\sim 10^{15}$ W/cm². Besides this, Fig. 2 shows an interesting behavior at field strength of 10^{13} W/cm², indicating that there begins a process of an intensive tunnel ionization, after which Z becomes greater than 10, which justifies our using of methods of ADKtheory. Fig. 2 also shows the saturation but at field strength of 10¹⁴ W/cm², differing from Fig. 1 by the order of magnitude.

299

On both our figures, the value of Z was switched from 1 to 10 at field intensity of 5×10^{13} W/cm² (atomic unit system), which is done for the very first time in such calculations. Reasoning for this goes as follows: Fig. 2 indicates that there exists a rapid growth in number of ejected electrons at intensity 10^{13} W/cm², due to the process of tunnel ionization, for which is known, as it is a well studied semi-classical phenomenon, that it becomes more intense for the energies near the barrier. Of course, the value of Z does not become 10 immediately (so we switched the value of Z at intensity $\sim 5 \times 10^{13}$ W/cm²), though it seems to gain that value pretty fast. On the y-axis of the graphs arbitrary units were used.

3. Final remarks

Now we can conclude that in this paper results were obtained which are pretty exciting. For it seems that after this analysis the idea of tunnel ionization of atoms in low frequency strong laser fields is gaining in its reliability, and after being indirectly confirmed through many experiments (see, for instance, [1] and the references thereof), now is helping in understanding the processes that are developed in the interaction of atoms with a low frequency superstrong laser field.

Finally, it can be noticed that, for the fields whose intensities vary from 10^{10} W/cm² up to 10^{17} W/cm², transition rates, given by Figs. 1 and 2, show behavior on Fig. 1, which was predicted many times, i.e. on Fig. 1 is shown the sudden decrease at laser-field intensities near 10^{15} W/cm² and saturation further on. On Fig. 2, which shows the case of the ADK-theory with the turning point corrected to the Coulomb interaction, this occurs near intensities of order 10^{14} W/cm². But this figure also indicates an enormous activity at fields 10^{13} W/cm², which is due to strong tun-

neling effect for these energies of laser field. After that, there is a saturation at a very low level of transition rate for fields from 10^{15} W/cm² to 10^{17} W/cm², which is, let us stress, all applicable only to multi-charged ions with the ion charge larger or equal to ten and with at least one electron left in a bound state. In our model the using of a laser whose intensity grows was assumed, thus giving atoms the opportunity to be multiply ionized via tunneling effect, while obtaining nucleus charge of $Z \ge 10$. This enabled us to apply the methods of ADK-theory in this paper.

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