# OPTIMAL TUNING OF PID CONTROLLERS FOR A HYDRAULICALLY DRIVEN PARALLEL ROBOT PLATFORM BASED ON FIREFLY ALGORITHM 

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#### Abstract

The positions of moving platform are changed by extension or shortening of the six pneumatic cylinders. A PID control technique has been applied in practice for control of a 6-DOF parallel robot platform. A parameter search based on Firefly Algorithm (FA) is suggested to effectively search the parameters of PID controllers. Simulation results show the advantages of the proposed optimal tuned PID controllers to solve the formulated tracking problem in relation to the classical tuned PID controllers.


Key words: optimal control, hydraulically control systems, firefly algorithm, parallel robot platform

## INTRODUCTION

The hydraulically driven parallel robot platform is obtained through a generalization of the mechanism proposed by Stewart [1] as a flight simulator. As shown in Fig.1, this spatial platform mechanism consists of a fixed base platform and an upper moving platform. The six extendable legs connect these platforms. Besides greater stiffness and accuracy, these robot platforms have high payload-weight ratio due to parallel linkage. The payload and positioning errors would be accumulated without parallel linkage.


Fig.1. Schematic diagram of a 6-DOF parallel robot platform
However, the upper moving platform of a parallel platform can move with the six desired degrees of freedom (DOF) if the lengths of the all legs are well controlled. A classical PID control technique has been applied in practice for a control of a 6 DOF parallel robot platforms.

An approach to optimal tuning the parameters of the PID controllers is presented. Namely, the PID controller is often poorly tuned owing to highly changing dynamics of parallel robot platform caused by large nonlinearities and changes of parameters during motion. Due to a change of the system parameters, the PID controllers result in suboptimal corrective actions and hence require retuning. In control design of continuous processes, the tuning of controller parameters could be done with traditional methodology. The models of such processes can be linearized in an equilibrium point. However, there are the systems that cannot be linearized around an equilibrium point, because there is no equilibrium point. If a linear approximation is found, the resulting model will be valid only for a small region around the linearization point. Therefore, it is necessary to use non classical tuning methods to achieve the
best overall control strategy for the entire operating envelope of the given system. As an alternative, metaheuristic algorithms are applied for the quality controller tuning. Authors use Firefly Algorithm (FA) to tuning the parameters of PID controllers. FA represents a new metaheuristic algorithm which is nature-inspired. The FA was developed by Yang [23], and it was based on the idealized behaviour of the flashing characteristics of fireflies.

## A MODEL OF PARALLEL ROBOT PLATFORM

A spherical joint is employed to connect the upper part of each leg by the upper moving platform while its lower part is connected to the base platform by an universal joint. A prismatic joint is employed for translational motion of the actuator. Well controlled lengths of six actuators make the upper moving platform follow the desired trajectory [5].

## KINEMATICS ANALYSIS

As shown in Fig.1, the legspace frame $\left(x_{B}, y_{B}, z_{B}\right)$ is located at the center of the base platform, and the workspace frame $\left(x_{P}, y_{P}, z_{P}\right)$ is located at the center of the moving platfrom. The angle between the fixed $x_{B}$ axis of the base platform and the line of the joint $J_{B i}$ is denoted by $\alpha_{i}$, as shown in Fig. 2a. In the same manner, an angle $\beta_{i}$ is defined between the local $x_{P}$ axis of the moving platform and the line of the joint $J_{P i}$, as shown in Fig.2b.

a) Base Platform

b) Upper Platform

Fig.2. Top view of joint positions

The upper platform has an ability of 6 degrees of freedom (DOF) motion (three for rotational motion and three for translational motion). The rotation matrix $R_{P}$ corresponding to a rotation of the upper moving platform with three Euler angles, $\psi$ (yaw), $\theta$ (pitch), and $\varphi$ (roll), in relation to $x_{B}, y_{B}, z_{B}$, is given by:
$R_{P L}=\left[\begin{array}{ccc}C_{\theta} C_{\psi} & S_{\varphi} S_{\theta} C_{\psi}-C_{\varphi} S_{\psi} & C_{\varphi} S_{\theta} C_{\psi}+S_{\varphi} S_{\psi} \\ C_{\theta} S_{\psi} & S_{\varphi} S_{\theta} S_{\psi}+C_{\varphi} C_{\psi} & C_{\varphi} S_{\theta} S_{\psi}-S_{\varphi} C_{\psi} \\ -S_{\theta} & S_{\varphi} C_{\theta} & C_{\varphi} C_{\theta}\end{array}\right\rfloor$
where $C$ denotes the cosine of an angle and $S$ denotes the sine of an angle. If the trajectory of the upper platform center is defined by $x_{B}(t), y_{B}(t)$ and $z_{B}(t)$ then the translation vector $T_{P}$ of the upper platform, in relation to legspace frame, has the form
$T_{P}=\left[\begin{array}{lll}x_{B}(t) & y_{B}(t) & z_{B}(t)+h\end{array}\right]^{T}$
where $h$ is the nominal height of the upper platform. It is easily shown that the angular velocity $\omega_{P}$ of the upper moving platform has the form, see [4]
$\omega_{P}=\left[\begin{array}{ccc}-S_{\theta} & 1 & 0 \\ S_{\varphi} C_{\theta} & 0 & C_{\varphi} \\ C_{\varphi} C_{\theta} & 0 & -S_{\theta}\end{array}\right]\left[\begin{array}{c}\dot{\psi} \\ \dot{\varphi} \\ \dot{\theta}\end{array}\right]$
Each leg has three degrees of freedom: two rotational and one translational motion. A spherical joint is employed to connect the upper part of each leg by the upper moving platform while its lower part is connected to the base platform by an universal joint. The leg is composed from the cylinder which can rotate around $x_{L i}$ axis with an angle $\delta_{i}$, and $z_{L i}$ axis with an angle $\gamma_{i}$, of the universal joint, and the piston which performs translational motion inside the cylinder by an actuating load force $F_{L i}$, as shown in Fig.3.


Fig.3: Leg motion of the parallel robot platform
This rotational angles $\gamma_{i}$ and $\delta_{i}$ of the leg are defined as in [4]. The motion of the each leg is considered by two frames: a leg fixed frame $\left(x_{L i}, y_{L i}, z_{L i}\right)$ located at the joint $J_{B i}$ parallel to the fixed legspace frame and the leg body frame $\left(x_{P L i}, y_{P L i}, z_{P L i}\right)$ located at the joint $J_{P i}$.

It is easily shown that the angular velocity $\omega_{i}$ of the each leg has the form [4]

$$
\omega_{i}=\left[\begin{array}{lll}
-\dot{\gamma}_{i} \sin \left(\delta_{i}\right) & \dot{\delta}_{i} & \dot{\gamma}_{i} \cos \left(\delta_{i}\right) \tag{4}
\end{array}\right]^{T}, i=\overline{1,6}
$$

The rotation matrix $R_{L i}$ from the leg body frame $\left(x_{P L i}, y_{P L i}, z_{P L i}\right)$ to the leg fixed frame $\left(x_{L i}, y_{L i}, z_{L i}\right)$ is defined as
$R_{L i}=\left\lfloor\begin{array}{ccc}C_{\delta_{i}} C_{\gamma_{i}} & -S_{\gamma_{i}} & S_{\delta_{i}} C_{\gamma_{i}} \\ C_{\delta_{i}} S_{\gamma_{i}} & C_{\gamma_{i}} & S_{\delta_{i}} S_{\gamma_{i}} \\ -S_{\delta_{i}} & 0 & C_{\delta_{i}}\end{array}\right\rfloor$

DYNAMIC ANALYSIS

The moment equation around the universal joint $J_{B i}$ is:

$$
\begin{align*}
& m_{C i} \vec{r}_{C i} \times\left(-\vec{g}+\vec{a}_{C i}\right)+m_{P i} \vec{r}_{P i} \times\left(-\vec{g}+\vec{a}_{P i}\right)+ \\
& \left(\left[I_{C i}\right]+\left[I_{P i}\right]\right) \vec{\varepsilon}_{i}+\vec{\omega}_{i} \times\left(\left[I_{C i}\right]+\left[I_{P i}\right]\right) \vec{\omega}_{i}-L_{i} \times f_{i}=0, i=\overline{1,6} \tag{6}
\end{align*}
$$

where $m_{C i}$ is the cylinder mass, $m_{P i}$ is the piston mass, $\left[I_{C i}\right]$ is the inertia matrix of the cylinder, $\left[I_{P_{i}}\right]$ is the inertia matrix of the piston, $\vec{a}_{C i}$ is the acceleration of the cylinder, $\vec{a}_{P i}$ is the acceleration of the piston, $\vec{g}$ is the gravitational acceleration, $\vec{\varepsilon}_{i}$ is the angular acceleration, $\vec{r}_{C i}$ is the position vector of the cylinder of the leg from universal joint $J_{B i}, \vec{r}_{P i}$ is the position vector of the piston of the leg from universal joint $J_{B i}$, and $\vec{f}_{i}$ is the reaction force between the spherical joint $J_{P i}$ and the upper platform. The force equation in $x_{P L i}$-direction of the sliding mechanism is given as:

$$
\begin{equation*}
m_{C i} \vec{r}_{C i} \times\left(-\vec{g}+\vec{a}_{C i}\right)=F_{L i}-f_{x_{P L i}}, \quad i=\overline{1,6} \tag{7}
\end{equation*}
$$

The dynamic equations of plate have three moment equations and three force equations as:
$m_{P} \vec{a}_{P}=m_{P} \vec{g}+\sum_{i=1}^{6} R_{P}^{T} R_{L i} \vec{f}_{i}$
$\left[I_{P}\right] \vec{\varepsilon}_{P}+\vec{\omega}_{P} \times\left[I_{P}\right] \vec{\omega}_{P}=\sum_{i=1}^{6} \vec{r}_{B i} \times R_{P}^{T} R_{L i} \vec{f}_{i}$
where $m_{P}$ is the mass of the platform with an external load, $\left[I_{P}\right]$ is the inertia matrix of the upper platform, $\vec{a}_{P}$ is the acceleration of the upper platform, $\vec{\omega}_{P}$ and $\vec{\varepsilon}_{P}$ are the angular velocity and acceleration of the upper platform, and $\vec{r}_{B i}$ the position vector from the center of platform to the joint $J_{B i}$.

## OPTIMAL TUNING OF PID CONTROLLERS

Firefly Algorithm (FA) is a nature inspired algorithm, which is based on the flashing light of fireflies. The swarm of fireflies will move to brighter and more attractive locations by the flashing light intensity that associated with the objective function of problem considered in order to obtain efficient optimal solutions. For simplicity, we can idealize these flashing characteristics as the following three rules
i) all fireflies are unisex so that one firefly is attracted to other fireflies regardless of their sex
ii) Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less brighter one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If no one is brighter than a particular firefly, it moves randomly;
iii) The brightness or light intensity of a firefly is affected or determined by the landscape of the objective function to be optimised.

The basic steps of the FA can be summarized as the pseudo code shown in next Table 1.

Table 1. Pseudo code of FA algorithm

## BEGIN FA

Objective function $f(x), x=\left(x_{1}, x_{2}, \ldots, x_{d}\right)$;
Initialize a population of fireflies $x_{i}, i=\overline{1, n}$
Define light absorption coefficient $\gamma$
WHILE (stop criterion)
FOR $i=1: n$
FOR $j=1: i$
Light intensity $I_{i}$ at $x_{i}$ is determined as $f\left(x_{i}\right)$
IF $I_{i}>I_{j}$
Move firefly $i$ towards $j$ in all $d$ dimensions replace $j$ by the new solution;

## END IF

Attractiveness varies with distance $r$ via $e^{-\gamma r}$
Evaluate new solutions and update light intensity
END FOR $j$

## END FOR $i$

Rank the fireflies and find the current best

## END WHILE

Display the obtained results.

## END FA

In the FA, there are two important issues: the variation of light intensity and formulation of the attractiveness. In the simplest form, the light intensity $I(r)$ varies with the distance r monotonically and exponentially. That is

$$
\begin{equation*}
I=I_{0} e^{-\gamma r} \tag{10}
\end{equation*}
$$

where $I_{0}$ is the original light intensity and $\gamma$ is the light absorption coefficient. As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the attractiveness $\beta$ of a firefly by

$$
\begin{equation*}
\beta=\beta_{0} e^{-\gamma r^{2}} \tag{11}
\end{equation*}
$$

where $\beta_{0}$ is the attractiveness at $r=0$. The distance between any two fireflies $i$ and $j$ at $x_{i}$ and $x_{j}$ can be the Cartesian distance $r_{i j}=\left\|x_{i}-x_{j}\right\|_{2}$ or the $\ell_{2}$ norm. The movement of a firefly $i$ is attracted to another more attractive (brighter) firefly $j$ is determined by

$$
\begin{equation*}
x_{i}=x_{i}+\beta_{0} e^{-\gamma r_{i j}^{2}}\left(x_{j}-x_{i}\right)+\alpha \varepsilon_{i} \tag{12}
\end{equation*}
$$

where the second term is due to the attraction, while the third term is randomization with the vector of random variables $\varepsilon_{i}$ being drawn from a Gaussian distribution. For most cases in our implementation, we can take $\beta_{0}-1, \alpha \in[0,1]$ and $\gamma-1$.

## SIMULATION RESULTS

The proposed control law is simulated using Matlab's Simulink model. To verify the effectiveness of the proposed optimal PID scheme, it is made a comparative study between optimal tuned and well-tuned PID controllers, which is widely used in practice $[6,7]$.

The system parameters are selected based upon their actual values and are given: areas of the head and rod side of the piston $A_{1}=5 \cdot 10^{-3} \mathrm{~m}^{2}$ and $A_{2}=2.3 \cdot 10^{-3} \mathrm{~m}^{2}$, respectively, piston stroke $L=0.7 \mathrm{~m}$, bulk modulus of the fluid $\beta_{e}=700 \mathrm{MPa}$, supply pressure $p_{s}=2.5 M P a$, mass of the upper platform $m_{p}=550 \mathrm{~kg}$, mass of the piston $m_{p i}=2 \mathrm{~kg}$, mass of the cylinder $m_{C i}=45 \mathrm{~kg}$, moment of inertia of the cylinder $I_{C x i}=0.2 \mathrm{kgm}^{2}, I_{C y i}=6 \mathrm{kgm}^{2}, I_{C z i}=6 \mathrm{kgm}^{2}$, moment of inertia of the piston $I_{P x i}=0.03 \mathrm{kgm}^{2}, I_{P y i}=3.5 \mathrm{kgm}^{2}, \quad I_{P z i}=3.5 \mathrm{kgm}^{2}$, moment of inertia of the upper platform $I_{P x}=3 \mathrm{kgm}^{2}, I_{P y}=28 \mathrm{kgm}^{2}, I_{P z}=28 \mathrm{gm}^{2}$, radius of the base platform $R_{B}=1.3 \mathrm{~m}$, radius of the upper platform $R_{P}=0.8 \mathrm{~m}$, coefficients of reference generator $k_{p}=2.8 \cdot 10^{3} \mathrm{~N} / \mathrm{m}$, $k_{v}=2 \cdot 10^{4} \mathrm{Ns} / \mathrm{m}$. The reference trajectory is chosen to be $x_{\text {ref }}=0.17 \sin (2 \pi t)[m]$,
$y_{\text {ref }}=0.14 \sin (2 \pi t+\pi / 2)[m]$
$z_{\text {ref }}=1+0.2 \sin (2 \pi t)[m]$,
$\varphi_{\text {ref }}=\theta_{\text {ref }}=-\psi_{\text {ref }}=0.2 \sin (2 t)[\mathrm{rad}]$
The total number of parameters whose values are determined in the process of optimal parameter tuning is 18 (three parameters for each leg). Search space for controller parameters $P, I$ and $D$ is $\left[10^{3}-10^{6}\right]$.

Table 2. Final values of design variables

| Leg 1 |  | $P$ | $I$ |
| :---: | :---: | :---: | :---: |
| Leg 2 | $4.373 \cdot 10^{5}$ | $4.715 \cdot 10^{3}$ | $1.811 \cdot 10^{4}$ |
| Leg 3 | $5.954 \cdot 10^{5}$ | $3.451 \cdot 10^{3}$ | $5.354 \cdot 10^{3}$ |
| Leg 4 | $6.745 \cdot 10^{5}$ | $3.541 \cdot 10^{3}$ | $1.517 \cdot 10^{4}$ |
| Leg 5 | $4.541 \cdot 10^{5}$ | $6.022 \cdot 10^{3}$ | $2.422 \cdot 10^{4}$ |
| Leg 6 | $5.672 \cdot 10^{5}$ | $4.551 \cdot 10^{3}$ | $3.325 \cdot 10^{4}$ |

FA algorithm randomly chooses parameters $P, I, D$, after which, it calculates the system response, and determines the position error. Then, the given optimization criterion, which represents objective function, is applied. In this paper, we used integral of absolute error (IAE) criterion:

$$
\begin{equation*}
J=\int_{0}^{T}|\varepsilon(t)| d t \tag{14}
\end{equation*}
$$

where $\varepsilon(t)$ represents position error (difference between system response and set points). The values of controller parameters, for which the objective function (14) is minimum, are chosen. The optimal tuned parameters of the cascade controllers are given in the Table 2.
The position errors of six legs, using classical classical tuned PID controllers, and optimal-tuned PID controllers are shown in Fig. 4 and Fig.5. From above figures, it can be seen that the developed optimal PID controllers perform position tracking much better than classical tuned PID control algorithm. Detailed analysis of the position errors, have shown that with classical PID control strategy, position errors are $8 \%$, but after applying proposed optimal algorithm, position errors are only $1.2 \%$.


Fig.4. Position error of the extensible actuators using classical tuned PID controller


Fig.5. Position error of the extensible actuators, using optimal tuned PID controller

## CONCLUSION

In order to realize reference trajectory of the 6-DOF parallel robot platform, we have proposed an optimal tuned PID control strategy in the legspace. We have presented an optimal parameter search based on Firefly Algorithm. Detailed simulation results have shown that the such optimal tuned PID controllers outperform the widely used classical tuned PID controllers and exhibit satisfactory tracking performance.

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