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DETERMINATION OF THE DESCRIBING FUNCTION OF NOZZLE-FLAPPER TYPE PNEUMATIC VALVE WITH TWO PORTS

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Abstract: *In many pneumatic systems with a wider bandwidth, nozzle-flapper type valves are usually used for flow rate control. Because of their inherently nonlinear behavior and parameters variation, modelling of the servo valve is important for analysis and design purpose. The describing function was used for modelling of the mass flow rate characteristic of the servo valve. The paper presents two describing functions. One is for the fixed orifice with the subsonic flow regime, and the other is for the nozzle with the sonic flow regime. In the first case, the mass flow rate function is approximated by a polynomial.*

Key words*: nozzle-flapper type pneumatic valve, nonlinear model, describing function, mass flow rate*

1. INTRODUCTION

Pneumatic servosystems are widely used in industrial applications because of the favourable performances/price ratio. However, high precision control of such systems is difficult due to their complex physical nature. The main causes of that complexity are: air compressibility, friction between the contact surfaces, nonlinear flow-pressure characteristics of the orifice type restriction and parameter variations [1-3]. In order to solve the problem of design and control of such systems, it is necessary to have better understanding of their nonlinear characteristics. A mathematical model which should clarify the most relevant dynamic and nonlinear behavior in the pneumatic system is used for that purpose.

Nozzle-flapper type valves are frequently used in pneumatic systems because of their simple structure, high sensitivity and a broad bandwidth [4-6]. As the nonlinear characteristics of the valve reflect in the operation of the whole pneumatic system, it is observed and modelled as a separate subsystem. The paper presents and analyzes the characteristics of the nonlinear mass flow rate-pressure ratio of a 2-port nozzle-flapper type pneumatic servo valve with one fixed orifice and one nozzle (Fig.1).

One of the methods of analysis of nonlinear-systems is the quasi-linearization method [7, 8]. Linearization in the ordinary sense is not valuable in the case when nonlinearity inputs exceed the limits of acceptable linear approximation or when there is discontinuity at the nominal operating point. The advantages of true linearization are kept in the case of quasi-linearization but there is no limit to the range of input signal magnitudes or to the selection of the operating point. The constraint is that linear description of the system depends on some properties of the input signal. The system description thus depends not only on the system itself, but also on the signals passing through the system (which is a property of nonlinear systems). In other words, quasi-linearization is performed for a certain form of input signal. The problem with nonlinear systems with feedback configurations is in difficult determination of the signal form which occurs on entering the nonlinearity. This is the main constraint of the method. It is not always possible to reduce the nonlinearity input signal to a simple form. The practical solution of the problem is to assume the form of the input signal in advance. In practice, three forms of input signals are used in quasi-linearization [7, 8]: bias, sinusoid and Gaussian process.

The quasi-linear function which approximatively describes nonlinearity is called the describing function (DF). As the design of control systems is frequently realized in the frequency domain, the Sinusoidal Input Describing Function (SIDF) is used in this paper. Assuming that the linear part of the system filters high order harmonics (low-pass filter), every periodic signal is reduced to a basic periodic function on entering the nonlinearity. In the case of memoryless nonlinearity, the SIDF represents the gain which is changed depending on the amplitude of the input signal.

The paper determines the SIDF of the nonlinear mass flow rate characteristic of the nozzle-flapper type pneumatic valve with two ports.

2. MASS FLOW RATE CHARACTERISTIC OF NOZZLE - FLAPPER TYPE PNEUMATIC VALVE

Figure Fig.1 presents the functional scheme of the nozzle-flapper type pneumatic valve with two ports. The valve consists of a fixed orifice-type restriction (*Or*) and a nozzle (*Nz*). Moving of the flapper (*Fl*) changes its distance from the nozzle, i.e. the mass flow rate $\dot{M}_n$. As a result, there occurs a change of the pressure P at the control port. A detailed mathematical description of the nozzle-flapper type pneumatic servo valve with four ports can be found in [4-6]. This paper deals primarily with the flow-rate characteristic of the valve. Since the flow-rate through the valve depends on the pressure at the control port, the figure also shows the chamber (*Ch I*) which is not a part of the valve but acts as a load.

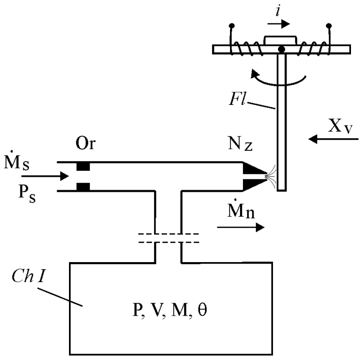

Fig. 1. The nozzle-flapper type pneumatic valve with the load chamber

The mass flow rate through the restriction can be in sonic or subsonic conditions depending upon the ratio of upstream-downstream pressure. According to the standard theory, mass flow rate can be presented in the form [9]:

$$\dot{M} = A_e \varphi(P_u, P_d, \theta_u) \tag{1a}$$

where the function φ is defined as:

$$\varphi(P_u, P_d, \theta_u) = \begin{cases} C_1 \dfrac{P_u}{\sqrt{\theta_u}} & \text{if} \quad \dfrac{P_d}{P_u} \le P_{cr} \\ C_2 \dfrac{P_u}{\theta_u} \left(\dfrac{P_d}{P_u}\right)^{\frac{1}{\kappa}} \sqrt{1 - \left(\dfrac{P_d}{P_u}\right)^{\frac{\kappa-1}{\kappa}}} & \text{if} \quad P_{cr} < \dfrac{P_d}{P_u} \le 1 \end{cases} \tag{1b}$$

while the parameters C_1, C_2 and P_{cr} are determined by:

$$C_1 = \sqrt{\frac{\kappa}{R}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}, \quad C_2 = \sqrt{\frac{2\kappa}{R(\kappa-1)}}$$
$$P_{cr} = \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa}{\kappa-1}} \tag{1c}$$

If the downstream/upstream pressure ratio is smaller than a critical value P_{cr} (0.528 for air), the flow is sonic and the function of upstream pressure is linear. If the pressure ratio is higher than P_{cr}, the flow is subsonic and depends nonlinearly on both pressures. In order to determine the flow regimes at the valve orifices, it is necessary to analyze the flow in the nominal regime for which it holds that:

$$\dot{M}_{sN} = \dot{M}_{nN} \tag{2}$$

If it is assumed that the air temperature in the chamber is constant (isothermal chamber) and equal to the ambient temperature:

$$\theta = \theta_a = const. \tag{3}$$

then, based on (1) and (2), it can be written that:

$$A_{es} \varphi(P_s, P_N, \theta_a) = A_{en} \varphi(P_N, P_a, \theta_a) \tag{4}$$

Numerical solution of Equations (1) and (4) results in the static characteristic (steady state characteristic) of the valve which shows dependence of the load pressure (P_N) on the supply pressure (P_S) for different ratios of effective flow areas (A_{enN} / A_{es}).

The flow regime which is established depends on the supply pressure and the ratio between the effective flow areas. It is assumed that the flow at the fixed orifice (*Or*) is in the subsonic regime and that the flow has sonic velocity at the nozzle. In other words, for the fixed orifice it can be written that:

$$\dot{M}_s = \dot{M}_s(P) = A_{es} C_2 \frac{P_S}{\theta_a} \left(\frac{P}{P_S}\right)^{\frac{1}{\kappa}} \sqrt{1 - \left(\frac{P}{P_S}\right)^{\frac{\kappa-1}{\kappa}}} \tag{5}$$

The flow through the nozzle is determined by the following expression:

$$\dot{M}_n = \dot{M}_n(A_{en}, P) = A_{en} C_1 \frac{P}{\sqrt{\theta_a}} \tag{6}$$

and it is the function of two values, the effective area of restriction A_e and the working pressure P.

If it is assumed that the area is a linear function of the distance between the flapper and the nozzle, it can be written that [4-6]:

$$A_{en} = \frac{X_{vN} - X_v}{X_{vN}} A_{enN} \tag{7}$$

Now, based on (6) and (7), it can be written that:

$$\dot{M}_n = \dot{M}_n(X_v, P) = \frac{X_{vN} - \tilde{X}_v}{X_{vN}} A_{en\,N} C_1 \frac{P}{\sqrt{\theta_a}} \tag{8}$$

It should be noted that the effective area of the nozzle in the nominal regime depends on the nominal working pressure P_N. Namely, from (5) and (6) in the nominal regime:

$$\frac{A_{enN}}{A_{es}} = A_{es} \frac{C_2}{C_1} \sqrt{\left(\frac{P_N}{P_S}\right)^{\frac{1-\kappa}{\kappa}} - 1} \tag{9}$$

It means that the position of the nominal point is determined by the working pressure in the nominal regime.

3. DESCRIBING FUNCTION OF THE VALVE

Behavior of the system from Fig.1 is described by the block diagram in Fig.2.

It was assumed that the load behavior was described by the linear dynamics (W(s)), while the valve was described by the static nonlinearities $\dot{M}_n(X_v, P)$ and

$\dot{M}_s(P)$. Describing functions of those nonlinearities will now be determined.

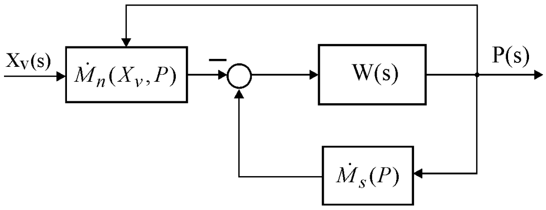

Fig.2. Block diagram of the pneumatic system

Let $\dot{m}_n$, x_v, p be relative changes of the values $\dot{M}_n$, X_v, P, respectively. Then, based on (8), it can be obtained that:

$$\dot{m}_n = p - x_v - x_v p \tag{10}$$

Let us now assume that the following holds:

$$x_v = x_{vA}\sin(\omega t) \tag{11a}$$

$$p = p_A \sin(\omega t + \alpha) \tag{11b}$$

In other words, relative changes of input values have the character of basic periodic oscillations with the amplitudes x_{vA} and p_A, and the frequency ω. The values are phase shifted for α.

Let us now introduce the assumption that for inputs (11) the relative change of flow $\dot{m}_n$ is also a periodic function which can be represented by the Fourier series:

$$\dot{m}_n = \frac{a_o}{2} + \sum_1^{\infty} a_n\cos(n\omega t) + b_n\sin(n\omega t) \tag{12a}$$

where:

$$a_o = \frac{1}{\pi}\int_{-\pi}^{\pi} \dot{m}_n\, d\psi \tag{12b}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \dot{m}_n \cos(n\psi)\, d\psi \tag{12c}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \dot{m}_n \sin(n\psi)\, d\psi \tag{12d}$$

Fourier coefficients can be obtained based on the expressions (10), (11) and (11). Table 1 gives the values for the first nine members of the series.

Table 1. Fourier coefficients for the mass flow rate approximation $\dot{m}_n$

$a_o/2$	$-\frac{1}{2}x_{vA}p_A\cos(\alpha)$		
a_1	$p_A\sin(\alpha)$	b_1	$p_A\cos(\alpha) - x_{vA}$
a_2	$\frac{1}{2}x_{vA}p_A\cos(\alpha)$	b_2	$-\frac{1}{2}x_{vA}p_A\sin(\alpha)$
a_3	0	b_3	0
a_4	0	b_4	0

It can be seen from the table that the values of Fourier coefficients depend on the amplitudes of input signals and the phase shifts between them.

It can be seen that the value of coefficients, i.e. phase delay, depends on load dynamics. However, if:

$$\alpha \neq 2k\pi,\ k = 0,1,... \tag{13a}$$

and if

$$x_{vA} < 1 \text{ and } p_A < 1 \tag{13b}$$

then

$$\begin{aligned} \dot{m}_n &\approx a_1\cos(\omega t) + b_1\sin(\omega t) \\ &= p_A\sin(\alpha)\cos(\omega t) + (p_A\cos(\alpha) - x_{vA})\sin(\omega t) \\ &= p_A\sin(\omega t + \alpha) - x_{vA}\sin(\omega t) = p - x_v \end{aligned} \tag{14}$$

which means that the nonlinearity defined by (10) for the case of inputs defined by (11) can be represented as a linear element with the unit gain. In other words, the describing function for both inputs has the value one if the conditions (13) are fulfilled.

Equation (5) can be the basis for determination of dependence of the relative change of flow through the fixed orifice on the relative change of pressure p:

$$\dot{m}_s = (1+p)^{\frac{1}{\kappa}}\left(1 - \left(P_N(1+p)/P_S\right)^{\frac{\kappa-1}{\kappa}}\right)^{\frac{1}{2}} \left(1 - \left(P_N/P_S\right)^{\frac{\kappa-1}{\kappa}}\right)^{-\frac{1}{2}} \tag{15}$$

Depending on the desired accuracy and the range of changes for p, the relative change of flow $\dot{m}_s$ (16) can be approximated by the polynomial:

$$\dot{m}_s = c_1 p + c_2 p^2 + c_3 p^3 \tag{16}$$

In Fig.3, the dashed line shows the value of the mass flow rate $\dot{m}_s$ approximated by the polynomial of the third degree (16).

If it is assumed that the value p is changed in the manner defined by Equation (11b) and that the relative change of flow $\dot{m}_s$ has the form defined by (12a), then, by using Equations (12b), (12c) and (12d) (the index *s* is used instead of the index *n*) as well as Equation (16), the Fourier coefficients given in Table T4 can be determined.

It is seen that the value of coefficients depends both on the input signal (amplitude and phase) and the nominal point around which harmonic linearization (coefficients c_1, c_2, c_3) is performed.

Table 2. Fourier coefficients for mass flow rate aproximation $\dot{m}_s$.

a_1	$\left(c_1 + \frac{3}{4}c_3 p_A^2\right)p_A\sin(\alpha)$
a_2	$\frac{1}{2}\left[2\sin^2(\alpha) - 1\right]p_A^2 c_2$
a_3	$-\frac{1}{4}p_A^3\sin(3\alpha)c_3$

b_1	$\left(c_1 + \frac{3}{4}c_3 p_A^2\right) p_A \cos(\alpha)$
b_2	$\frac{1}{2}\sin(2\alpha) p_A^2 c_2$
b_3	$-\frac{1}{4} p_A^3 \cos(3\alpha) c_3$
a_4, b_4	0

If the mass flow rate is approximated by the fundamental harmonics, then:

$$\dot{m}_s \approx \left(c_1 + \frac{3}{4}c_3 p_A^2\right) p_A \sin(\alpha) \qquad (17)$$

and hence the describing function of the nonlinear function (15) is:

$$N_s = \left(c_1 + \frac{3}{4}c_3 p_A^2\right) \qquad (18)$$

The results of simulation show that the accuracy of approximation (17) is most influenced by the amplitude of the input signal. Fig.3 (a, b) presents the exact values of relative changes of the mass flow rate $\dot{m}_s$ (15) and their approximations (17) for two different values of the amplitude of the input signal $p_A = \{0.1, 0.2\}$ (11b).

4. CONCLUSION

At the orifices of the nozzle flapper type valve all four combinations of flow regimes (sonic/subsonic) are possible. However, for standard working conditions, with the constant volume chamber load type, the flow regime through the fixed orifice is subsonic while the flow state through the nozzle is sonic.
If the changes of pressure and position of the flapper have a periodic character and the same frequency but are phase shifted, the mass flow rate of the nozzle can be approximated by a constant, unit gain for both values. In other words, the mass flow rate of the nozzle can be approximated by true linearization. Nonlinearity of the mass flow rate becomes more pronounced at the fixed orifice. The describing function can be determined analytically with the previous determination of the mass flow rate by a polynomial. The value of the describing function depends on the selection of the nominal operating point and the amplitude of the input signal.

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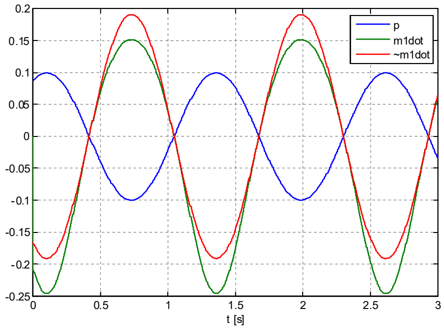
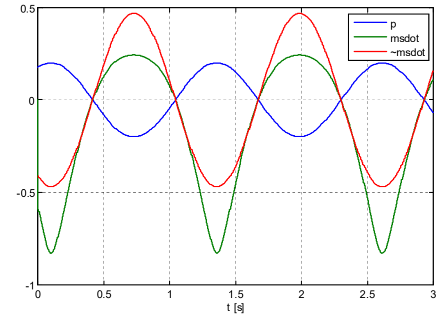


(a)



(b)

Fig.3. Exact and approximated values of the mass flow rate for two values of the amplitude of the input signal

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