Adaptive dynamic programming based optimal control for hydraulic servo actuator

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This paper considers optimal tracking control for hydraulic servo actuator with unknown dynamics. The aim is to achieve asymptotic tracking and disturbance rejection by minimizing some predefined performance index. Through the combination of adaptive dynamic programming (ADP) and internal model principle, an approximate optimal controller is iteratively learned online using measurable input/output data. Unmeasurable states are also reconstructed from input/output data. The discrete-time algebraic Riccati equation is iteratively solved by ADP approach. Simulation results demonstrate the feasibility and effectiveness of the proposed approach.

Keywords: Adaptive dynamic programming, Optimal control, Hydraulic servo actuator, Unknown dynamics, Algebraic Riccati equation

1. INTRODUCTION

The performance of hydraulic servo systems strongly depends on the control valve and spool geometry as well as their manufacturing tolerances. Without a proper model, accurate analysis of hydraulic system performance is not possible. It is well known that it is very difficult to determine a large number of physical parameters which are integral part of complex systems. Despite the fact that many system parameters are available with some reasonable accuracy, a large number of parameters are known within a certain range, while some parameters are entirely unknown because manufacturers consider these data as proprietary information. For example, precise determination of system parameters such as dimensions of certain components, leakage and friction coefficients, as well as static and dynamic friction forces due to impossibility of direct measurement or calculation causes great difficulty in control of servo actuators [1]. More precise knowledge of the system parameters increases the model quality, which causes better control performances. Recent reviews dedicated to the influence of disturbances, modeling errors, various uncertainties in the control systems in the real systems[2-3].

Adaptive dynamic programming (ADP) provides a feasible and effective way to achieve optimal control performance based on traditional or intelligent control methods. It combines the theories of dynamic programming and neural networks, trying to solve optimal control problems in dynamic programming problems using the approximating characteristic of neural networks [4-5]. Recent years the ADP also has been extended and applied to many different areas, such as robots, spacecraft and so on [6-7].

For the reason that ADP not only has unique algorithm and structure, but also overcomes the disadvantage that classical variational theory cannot deal with the optimal control problem of control variables with closed-set constraints, and also solves the curse of dimensionality problems, more and more research and application attentions have focused on it.

When the state of a system is not directly measurable and the system matrices are unknown, it is thus meaningful to resort to output feedback based ADP. An output feedback ADP methodology for discrete-time linear systems is proposed in [8]. The measurable input/output data are used to describe the state of the discretized model, and then use policy iteration (PI) and value iteration (VI) to obtain optimal control policy. Nevertheless, in order to get the unique solution in each iteration step, some exploration noise need to be added, which may influence the accuracy of solutions.

It is usually too expensive to measure directly the system states. Self-applied state estimation methods assume that the system parameters are constant. In the real world, these parameters can’t know exactly (e.g., friction coefficients, temperature, pressure, or flow). It’s also known that the dynamic behavior of complex systems can be described by a linear stochastic state-space model with online estimated dynamics [9]. Precise knowledge of system parameters and states is crucial for successful realization of many control techniques. Many modern engineering applications such as intelligent vehicles[10], microphone sensing[11], maintaining security of cyber–physical systems[12], robotic manipulation tasks[13], 2-degree-of-freedom helicopter[14], require real-time control based on linear models.

In this paper, the continuous-time linear plant is discretized due to easier practical implementation, and then, the optimal control problem is considered. An adaptive optimal output feedback strategy for the discretized model is applied for optimal control of hydraulic actuator with unknown dynamics. Simulation results demonstrate the validity and effectiveness of the proposed control approach, in which the exploration noise does not affect accuracy of the solution of discrete Riccati equation.

2. HYDRAULIC SERVO ACTUATOR

Hydraulic systems are often employed in high performance applications that require fast response and high power. These applications include high bandwidth

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control position and force. The problem is that these systems contain non-smooth nonlinearities caused by variable geometry and variable working conditions. The external load consists of the mass of external mechanical elements connected to the piston and a force produced by an environmental interaction. Schematic view of double acting, asymmetric hydraulic cylinder with connected four-way spool valve is shown on Figure 1.

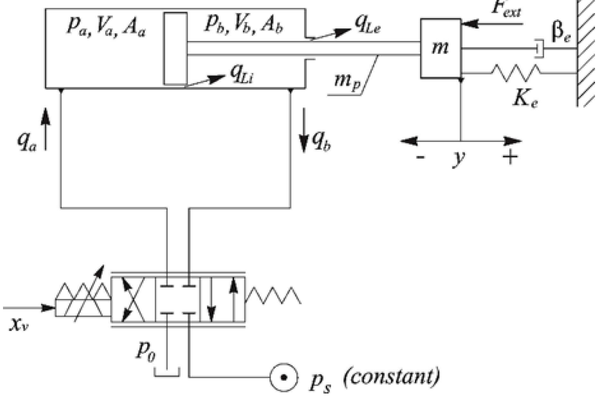

Figure 1: Schematic representation of the valve-controlled asymmetric piston

The total mass of the piston m_t includes the mass of piston rod m_p and the mass of the load m referred to the piston.

A detailed mathematical model derivation of the hydraulic servo actuator is given in [1]. Using this model, we can express the system model in the state space.

2.1. Force balance equation for piston

Applying Newton's second law to the forces on the piston, the resulting force equation is:

$$A_a p_a - A_b p_b = m_t \ddot{y} + F_f(\dot{y}) + K_e y + F_{ext} \tag{1}$$

The spool valve displacement is denoted as x_v. Pressures p_a and p_b denote the forward and the return pressure, respectively, the corresponding flows are q_a and q_b, y is the piston displacement, K_e denotes the load spring gradient, p_S is the supply pressure, and p_0 is the tank pressure. The piston displacement depends on the action of fluid pressures as well as on the load referred to the piston. This load can be seen as summing effects of inertia which comes from the total piston mass m_t, friction forces F_f, spring load forces $K_e y$ and disturbance forces F_{ext}. The total mass of the piston m_t in addition to the mass of piston rod m_p also includes the mass m of the load referred to the piston. The mass of the piston is considered together with the load mass due to the fact that, in every moment, the load directly affects on the piston. The term F_f in equation (1) describes the summing nonlinear effects of viscous, static and Coulomb friction forces of the system. The detailed analysis for the influences of friction forces can be found in [15]. The area ratio of the asymmetric piston is $\alpha = A_b / A_a$, where A_b is the effective area of the head side of the piston, and A_a is the effective area of the rod side of the piston, see Figure 1.

2.2. Pressure dynamics in cylinder chamber

Applying the continuity equation to each of the cylinder chambers yields:

$$q_a - q_{Li} = \dot{V}_a + \frac{V_a}{\beta_e} \dot{p}_a \tag{2}$$

$$q_b + q_{Li} - q_{Le} = \dot{V}_b + \frac{V_b}{\beta_e} \dot{p}_b \tag{3}$$

where β_e is the bulk modulus of the fluid, q_{Li} and q_{Le} denote the internal leakage flow and the external leakage flow, respectively. The internal leakage flow can be calculated by:

$$q_{Li} = c_{Li}(p_a - p_b) \tag{4}$$

where c_{Li} is the internal leakage flow coefficient. External leakage (leakage from each cylinder chamber to case drain or to tank) is usually neglected, $q_{Lea} = q_{Leb} = 0$.

The total fluid volumes of two cylinder sides, V_a and V_b, are given as:

$$V_a = V_{a0} + yA_a \tag{5}$$

$$V_b = V_{b0} + (L - y)\alpha A_a \tag{6}$$

where L is the piston stroke and V_{a0} and V_{b0} represent initial chamber volumes. Equations (2) and (3) can be rearranged to yield the pressure dynamics equations

$$\dot{p}_a = \frac{\beta_e}{V_a(y)}\left(q_a - A_a \dot{y} - q_{Li} - q_{Lea}\right) \tag{7}$$

$$\dot{p}_b = \frac{\beta_e}{V_b(y)}\left(q_b + \alpha A_a \dot{y} + q_{Li} - q_{Leb}\right) \tag{8}$$

2.3. Valve flow equatins

The flow through the ith valve orifice q_{svi} is described by next relation, which takes the direction of the pressure drop into account:

$$q_{svi} = q(x_v, \Delta p) = c_{vi}\, \mathrm{sg}(x_v) sign(\Delta p)\sqrt{|\Delta p|} \tag{9}$$

where $i = 1, 2, \ldots, 4$.

The function $sg(x)$ is defined by:

$$sg(x) = \begin{cases} x, & x \ge 0 \\ 0, & x < 0 \end{cases} \tag{10}$$

where $i = 1, 2, \ldots, 4$.

Discharge coefficients of valve orifices $c_{vi} > 0$, $i = 1, 2, 3, 4$ represent valve constants, which will be equal if all orifices are identical. Consider the four-way spool valve as shown in Figure 2.

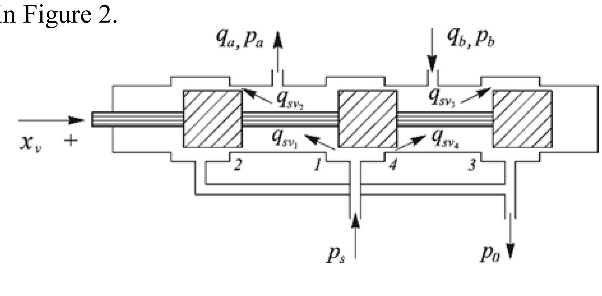

Figure 2: Four-way spool valve

The corresponding flow equations for two valve chambers can be written as:

$$q_a = q_{sv1} - q_{sv2} = c_{v_1} sg(x_v) sign(p_s - p_a)\sqrt{|p_s - p_a|} - \\ -c_{v_2} sg(-x_v) sign(p_a - p_0)\sqrt{|p_a - p_0|} \tag{11}$$

$$q_b = q_{sv3} - q_{sv4} = c_{v_3} sg(-x_v) sign(p_s - p_b)\sqrt{|p_s - p_b|} - \\ -c_{v_4} sg(x_v) sign(p_b - p_0)\sqrt{|p_b - p_0|} \tag{12}$$

2.4. Dynamic model of the hydraulic servo actuator

If state variables and input variables are defined as:

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T \triangleq [y \quad \dot{y} \quad p_a \quad p_b]^T \tag{13}$$

$$u = x_v \tag{14}$$

then a completely nonlinear model of the hydraulic system, can be expressed in a state-space form as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m_t}\left(A_a x_3 - \alpha A_a x_4 - F_f(x_2) - K_e x_1\right), \\ \dot{x}_3 &= \frac{\beta_e}{A_a x_1 + V_{a0}}\left(c_{v_1} sg(u) sign(p_s - x_3)\sqrt{|p_s - x_3|} - c_{v_4} sg(-u) sign(x_3 - p_0)\sqrt{|x_3 - p_0|} - A_a x_2 - c_{Li}(x_3 - x_4)\right) \\ \dot{x}_4 &= \frac{\beta_e}{\alpha A_a (L - x_1) + V_{b0}}\left(c_{v_3} sg(-u) sign(p_s - x_4)\sqrt{|p_s - x_4|} - c_{v_2} sg(u) sign(x_4 - p_0)\sqrt{|x_4 - p_0|} + \alpha A_a x_2 + c_{Li}(x_3 - x_4)\right) \end{aligned} \tag{15}$$

It is known that there are coefficients with random characters in the nonlinear state space model of a hydraulic servo system [16]. Taking into account that some parameter changes have random character, as well as the possibility of approximation of nonlinear models with a model with time-varying parameters, see [17], this paper proposes a linear stochastic model with time-varying parameters.

It is now more convenient to define the pressure drop across the load, or simply the load:

$$p_L = p_a - \alpha p_b \tag{16}$$

which can be seen as the "virtual" pressure required to counterbalance the friction and load forces.

Finally, after linearization of nonlinear equations (15), using previous notation which allows us to present the hydraulic servo system in more compact form, with new state vector $[x_1 \quad x_2 \quad x_3]^T \triangleq [\Delta y(k) \quad \Delta\dot{y} \quad \Delta p_L]^T$, the continuous time state-space description of the reduced order, can be obtained as [16]:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{17}$$

$$y(t) = Cx(t) \tag{18}$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \dfrac{-1}{T_m} & \dfrac{A_a}{m_p} \\ 0 & -K_d & \dfrac{-1}{T_h} \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 & K_Q \end{bmatrix}^T$.

The damping of the resonance frequency is determined by the viscous friction ($-1/T_m = -B_C/m_p$) and the leakage ($-1/T_h$), where $T_m = m_t/B_C$. Other useful quantities are the hydraulic capacitance $C_h = \left(\dfrac{\beta_e}{V_A} + \alpha^2 \dfrac{\beta_e}{V_B}\right)$, $K_d = \dfrac{A_1}{C_h}$, valve flow-pressure coefficients K_{Qp} , valve flow gains K_{Qx} , as well as pressure sensitivities $K_{px} = K_{Qx}/K_{Qp}$ and damping ratio:

$$K_j = \left(R + B_d^T P_{j-1} B_d\right)^{-1} B_d^T P_{j-1} A_d \tag{19}$$

3. OPTIMAL PROBLEM FORMULATION

For practical implementation in the hydraulic actuator control system, we will consider the discretized system described by:

$$x_{k+1} = A_d x_k + B_d u_k \tag{20}$$

$$y_k = Cx_k \tag{21}$$

in which $A_d = e^{Ah}$, $B_d = \int_0^h \left(e^{A\tau} d\tau\right) B$ and $h > 0$ is the sampling period, assuming $\omega_h = 2\pi/h$ is non-pathological sampling frequency [18]. In other words, one cannot find any two eigenvalues of A with equal real parts and imaginary parts that differ by an integral multiple of ω_h. The state, input, and output vector at the sampled instant kh are x_k , u_k , y_k , respectively. Then, both (A_d, C) and $(A_d, Q^{1/2}C)$ are observable and (A_d, B_d) is controllable. Cost for (20)-(21) is:

$$J_d(x_0) = \sum_{j=0}^{\infty} y_j^T Q y_j + u_j^T R u_j \tag{22}$$

The optimal control law minimizing (un) is

$$u_k = -K_d^* x_k \tag{23}$$

where discrete optimal feedback gain matrix is $K_d^* = \left(R + B_d^T P_d^* B_d\right)^{-1} B_d^T P_d^* A_d$, and P_d^* is the unique symmetric positive definite solution to

$$A_d^T P_d^* A_d - P_d^* + C^T Q C - A_d^T P_d^* B_d K_d^* = 0 \tag{24}$$

Up to now, this known optimal control design method is mainly applicable to low order simple linear systems. In fact, for high order large scale systems, it is usually difficult to directly solve P_d^* from (24), which is nonlinear in P_d. Nevertheless, many efficient algorithms have been developed to numerically approximate the solution of (24). One of such algorithms was developed by Hewer [19]. By iteratively solving the Lyapunov equation

$$\left(A_d - B_d K_j\right)^T P_j \left(A_d - B_d K_j\right) + C^T Q C + K_j^T R K_j = 0 \tag{25}$$

which is linear in P_j , and updating K_j by

$$K_j = \left(R + B_d^T P_{j-1} B_d\right)^{-1} B_d^T P_{j-1} A_d \tag{26}$$

the solution to the nonlinear equation (24) is numerically approximated. It has been concluded that sequences $\{P_j\}_{j=0}^{\infty}$ and $\{K_j\}_{j=0}^{\infty}$ computed from this algorithm converge to P_d^* and K_d^*, respectively. Moreover, for $j = 0,1,\ldots$, $A_d - B_d K_j$ is a Schur matrix.

It should be noted that Hewer’s algorithm is model based policy iteration (PI) algorithm which cannot be implemented when the system matrices are all unknown, since it is an offline algorithm relying on system parameters. In order to implement it online, we will develop an adaptive optimal control algorithm for the discretized system (20)-(21) via output feedback which does not rely on the knowledge of the system matrices.

4. ADAPTIVE OPTIMAL CONTROLLER DESIGN

Just like in [8] the extended state equation using input/output sequences on time horizon can be written as $[k-N, k-1]$:

$$\begin{aligned} x_k &= A_d^N x_{k-N} + V(N)\bar{u}_{k-1,k-N} \\ \bar{y}_{k-1,k-N} &= U(N)x_{k-N} + T(N)\bar{u}_{k-1,k-N} \end{aligned} \tag{27}$$

where

$$\bar{u}_{k-1,k-N} = [u_{k-1}^T \quad u_{k-2}^T \quad \ldots \quad u_{k-N}^T]^T$$
$$\bar{y}_{k-1,k-N} = [y_{k-1}^T \quad y_{k-2}^T \quad \ldots \quad y_{k-N}^T]^T$$
$$V(N) = [B_d \quad A_d B_d \quad \ldots \quad A_d^{N-1} B_d]$$
$$U(N) = [(CA_d^{N-1})^T \quad (CA_d)^T \quad \ldots \quad C^T]^T$$
$$T(N) = \begin{bmatrix} 0 & CB_d & CA_dB_d & \cdots & CA_d^{N-2}B_d \\ 0 & 0 & CB_d & \cdots & CA_d^{N-3}B_d \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & CB_d \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

and $N = \max(\rho_u, \rho_v)$ is the observability index, where ρ_u is the minimum integer which can make $U(\rho_u)$ full column rank, ρ_v is the minimum integer which can make $V(\rho_v)$ full row rank.

A lemma about the uniqueness of state reconstruction is shown below.

Lemma 1. Given a controllable and observable system (20) -(21), the system state is obtained uniquely in terms of measured input/output sequences by

$$x_k = \Theta z_k \tag{28}$$

where $M_u = V(N) - M_y T(N)$, $M_y = A_d^N U^+(N)$, $\Theta = \begin{bmatrix} M_u & M_y \end{bmatrix}$, $z_k = \begin{bmatrix} \bar{u}_{k-1,k-N}^T & \bar{y}_{k-1,k-N}^T \end{bmatrix}^T \in \mathbb{R}^q$, where $q = N[\dim(u) + \dim(y)]$.

Now, based on (25)-(26) online output feedback learning strategy for linear discretized system (20)-(21) can be derived. The discrete model (20) can be rewritten as

$$x_{k+1} = A_j x_k + B_d\left(K_j x_k + u_k\right) \tag{29}$$

where $A_j = A_d - B_d K_j$. Letting $\bar{K}_j = K_j \Theta$ and $\bar{P}_j = \Theta^T P_j \Theta$, from (25) and (29) it follows that,

$$\begin{aligned} &z_{k+1}^T \bar{P}_j z_{k+1} - z_k^T \bar{P}_j z_k = \\ &\phi_\kappa^1 vec(\bar{H}_j^1) + \phi_\kappa^2 vec(\bar{H}_j^2) - \left(y_k^T Q y_k + z_k^T \bar{K}_j^T R \bar{K}_j z_k\right) \end{aligned} \tag{30}$$

in which $\bar{H}_j^1 = B_d^T \bar{P}_j B_d$, $\bar{H}_j^2 = B_d^T \bar{P}_j A_d \Theta$,

$\phi_\kappa^1 = u_k^T \otimes u_k^T - (z_k^T \otimes z_k^T)(\bar{K}_j^T \otimes \bar{K}_j^T)$,

$\phi_\kappa^2 = 2\left[(z_k^T \otimes z_k^T)(I_q \otimes \bar{K}_j^T) + (z_k^T \otimes u_k^T)\right]$

This assumption is related to the condition of persistent excitation in adaptive control theory [20]. Then, $\bar{K}_{j+1}$ can be computed as

$$\bar{K}_{j+1} = \left(R + \bar{H}_j^1\right)^{-1} \bar{H}_j^2 . \tag{31}$$

Here, (30) is called policy evaluation, which is used to uniquely solve $\bar{P}_j$, and (31) is policy improvement (PI), which is used to update control gain $\bar{K}_{j+1}$. Then, we present our output feedback adaptive optimal control algorithm.

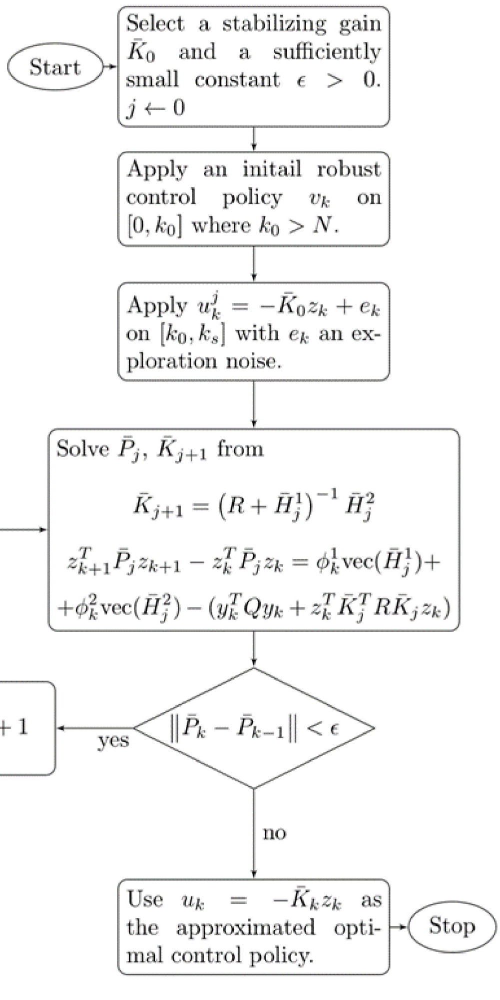

Figure 3: Flowchart of adaptive optimal controller design

5. SIMULATION RESULTS

A basic prerequisite for energy savings in processes of production, transportation, and energy consumption is a high-quality synthesis of optimal control algorithm. In this section, we conduct simulations on the valve-controlled hydraulic actuator to show the effectiveness of the output-feedback ADP control algorithm in the case with unknown system matrices and unmeasurable states.

Through iterative calculation, the approximated optimal control gain and performance index for the discrete-time system can be obtained. Furthermore, the discrete control policy is implemented on the continuous plant by zero-order holder. Adopted sampling time is $h = 0.1s$.

The model parameters are: $\beta_e = 2.1 \cdot 10^8 Pa$ is the bulk modulus of the fluid, $K_e = 10^{-1}$ denotes the load spring gradient, F_{ext} represents the load force disturbance on the piston, $p_S = 40bar$ is the supply pressure, and $p_0 = 1.7bar$ is the tank pressure, $V_{a0} = V_{b0} = 8 \cdot 10^{-6} m^3$ represent initial chamber volumes, $L = 1m$ is the piston stroke, $m = 20kg$ is the piston mass.

The area ratio of the asymmetric piston is $\alpha = A_b / A_a$, where $A_b = 2.36 \cdot 10^{-4} m^2$ is the effective area of the head side of the piston, and $A_a = 4.91 \cdot 10^{-4} m^2$ is the effective area of the rod side of the piston. Discharge coefficients of valve orifices $c_{vi} > 1.14$, $i = 1,2,3,4$ represent valve constants, and $c_{Li} = 5 \cdot 10^{-14}$ is the internal leakage coefficient.

Behavior of the control algorithm will be considered on discretized continuous linear model (17)-(18). For the purpose of simulation, Q and R are chosen to be identity matrices. The observability index is $N = 3$ and the stopping criterion is $\varepsilon = 0.1$. By solving of the discrete-time Riccati equation (24), we get the optimal values $\bar{P}_d^*, \bar{K}_d^*$. More precisely

$$\bar{P}_d^* = \begin{bmatrix} 269.1657 & -377.7014 & 139.5220 & 7.7063 & -276.6863 & 293.3140 \\ -377.7014 & 535.9737 & -199.3177 & -10.0756 & 387.6731 & -419.0086 \\ 139.5220 & -199.3177 & 74.4521 & 3.7667 & 3.7667 & 156.5180 \\ 7.7063 & -10.0756 & 3.7667 & 1.5862 & -7.2146 & 7.9186 \\ -276.6830 & 387.6731 & -142.9420 & -7.2146 & 284.9547 & -300.5013 \\ 293.3140 & -419.0086 & 156.5180 & 7.9186 & -300.5013 & 329.0404 \end{bmatrix} \quad (32)$$

$$\bar{K}_d^* = \begin{bmatrix} 3.3615 & -4.4696 & 1.6358 & 0.3700 & -3.3307 & 3.4388 \end{bmatrix} \quad (33)$$

The input/output data are collected from 0.8 to 4 seconds, and the PI is started from t = 4s. The online information of input and output are collected in the whole process and the adaptive optimal controller is also computed iteratively. After 6 iterative iterations, we obtain the derived approximate optimal values as shown below:

$$\bar{P}_6^* = \begin{bmatrix} 269.1659 & -377.7057 & 139.5221 & 7.7082 & -276.6863 & 293.3106 \\ -377.7057 & 535.9646 & -199.3158 & -10.0757 & 387.6722 & -419.0119 \\ 139.5221 & -199.3158 & 74.4526 & 3.7667 & -142.9427 & 156.5181 \\ 7.7082 & -10.0757 & 3.7667 & 1.5861 & -7.2146 & 7.9186 \\ -276.6863 & 387.6722 & -142.9427 & -7.2146 & 284.9548 & -300.5016 \\ 293.3106 & -419.0119 & 156.5181 & 7.9186 & -300.5016 & 329.0404 \end{bmatrix} \quad (34)$$

$$\bar{K}_6^* = \begin{bmatrix} 3.3615 & -4.4695 & 1.6358 & 0.3700 & -3.3306 & 3.4387 \end{bmatrix} \quad (35)$$

Figures 4-6 depict the plots of input, output and states of the hydraulic actuator. At t = 4s, the approximated optimal control gain is thus obtained by applied optimal ADP and is implemented online.

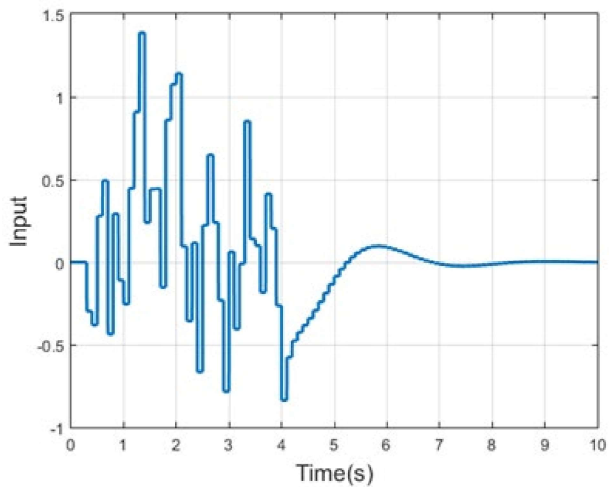

Figure 4: Trajectory of input

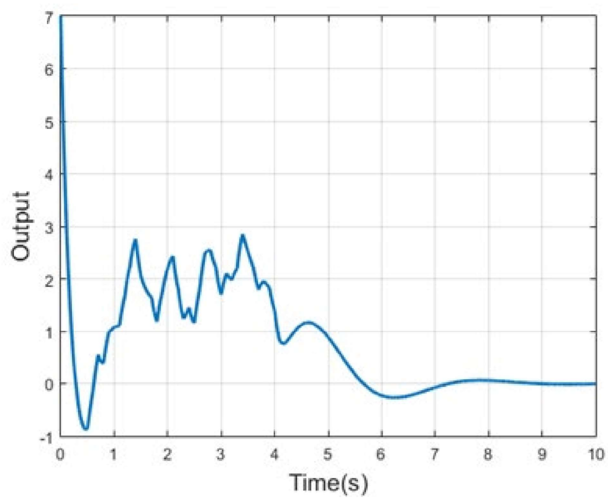

Figure 5: Trajectory of output

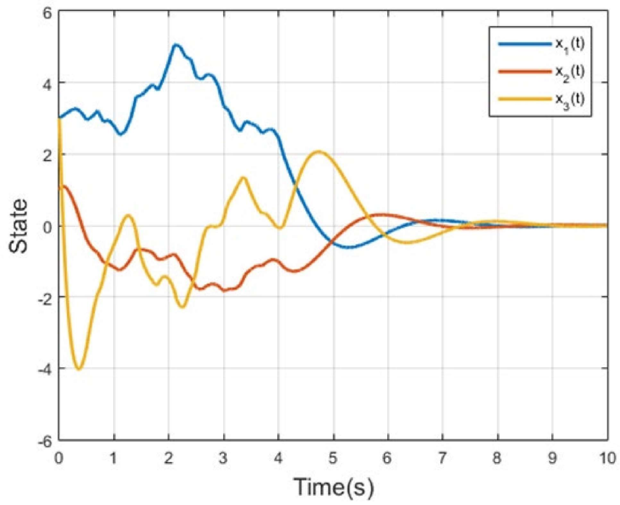

Figure 6: Trajectory of states

6. CONCLUSION

In this paper, ADP based optimal controller design has been considered for the hydraulic servo actuator with completely unknown dynamics. Applied sampled-data adaptive optimal control strategy based on the discretized model and output feedback has been shown as useful tool in this cases. It should be noted that exploration noise does not affect accuracy of the solution of discrete Riccati equation. Simulation results demonstrate the validity and effectiveness of the proposed control approach.

ACKNOWLEDGEMENTS

This research has been supported by the Serbian Ministry of Education, Science and Technological Development under grant No. 451-03-9/2021-14/200108 and CNPq under grant No. 304032/2019-0.

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