Design of Fixed Order H_∞ Controllers with Specified Settling Time using D-Decomposition

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Abstract: *The H_∞ control theory has the high level. But application in industry, owing the complexity of theory, is limited. Controller design is based on complex numerical procedure (Nevannlina–Pick algorithm) and given parameters does not have physical meaning. Also, the order of the controller is high (at least as an order of system). Owing that facts in this paper we propose simple interactive procedure for H_∞ PI controller design. The procedure is simple and is based on D-decomposition. In comparison with original H∞ controller here we introduce constraint in the form of specification of settling time. The program for procedure of H∞ PI controller design is simple and based on MATLAB (Robust Control Tool Box). Given PI controller is applied for control of CSTR (continuous stirred–tank reactor). The process is described as linear model with time delay. In this paper the time-delay approximated with second order Pade approximation. Finally, designed controller provides nominal performance for control systems.*

Keywords: H_∞ Controller, Settling time, D- Decomposition, H_∞ PI Controller

1. INTRODUCTION

Robust control theory based on the mathematical theory of infinite-dimensional Hardy space has reached a high level [1-2]. There are applications of this theory in different areas. However, there are quite serious difficulties in including this methodology in a wide application in the industry. First, the controller design procedure is very complicated and it is based on numerical procedures (Nevanlinna-Pick procedure [3-4]). Secondly, the controller is of high order (as a rule, it is higher order than the order of the model) and its parameters have no physical significance. Third, the dispersion of controller parameters is large (parameter values differ by many size orders). The result of this fragility is controller because the numerical implementation introduces errors in the implementation of digital controllers. For these reasons and the fact that staff in the industry is not able to understand H∞ control theory there is a natural resistance to the application of this theory.

On the other hand, there is dominant application of PI (PID) controllers in the industry. Here, the regulator parameters have physical meaning, they are easy to set up and suitable, in complexity, for personnel which lead industrial processes. Therefore, the design procedure of robust PI controllers, which satisfy the optimization criterion H∞, is proposed in this paper. Also, it is introduced the restriction which the controller must satisfy, and it is the settling time. The design procedure of H∞ PI controller is based on the application of D - decomposition and it is independent of the model order. A network of points for the stability area of the system is constructed, provided that it is a PI controller. At each point of the network, the PI controller is designed and using the MATLAB (Robust Control Toolbox), the minimum of H_∞ criteria is determined. Based on this, it is formed a catalog of a reference response, a catalog of a load disturbance response and minimum criteria on which basis the operator, through an interactive procedure, selects the controller that suits the best.

A simplified design procedure of the H∞ controller based on the input-output model is proposed in reference [5]. The problem is reduced to solving two Diphantine equations. A similar procedure to this one presented in the paper is proposed in the reference [6]. The difference is that the approximation of the delay is not carried out. Reference [6] is dedicated to the design of H∞ controller for communication computer networks. For the design methods of the H∞ controller, a suitable model [8-9] is also needed. A new approach to machine learning – based identification is proposed in the reference [10].

This paper discusses the regulation of the CSTR (Continuous Stirred-Tank Reactor) using the H∞ PI controller. The process is described as a linear process with a delay [7, 11-14]. The problem is, subject to a limitation on relative stability, considered in the reference [15]. When at the same time, there are limitations on the relative stability and the settling time, the design of standard PI controllers, is discussed in references [16-17]. In all of these cases, D-decomposition was used [18-20].

2. MATHEMATICAL MODEL OF A CSTR

Continuous stirred-tank reactors have widespread application in industry and embody many features of other types of reactors [21].

Consider a simple liquid-phase, irreversible chemical reaction where chemical species A reacts to form species B. The reaction can be written as

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$$A \longrightarrow B \tag{1}$$

It is supposed that the rate of reaction is first-order with respect to the component A

$$r = kc_A \tag{2}$$

where r is the rate of reaction A per unit volume, k is the reaction rate constant (with units of reciprocal time and c_A is the molar concentration of species A. For single phase reactions, the constant k is given by the Arrhenius equation

$$k = k_0 \exp\left\{-\frac{E}{RT}\right\} \tag{3}$$

where k_0 is constant, E is the activation energy and R is the gas constant. The parameters k_0 and E are determined by fitting experimental data. The last two equations can be considered to be semi-empirical relations.

The schematic diagram of the CSTR is shown in Fig. 1.

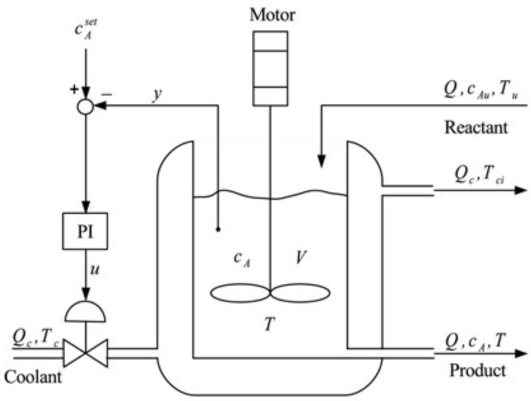

Figure 1. CSTR with cooling jacket

The designations in the figure have the following meanings:

c_{Au} - concentraction of the reactant in the feed CSTR [kg·mole/m^3]

c_A - concentraction of the reactant in the feed CSTR [kg·mole/m^3]

c_A^{set} - set point for the concentraction [kg·mole/m^3]

T_u - temperature of the feed [°C]

T_c - temperature of the coolant [°C]

T_{ci} - output temperature of the coolant [°C]

T - temperature in the CSTR [°C]

Q - flow of the reactant and product [m^3/s]

Q_c - flow of the coolant [m^3/s]

y - measurement of the concentration [kg·mole/m^3]

u - input signal

The CSTR model development [13-14] is based on three assumptions:

(i) The CSTR is perfectly mixed

(ii) the mass densities of the feed and product streams are equal and constant

(iii) the liquid volume V in the CSTR is kept constant

The mathematical model of CSTR is taken from [11-12] and has the form

$$G(s) = \frac{Y(s)}{U(s)}$$
$$G(s) = \frac{1.308}{(13.515s+1)(6.241s+1)}e^{-4.896s} \tag{4}$$

The goal of the control systems is to control the CSTR composition (y) by manipulating the cool rate through the control signal (u). Different control strategies are presented in [13].

In this paper we shall approximate the element of delay by a second-order Padé approximation. The general form of Padé approximation, according to [22-23], is

$$e^{-T_d s} = \left(\frac{1 - \frac{sT_d}{2n} + \frac{1}{3}\left(\frac{sT_d}{2n}\right)^2}{1 + \frac{sT_d}{2n} + \frac{1}{3}\left(\frac{sT_d}{2n}\right)^2} \right)^n, T_d = -4.896 \tag{5}$$

In this paper, the approximation for n=2 is used and, in that case, the transfer function or the process is:

$$G(s) = \frac{1.308}{(13.515s+1)(6.241s+1)} =$$
$$= \left(\frac{1 - \frac{sT_d}{2n} + \frac{1}{3}\left(\frac{sT_d}{2n}\right)^2}{1 + \frac{sT_d}{2n} + \frac{1}{3}\left(\frac{sT_d}{2n}\right)^2} \right)^2, T_d = -4.896 \tag{6}$$

The application of Padé approximation for delay is widely used in chemical industry [20].

Note 1. Possible approximations for delay are:

A) Laguerre shift

$$e^{-T_d s} = \lim_{n\to\infty} \left(\frac{1 - \frac{sT_d}{2n}}{1 + \frac{sT_d}{2n}} \right)^n \tag{7}$$

This type of approximation is applied in robust control theory [19].

B) Kautz shift

$$e^{-T_d s} \cong \left(\frac{1 - \frac{sT_d}{2n} + \frac{1}{2}\left(\frac{sT_d}{2n}\right)^2}{1 + \frac{sT_d}{2n} + 2\left(\frac{sT_d}{2n}\right)^2} \right)^n \tag{8}$$

It was analytically shown that this type of approximation is more accurate than the Laguerre one [19].

3. DESIGN OF H∞ PI CONTROLLERS BY APPLYING D-DECOMPOSITION

We will now expose the design procedure for the H∞ PI controller that will be used for regulating the CSTR process. The purpose of the constraints is to specify an area, in z-plane, in which the complex-conjugate roots of the characteristic system equation are located.

For the system to possess the required settling time, it is necessary that all the real parts of the poles of the transfer function of the closed loop are located left to the real line σ_m = const., from Figure 2. In this way, we map the area from the "s" plane, left of the line σ_m = const., (Fig. 2), in the area of the corresponding settling time σ_{min}= const., in the parameter plane of the adjustable controller parameters (K_p, K_i). Here, it is necessary to find the minimum undumped frequency that will satisfy the condition:

$$\left|\sigma_{min}\right| \geq \left|\sigma_m\right|; \quad \left|\xi \cdot \omega_{n\,min}\right| \geq \left|\sigma_m\right| \tag{9}$$

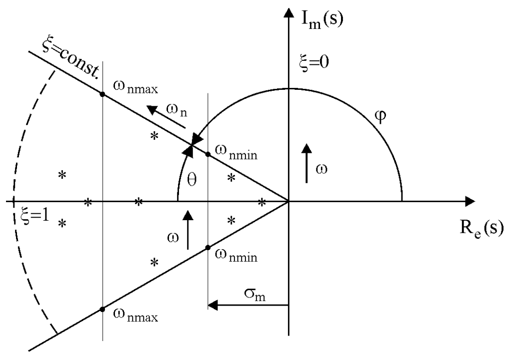

Figure 2. Area with the required settling time σ_m

Having in mind the constraint given by the equation (9), the following criterion is minimized

$$J = \sup_{0 \leq \omega \leq \omega_1} \left\| S(j\omega) \right\| \tag{10}$$

Let us note that, instead of the constraint (9), a constraint that relates to the settling time may also be introduced. It is possible to introduce both criteria simultaneously [16].

The procedure of design of H_∞PI controllers is as follows:

1. Select the area of settling time of the system in the s plane.
2. Choose the points on the curve ξ=const. and determine the parameters K_p and K_i PI of the controller for each point.
3. Based on the transfer function of the C(s) controller (obtained on the basis of the parameters K_p and K_i) and the transfer function of the G(s) process, calculate the sensitivity of the system S(s).
4. Compute the minimum of criterion J_{min} based on the equation (10), by applying MATLAB (Robust Control Toolbox).
5. Create a catalogue which contains:
a) graphical presentation of the reference responses
b) graphical presentation of suppression of disturbances
c) minimum of the criterion J
6. In accordance with engineering reasoning, choose the PI controller which provides nominal performances of the system.

The proposed procedure is an interactive graphical procedure which is simple and enables engineers in industry to use, in a comprehensible and easy way, the latest accomplishments in automatic control theory. In this case, it is H_∞ optimization.

The advantages of the proposed procedure are as follows:

a) Simple structure of the controller (PI controller), which allows physical interpretation of its parameters.

b) Superiority of thus obtained controller in comparison with the original H_∞ controller (which has a high order), which was shown for the case when there are no constraints (9).

c) Elimination of the need to determine the weight function W(s), which is a non-trivial problem.

The simulation results were done on the CSTR model.

Figure 3 shows the parameter plane with the limit values of the PI controller parameters for the required settling time.

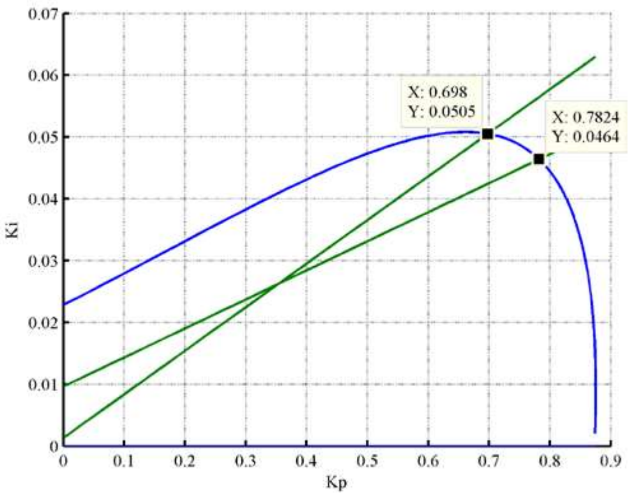

Figure 3. The parameter plane for the required settling time σ_m

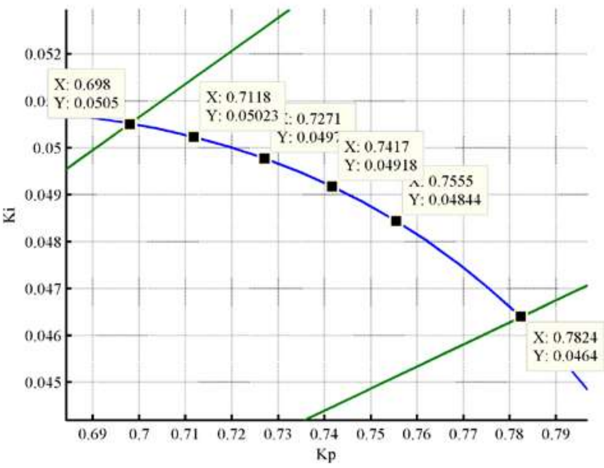

Figure 4. The parametrer plane with catalog of PI controllers for the required settling time

From Figure 4 it can be seen, viewed from left to right, as it is formed a catalog of 6 (C1-C6) PI controllers with a corresponding step. For each of these controllers, the minimum of J criteria is calculated and it is established a catalog of a reference response, and a catalog of a load disturbance response.

For a formed response catalog of controllers C1 to C6, a minimum of J criteria is determined based on the corresponding program written in Matlab, and its result is: J= min([J1,J2,J3,J4,J5,J6])= min([0.9968, 0.9968, 0.9969, 0.9969, 0.9970, 0.9971]) =(J1,J2)= 0.9968.

Based on this result, it can be seen that the two controllers, C1 and C2, meet the minimum of J criteria in the already defined interval of the parametric plane that meets the condition of the required settling time. Figures 5 and 6 show the reference system response and load disturbance response for the limit values of the C1 and C6 controllers as well as the C2 controller which fulfills the minimum of J criteria.

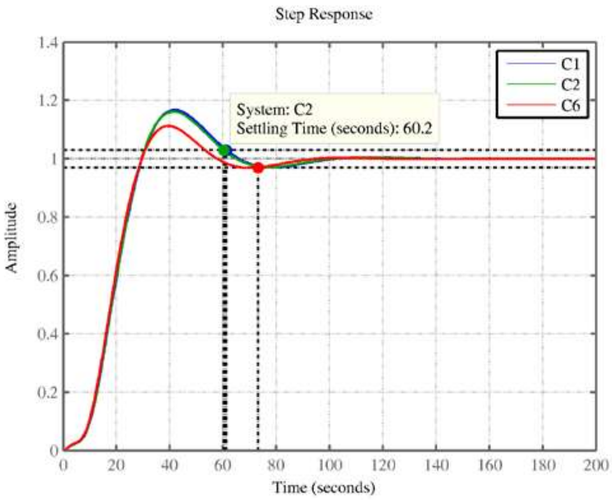

Figure 5. System response to reference for three controllers C1,C2 and C6

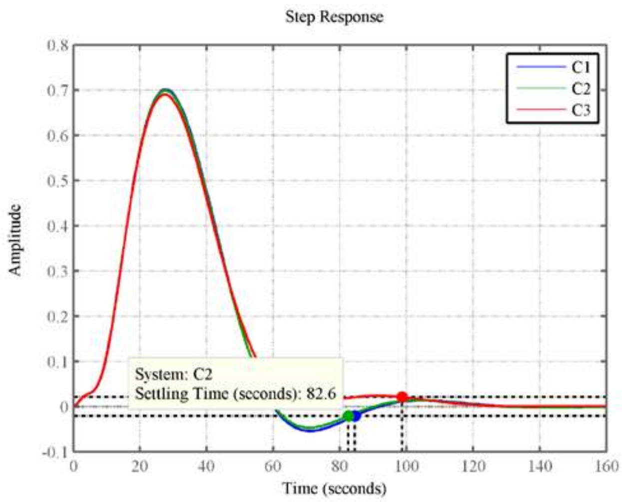

Figure 6. System response to load disturbance for trhree controllers C1,C2 i C6

On the basis of the results shown in Figures 5 and 6, it can be seen that C2 controller gives the best result from the aspect of the responses to the reference and to the load disturbance rejection.

Figure 7 shows Bode diagrams with system characteristics that are designed based on the values of the controller parameters C2. The picture shows that the system has the correct values of the phase margin and the gain margin.

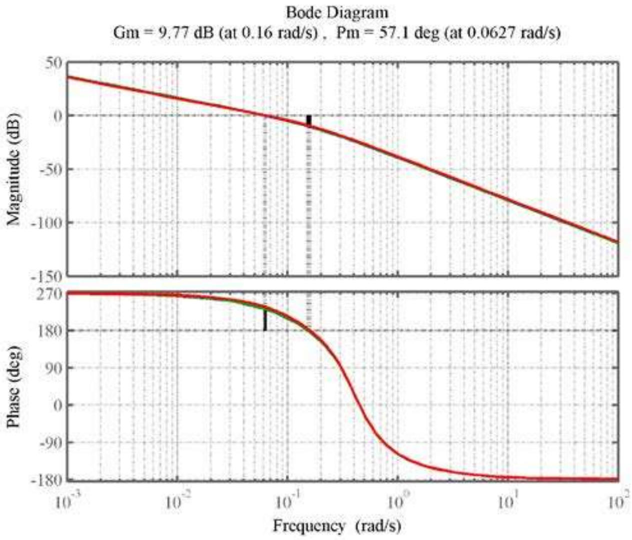

Figure 7. Bode diagrams for open loop designed with controller C2

CONCLUSIONS

In this paper, a simple interactive procedure, based on D-decomposition, is proposed for the design of the H∞PI controller. The design of the controller includes a limitation in the form of a given settling time. Through the simulation, it is formed a catalog of: response to reference, a response to suppression of load disturbance and minimum of H∞ control criteria. Based on the catalog, a controller is considered to be most suitable for the given process. The resultant controller was of a much lower order than the original H∞ controller that is obtained by using very complex numerical procedures. Also, the parameters of the H∞ PI controller have a physical meaning.

ACKNOWLEDGEMENTS

This paper was supported by the Republic of Serbia, Ministry of Education, Science and Technological developments, through project no. 33026-TR.

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