Latin American Journal of Solids and Structures
www.lajss.org

Improved Cuckoo Search (ICS) algorthm for constrained optimization problems

Abstract
Constrained optimization is very important issue in engineering design. The problem of constrained optimization contains the objective function with linear and nonlinear constraint equations. In this paper the Improved Cuckoo Search (ICS) algorithm has been applied for solving problems of constrained engineering optimization, which gives better solutions than the standard CS algorithm and the other optimization algorithms used in solving engineering problems from the literature. In the proposed ICS algorithm, dynamic change of parameters of probability and step size is introduced, which are constant in the standard algorithm.

Keywords
Improved Cuckoo Search algorithm, engineering, global optimization, dynamic change of parameters.

Radovan R. Bulatović* ,[a]
GoranBošković[a]
Mile M. Savković[a]
Milomir M. Gašić[a]

[a]The Faculty of Mechanical and Civil Engineering in Kraljevo of the University in Kragujevac, Dositejeva 19, 36000 Kraljevo, Serbia

*Author email: ukib@tron-inter.net

1 INTRODUCTION

The task of optimization theory is development of methods for finding a global minimum. How successful is the answer to the task depends on the characteristic of the objective function, type of constraint and selection of the optimization algorithm. Different optimization algorithms have been used in optimization problems in the existing literature. In Yang (2010a), author considered different optimization algorithms as well as their classification. Generally, all optimization algorithms can be classified, by their nature, into deterministic and stochastic, as well as the algorithms which are a mixture (hybrid) of deterministic and stochastic algorithms. Stochastic algorithms are classified into two types: heuristic and metaheuristic algorithms

Heuristics means "to find" or "to discover by trial and error" good solutions in complex optimization problems. However, in most cases it is not realistic to expect those quality solutions obtained by heuristic algorithms to be optimal.

The algorithms that are more advanced and efficient than heuristic algorithms are the so-called metaheuristic algorithms. Meta — means "beyond" or "higher level" and all metaheuristic algorithms use a certain tradeoff of randomization and local search. No agreed definitions of heuristics and metaheuristics exist in the literature so that the terms "heuristics" and "metaheuristics" are used interchangeably. The recent trend, according to Yang (2010a), tends to name all stochastic algorithms with randomization and local search as metaheuristic. Randomization provides a good way to move away from local search to the search on the global scale. Therefore, all metaheuristic algorithms intend to be suitable for global optimization.

More and more metaheuristic algorithms are being developed and they are mostly nature-inspired and have diverse applications. Some of them are: simulated annealing (SA), genetic algorithms (GA), ant colony optimization (ACO), bee algorithms (BA), differential evolution (DE), particle swarm optimization (PSO), harmony search (HS), the firefly algorithm (FA), Cuckoo Search (CS), and bat-inspired algorithm (BA).

Many of authors for the application in engineering design optimization have used different metaheuristic algorithms which are nature-inspired. Cuckoo Search Algorithm (CS) was applied by Yang and Deb (2009), (2010b), Gandomi (2013) for solving structural optimization problems. Valian et al. (2013) improved cuckoo search algorithm for reliability optimization problems and Bulatović et al. (2013) used cucko search algorithm for solving the problem of optimum synthesis of a six-bar double dwell linkage. Artificial bee colony algorithm (ABC) has been applied by: Akay and Karaboga(2012), Karaboga and Akay (2011), Grković and Bulatović (2013) for solving constrained and large-scale engineering optimization problems. Harmony search algorithm (HS) was defined by Lee and Geem (2004) and it was used for solving engineering optimization problems by Jaberipour and Khorram(2010). Mahdavi et al. applied improved harmony search algorithm (IHS) while Mun& Cho (2012) used modified harmony search algorithm(MHS). Datta and Figueira (2011) used a real-integer discrete-coded particle swarm optimization algorithm for solving design problems. Kayhan et al. (2010) used a new hybrid particle swarm optimization algorithm for solving continuous optimization problems, and Zahara and Kao (2009) applied hybrid Nelder-Mead simplex search and particle swarm optimization for constrained engineering design problems. Gandomi et al. (2011) used Firefly algorithm for solving structural optimization problems, Ashtari and Barzegar (2012) used accelerating fuzzy genetic algorithm (AFGA) for optimization of steel structures and Zhao et al. (2012) applied an effective hybrid genetic algorithm (HGA), Zou et.al. directedsaearching optimization algorithm (DSO), for solving constrained optimization problems, Lobato and Valder (2014) used Fish Swarm Optimization algorithm (FSO) applied to Engineering System design.

This paper is organized through 6 sections. Chapter 2 refers to objective function and the way of using constraints, in Section 3, the standard CS algorithm is shown, and Section 4 introduced the Improved Cuckoo Search algorithm with dynamic changes of probability parameters and step size in each generation, which are otherwise constant in CS algorithm. Section 5 discusses experimental results for several constrained benchmark functions against optimal solutions reported in the literature. Finnaly, in Section 6, conclusions based on the experimental results are discussed.

2 OBJECTIVE FUNCTION AND CONSTRAINTS

Optimization in engineering design is one complex optimization problem which is highly nonlinear, includes large number of projected variables as well as numerous constraints in the form of equations and inequalities, which are also nonlinear. The optimal solution must satisfy the given constraints. Generally speaking, a constrained optimization problem can be formulated as follows:

$$\text{minimize:} \quad f(\mathbf{X}) \tag{1}$$

$$\begin{aligned} \text{subject to:} \quad & g_i(\mathbf{X}) \le 0, \quad i = 1,\dots,n_g \\ & h_j(\mathbf{X}) = 0, \quad j = n_g,\dots,n_h \end{aligned} \tag{2}$$

where $f(\mathbf{X})$ is objective function, $\boldsymbol{X} = (x_1, x_2, \dots, x_n)^T$ is n-dimensional vector which represents the design variables, $g_j(\mathbf{X}) \le 0$ are constraints in form of inequalities, n_g represents the number of constraint inequalities, $h_j(\mathbf{X}) = 0$ are constraints in form of equations and $(n_h - n_g)$ is the number of constraint equations.

Design variables are the values which should be defined during the optimization procedure. Each design variable is defined by its lower and upper boundaries. By introducing the constraints (2) in the objective function (1), Eqs. (1) and (2) can be transformed into the following form:

$$\text{minimize:} \quad F(\boldsymbol{X}) = f(\boldsymbol{X}) + \boldsymbol{P}(\boldsymbol{X}, k_j) \tag{3}$$

where $\boldsymbol{P}(\boldsymbol{X}, k_j)$ are the penalty functions which can be presented in the following way:

$$\boldsymbol{P}(\boldsymbol{X}, k_j) = \sum_{j=1}^{n_g} k_j \cdot \left[max\left(0, g_j(\boldsymbol{X})\right) \right]^2 \tag{4}$$

When the solution is found outside the region condemned, then the current parameters of the solution are squared and multiplied by large enough positive numbers (penalty factors) , and then added to the numerical values of the objective function.Coefficient k_j takes the same value in all examples in this paper, and that value is 100.

3 ALGORITHM CS

The power of almost all modern metaheuristic algorithms comes from the fact that they imitate the best characteristics from nature, particularly biological systems evolved by natural selection for millions of years. Two very important characteristics are: selection of the most favorable species and adaptation to the environment. Numerically, it can be translated into two very important characteristics of modern metaheuristics: intensification and diversification. Intensification searches for the best current solutions, while diversification allows the algorithm to search the space efficiently.

Cuckoo Search (CS) represents a new optimization metaheuristic algorithm, which is also biologically inspired by the cuckoos' manner of looking for nests where they could lay eggs. This algorithm, as already said, was proposed by Yang and Deb (2009) and (2010).

Cuckoos lay their eggs in other birds' nests and the host birds later take care of cuckoo chicks. Cuckoos usually choose the nest of a bird that has just laid its eggs so that they can be sure that their eggs would hatch first because cuckoo eggs hatch earlier than their host eggs birds. Some types of cuckoos have adapted to laying their eggs in other birds' nests so that their eggs are quite similar to the eggs of the host birds. When a cuckoo chick is hatched, it instinctively pushes out of the nest the host bird chicks and eggs that have not yet hatched to receive all the food brought in. A cuckoo chick can mimic the call of host chicks. If the host birds realize that a cuckoo egg has been laid in, they either remove the egg or abandon the nest.

In this optimization algorithm, each nest represents a potential solution. The cuckoo reproduction process in the algorithm is simplified by three rules:

1. Each cuckoo lays an egg in a randomly chosen nest;
2. The best nests carry over to the next generation of cuckoos;
3. The number of available host nests is fixed (limited), and the egg laid by a cuckoo is discovered by the host bird with a probability p_a, which ranges $[0,1]$. Birds can detect only the worst nests so that they are losing from the population.

CS has a simple algorithm, and its code is given in Yang and Deb (2010). The initial population of nests with the size n, which are randomly distributed over the search space, is generated first. The randomly chosen initial solutions of design variables are defined in the search space by the lower and upper boundaries.

The new nest, for example i-th, is generated according to the following law

$$\mathbf{x}_i^{(t+1)} = \mathbf{x}_i^{(t)} + \alpha \oplus Levy(\lambda), \tag{5}$$

where $\alpha > 0$ is the step size whose value depends on the optimization problem, and t is the current generation. Step size is multiplied by the random numbers with Lévy's distribution, and such random motion is called Lévy flight.

In this research work (Yang and Deb 2010) a Levy flight in which the step-lengths are distributed acording to the following probability distribution:

$$Le'vy \sim u = t^{-\lambda}, \quad 1 < \lambda \leq 3. \tag{6}$$

Lévy flight represents a variation of random walk, in which the step length is determined by Lévy distribution. Lévy flight represents one of the ways of motion used by birds for searching for food in the environment. When there is some food in the environment, animals perform motion which is analogous to Brownian motion. If they cannot find any food, animals start moving in the manner analogous to Lévy flight, i.e., by combining short and long steps in different directions thus searching a considerably larger space. The numerical algorithm proposed by Mantegna (1994), using the exponential law, was used for generation of Lévy distribution in the CS algorithm. It is recommended that the step size should be $L/100$, where L is the size of the space which is searched. There is a danger that Lévy flight may become too "aggressive" for large values of the step size and that new solutions may go out of the space which is searched.

A detailed description of the CS algorithm can be seen in Yang and Deb (2009) and (2010b). Yang and Deb (2009) came to the conclusion that the CS algorithm finds the optimum solution for the values of the parameter n from 15 to 25 while for the parameter p_a it is from 0.15 to 0.25.

4 IMPROVED CUCKOO SEARCH (ICS) ALGORITHM

In standard CS algorithm, parameters p_a and α are very important in finetuning of solution vector and appropriate selection of their values can result to the global solutions. However, as noted in previous section, values of these parameters are constant in the standard CS algorithm. Valian et al. (2013) have introduced dynamic changes of these parameters in each generation, in solving complex engineering problem. If the value of probability p_a is small, and the value of parameter α, which represents step size, is large, such values can result in very slow convergency in CS algorithm. Otherwise, if the value of p_a is large and the value of α is small, the speed of convergence is very fast and algorithm cannot find the best solution.

In all examples presented in this paper, in each generation the parameter p_a is increasing, and parameter α is decreasing as follows:

$$\begin{aligned} p_a &= p_a + \ rand * p_a \ / gn, \\ \alpha &= \alpha \cdot \exp \ 1 / gn \ . \end{aligned} \tag{7}$$

Value of gn in expression (7) represent the current generation (iteration) and grows up from 0 to some maximum specified number. Maximum number of generations in all examples in this paper is 1000. The probability p_a that the host bird will find cuckoo's egg depends only from the random number generator and rarely it may happen that generator "allows" host bird to find the egg or doesn't find it at all. That is the reason why dynamic change of probability is introduced, which is than compared with the actual probability of finding the egg. Namely, if the new calculated probability is smaller than the random generated value, the "interloper" is discovered by host bird and cuckoo must find new nest.

With regard to step size α, its recommended value is between 1 and 3. In all examples in this paper the starting value of this parameter is 2. It starts with higher step size so different solutions are generated in order to explore the search space on the global level. By reducing the step size, focusing on the search in a region where currently found a good solution is performing. Thus in an appropriate manner manage the process of selecting the best solutions, which results in finer approximation of the optimum value of the objective function.

Because the number of generations is increasing during the optimization process, values of these two parameters will stabilize by achieving a certain number of generations and in this case the change of the objective function is slightly small, which achieves its optimum value approximation.

The idea of ICS algorithm is that these parameters are adjustable in each generation, because in that way better solutions of algorithm can be achieved. Dynamic change of these parameters is proposed also in this paper, which made the results for discussed examples from practice better than the results from the mentioned literature.

5 EXPERIMENTAL RESULTS AND ANALYSIS

In this section, results of well-known examples from literature related to the engineering design are shown. Comparative analysis of results from the literature and results obtained by the proposed ICS - algorithm is given.

5.1 Experimental results: the minimization of the weight of spring

The problem of minimization of the weight of tension/compression spring (Figure 1) firstly was described by Arora (1989). Different optimal solutions of this problem in various researches are obtained by (Mahdavi et al., 2007; Jaberipour and Khorram, 2010; Akay and Karaboga, 2012; Coello, 2000a; Coello, 2000b; Kayhan et al., 2010; Zou et al., 2011; Mun and Cho, 2012; Lobato and Valder 2014). Constraints in this problem are: the minimum deflection, shear stress, frequency impact and outer diameter constraint. The design variables are the wire diameter $d = x_1$; the mean coil diameter $D = x_2$ and the number of active coils $N = x_3$. Mathematical formulation of these problem is described in Eqs. (8-12).

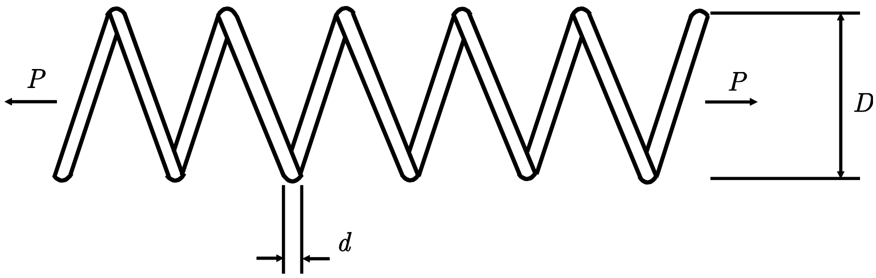

Figure 1. Tension/compression spring

In Table 1, optimal solutions which are obtained by (Mahdavi et al., 2007; Lobatoand Valder (2014); Zou et al., 2011; Mun and Cho, 2012) and two results obtained by proposed algorithm, are presented. The first result has minimum value of the objective function compared to the other results, but one constraint is not satisfied, while the second result is better than the results which are obtained by (Mahdavi et al., 2007; Lobatoand Valder (2014)) but worse than the results Zou et al. (2011), Mun and Cho (2012).

$$\text{Minimize} \quad f(\mathbf{x}) = (x_3 + 2)x_2 x_1^2 \tag{8}$$

$$\text{Subject to:} \quad g_1(\mathbf{x}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0, \tag{9}$$

$$g_2(\mathbf{x}) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \le 0, \tag{10}$$

$$g_3(\mathbf{x}) = 1 - \frac{140.45x_1}{x_2^2x_3} \le 0, \tag{11}$$

$$g_4(\mathbf{x}) = \frac{x_2 + x_1}{1.5} - 1 \le 0, \tag{12}$$

$$0.05 \le x_1 \le 2,\ 0.25 \le x_2 \le 1.3,\ \ 2.0 \le x_3 \le 15.0\,.$$

Table 1: Experimental results for minimization of the weight of spring

	Mahdavi et al. (2007) (IHS)	Lobato and Valder (2014) (FSO)	Zou et al. (2011) (DSO)	Mun and Cho (2012) (MHS)	Proposed method (a) (ICS)	Proposed method (b) (ICS)
x_1	0.05115438	0.051744	0.051711791	0.05171296	0.050000	0.05170254
x_2	0.34987116	0.357754	0.357264808	0.35729285	0.489169	0.35704214
x_3	12.0764321	11.56132	11.25696483	11.2553284	3.832967	11.26997226
$g_1(\mathbf{x})$	-0.0521995	-0.028697	-5.1563e-009	N/A	0.000000	-1.2854e-007
$g_2(\mathbf{x})$	0.0136707	-0.000645	-2.8087e-010	N/A	0.430373	0.000000
$g_3(\mathbf{x})$	-3.8601496	-3.911391	-4.0549	N/A	-6.656629	-4.054426
$g_4(\mathbf{x})$	-0.7326496	-0.727001	-0.7273	N/A	-0.960830	-1.091255
$f(\mathbf{x})$	0.0128874	0.012789	0.12665	0.0126652	0.0071332	0.01266524

5.2 Experimental results: the welded beam design

The welded beam design is shown in Figure 2. This problem has been used as an experimental benchmark to apply different optimization methods (Coello, 2000a; Mahdavi et al., 2007; Jaberipour and Khorram, 2010; Kayhan et al., 2010; Akay and Karaboga 2010; Gandomi et al., 2011; Zou et al., 2011; Moumen et al., 2011; Mun and Cho, 2012; Ashtari and Barzegar, 2012; Lobato and Valder 2014). The objective of this problem is to construct the welded beam, subjected to the constraint of shear stress τ, bending stress σ, buckling load P_C, beam deflection δ, so that it has minimum costs of production,. There are four design variables: $h = x_1$, $l = x_2$, $t = x_3$ and $b = x_4$. Mathematical formulation of the objective function is presented in Eqs. (13-20) and it represents total cost of production which is mainly comprised of the set-up, welding labor, and material costs as follows:

$$\text{Minimize} \qquad f(\mathbf{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \tag{13}$$

$$\text{Subject to:} \qquad g_1(\mathbf{x}) = \tau(\mathbf{x}) - \tau_{\max} \le 0\,, \tag{14}$$

$$g_2(\mathbf{x}) = \sigma(\mathbf{x}) - \sigma_{\max} \le 0\,, \tag{15}$$

$$g_3(\mathbf{x}) = x_1 - x_4 \le 0\,, \tag{16}$$

$$g_4(\mathbf{x}) = 0.1047x_1^2 + 0.04811x_3x_4(14 + x_2) - 5.0 \le 0\,, \tag{17}$$

$$g_5(\mathbf{x}) = 0.125 - x_1 \le 0\,, \tag{18}$$

$$g_6(\mathbf{x}) = \delta(\mathbf{x}) - \delta_{\max} \le 0\,, \tag{19}$$

$$g_7(\mathbf{x}) = P - P_C(\mathbf{x}) \le 0\,. \tag{20}$$

where:

$$\tau(\mathbf{x}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}\,, \tau' = \frac{P}{\sqrt{2}x_1x_2}\,, \tau'' = \frac{MR}{J}\,, M = P\left(L + \frac{x_2}{2}\right),$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}\,,\ \sigma(\mathbf{x}) = \frac{6PL}{x_4x_3^2}\,,\ \delta(\mathbf{x}) = \frac{4PL^3}{Ex_3^2x_4}\,,$$

$$J = 2\sqrt{2}x_1x_2\left(\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right),\ P_C(\mathbf{x}) = \frac{4.013E\sqrt{x_3^2x_4^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right).$$

$$P = 6000 \text{ lb}\,,\ L = 14 \text{ in}\,,\ \delta_{\max} = 0.25 \text{ in}\,,\ E = 30\times10^6 \text{ psi}\,,$$

$$G = 12\times10^6 \text{ psi}\,,\ \tau_{\max} = 13600 \text{ psi}\,,\ \sigma_{\max} = 30000 \text{ psi}\,,$$

$$0.125 \le x_1 \le 10\,,\quad 0.1 \le x_2 \le 10\,,\quad 0.1 \le x_3 \le 10\,,\ 0.1 \le x_4 \le 5\,.$$

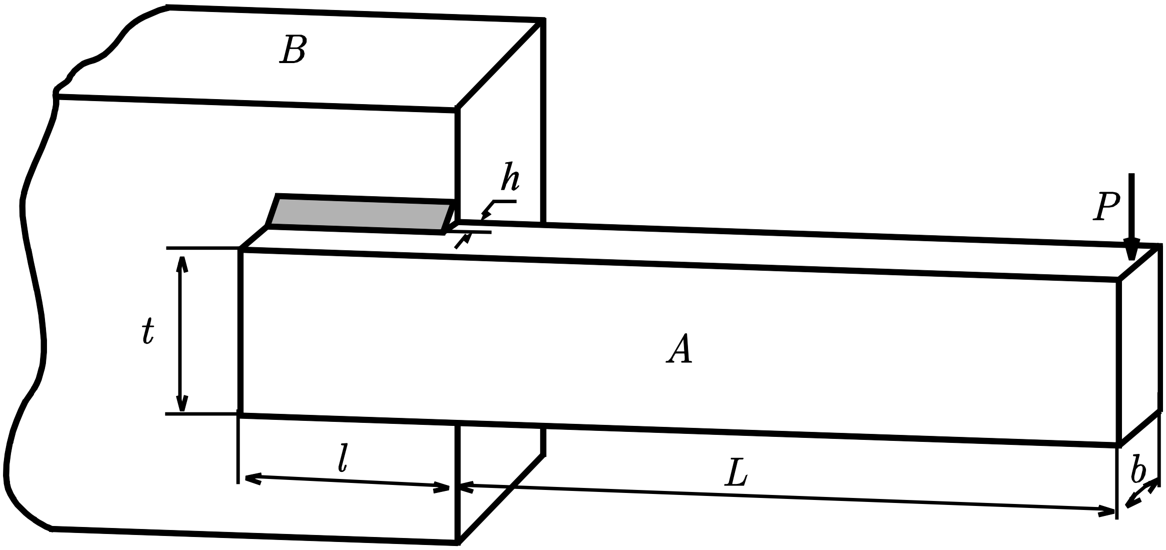

Figure 2. Welded beam structure

Based on Table 2, it can be seen that the results obtained by proposed algorithm are better than the results obtained by (Lobato and Valder 2014; Ashtari and Barzegar, 2012; Zou et al. 2011; Mun and Cho 2012).

Table 2. Experimental results for welded beam design.

	Lobato and Valder (2014) (FSO)	Ashtari and Barzegar (2012) (AFGA)	Zou et al. (2011) (DSO)	Mun and Cho (2012) (MHS)	Proposed method (ICS)
x_1	0.208796	0.2076	0.20571226	0.206704	0.227718
x_2	3.412545	3.4160	3.25344086	3.252355	1.611499
x_3	8.910044	9.0139	9.03661355	8.9759952	8.524100
x_4	0.210001	0.2057	0.20573012	0.208518	0.231704
g_1 $\boldsymbol{x}$	-23896.252	-74.220502	N/A	N/A	-10950.516240
g_2 $\boldsymbol{x}$	-230.95874	155.794377	N/A	N/A	-63.597209
g_3 $\boldsymbol{x}$	-0.001204	0.001900	N/A	N/A	-0.003985
g_4 $\boldsymbol{x}$	-3.384378	-3.441917	N/A	N/A	-3.511156
g_5 $\boldsymbol{x}$	-0.083796	-0.082600	N/A	N/A	-0.102719
g_6 $\boldsymbol{x}$	-0.235222	-0.235428	N/A	N/A	-0.234703
g_7 $\boldsymbol{x}$	-808.56989	12.521227	N/A	N/A	-2242.539283
f $\boldsymbol{x}$	1.7318117	1.71620746	1.69526699	1.707009	1.575729

5.3 Experimental results: the pressure vessel design

The third example is minimization of the pressure vessel production costsFigure3. The minimization of the total costs includes the cost of material, forming and welding of the cylindrical pressure vessel.In this example, there are four design variables: the shell thickness $T_s = x_1$, head thickness $T_h = x_2$, inner radius $R = x_3$ and length of cylindrical section of the vessel, not including the head $L = x_4$. Mathematical formulation of the objective function is given in Eq. (18-22). The variables x_1 and x_2 must be integer multipliers of $0.625in$ which are the available thicknesses of rolled steel plates. Many authors have dealt with this optimization problem in their research work (Coello, 2000a; Lobato and Valder 2014; Lee and Geem, 2004; Mahdavi et al., 2007; Akay and Karaboga, 2010; Kayhan et al., 2010; Jaberipour and Khorram, 2010; Gandomi et al., 2011, and 2013; Datta and Figueira, 2011; Zou et al., 2011; Mun and Cho, 2012).

$$\text{Minimize} \quad f(\boldsymbol{x}) = 0.6224x_1 x_3 x_4 + 1.7781x_2 x_3^2 + 3.1661x_1^2 x_4 + 19.84x_1^2 x_3 \tag{21}$$

$$\text{Subject to:} \quad g_1(\boldsymbol{x}) = -x_1 + 0.0193x_3 \leq 0\,, \tag{22}$$

$$g_2(\boldsymbol{x}) = -x_2 + 0.00954x_3 \leq 0\,, \tag{23}$$

$$g_3(\boldsymbol{x}) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leq 0\,, \tag{24}$$

$$g_4(\boldsymbol{x}) = x_4 - 240 \leq 0\,. \tag{25}$$

where: $\boldsymbol{X} = \left[x_1, x_2, x_3, x_4 \right]^T$

The ranges of the design parameters are: $0 \le x_1 \le 99$, $0 \le x_2 \le 99$, $10 \le x_3 \le 200$ and $10 \le x_4 \le 200$.

The results obtained by different methods from the cited literature are showed in Table 3 with four inequalities Eqs. (21-25), and in Table 4 with six inequalities Eqs.(21-25) and Eqs. (26,27). The minimum value of the objective function obtained by the proposed algorithm in this paper is the smallest in comparison to the values from the cited literature. Based on Table 3, it can be seen that the results obtained by proposed algorithm are better than the results obtained by (Lobato and Valder 2014; Gandomi et. al. 2013; Zou et al. 2011; Mun and Cho 2012).

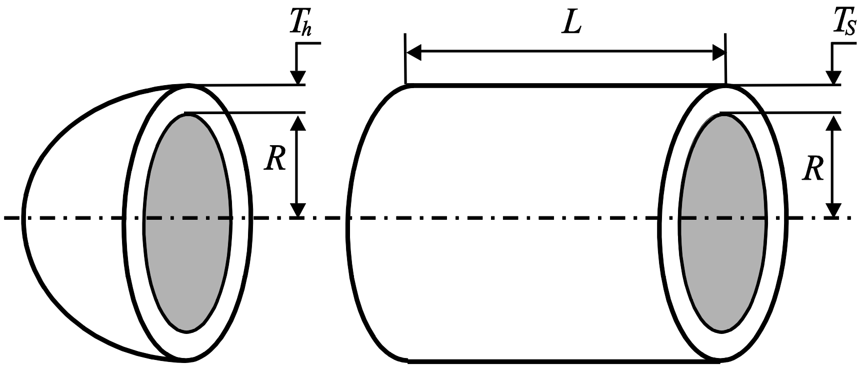

Figure 3. Shematic of pressure vessel

Table 3. Experimental results for pressure vessel design (four inequalities)

	Lobato and Valder (2014) (FSO)	Gandomi et al. (2013) (CS)	Zou et al. (2011) (DSO)	Mun and Cho (2012) (MHS)	Proposed method (ICS)
x_1	0.812500	0.8125	0.75	0.744060	0.738900
x_2	0.437500	0.4375	0.375	0.367789	0.365511
x_3	42.09127	42.0984456	38.860103	38.552312	38.279258
x_4	176.7466	176.6365958	221.365474	226.155260	230.508784
$g_1(\boldsymbol{x})$	-0.000139	N/A	-0.000004	N/A	-0.00001104
$g_2(\boldsymbol{x})$	-0.035949	N/A	-0.0043	N/A	-0.000327
$g_3(\boldsymbol{x})$	-116.3827	N/A	-0.0011	N/A	-71.6897153
$g_4(\boldsymbol{x})$	-63.25350	N/A	-18.6345	N/A	-9.491215
$f(\boldsymbol{x})$	6061.0778	6059.7143348	5850.38309	5829.54746	5823.372987

The second variant of the above mentioned problem, which has two additional inequalities is:

$$g_5(\boldsymbol{x}) = 1.1 - x_1 \le 0, \tag{26}$$

$$g_6(\boldsymbol{x}) = 0.6 - x_2 \le 0. \tag{27}$$

Table 4. Experimental results for pressure vessel design (six inequalities)

	Mahdavi et al. (2007) (IHS)	Lee and Geem (2004) (HS)	Zou et al. (2011) (DSO)	Mun and Cho (2012) (MHS)	Proposed method (ICS)
x_1	1.125	1.125	1.125	1.11157	1.102314
x_2	0.625	0.625	0.625	0.600	0.601033
x_3	58.29015	58.2789	58.290155	57.5944	57.113905
x_4	43.69268	43.7549	43.692656	47.5712	50.313880
$g_1(\mathbf{x})$	0.000000	-0.00022	-0.000000	N/A	-0.000016
$g_2(\mathbf{x})$	-0.06891	-0.06902	-0.0689	N/A	-0.056166
$g_3(\mathbf{x})$	-2.01500	-3.71629	-0.000148	N/A	-4.522062
$g_4(\mathbf{x})$	-196.307	-196.245	-196.3073	N/A	-189.686119
$g_5(\mathbf{x})$	N/A	N/A	N/A	N/A	-0.002314
$g_6(\mathbf{x})$	N/A	N/A	N/A	N/A	-0.001033
$f(\mathbf{x})$	7197.730	7198.433	7198.00542	7032.419	7028.064685

5.4 Experimental results: for speed reducer design

The objective of a speed reducer optimization shown in Figure 4 is weight minimization with constraints of the bending stress of gear teeth, surface stress, transverse deflections of shafts due to transmitted force, and stresses in shafts. The design parameters of a speed reducer are:face width $b = x_1$, module of teeth $m = x_2$, number of teeth on pinion $z = x_3$, length of shaft 1 between bearings $l_1 = x_4$, length of shaft 2 between bearings $l_2 = x_5$, diameter of shaft 1 $d_1 = x_6$, and diameter of shaft 2 $d_2 = x_7$. The third variable (number of teeth on pinion) is integer, while other values are continuous.

Mathematical formulation of the problem is presented in Eqs. (28-39):

$$\text{Minimize} \qquad f(\mathbf{x}) = 0.7850 x_1 x_2^2 \; 3.3333 x_3^2 + 14.9334 x_3 - 43.0934 \; - \\ -1.508 x_1 \; x_6^2 + x_7^2 \; + 7.477 \; x_6^3 + x_7^3 \; + 0.7854 \; x_4 x_6^3 + x_5 x_7^2 \tag{28}$$

$$\text{Subject to:} \qquad g_1(\mathbf{x}) = \frac{27}{x_1 x_2^2 x_3} - 1 \le 0 , \tag{29}$$

$$g_2(\mathbf{x}) = \frac{397.5}{x_1 x_2^2 x_3^2} - 1 \le 0 , \tag{30}$$

$$g_3(\mathbf{x}) = \frac{1.93 x_4^3}{x_2 x_3 x_6^4} - 1 \le 0 , \tag{31}$$

$$g_4(\mathbf{x}) = \frac{1.93 x_5^3}{x_2 x_3 x_7^4} - 1 \le 0 , \tag{32}$$

$$g_5(\boldsymbol{x}) = \frac{\sqrt{\left(745x_4 / (x_2x_3)\right)^2 + 16.9\times10^6}}{110x_6^3} - 1 \le 0\,, \tag{33}$$

$$g_6(\boldsymbol{x}) = \frac{\sqrt{\left(745x_5 / (x_2x_3)\right)^2 + 157.5\times10^6}}{85x_7^3} - 1 \le 0\,, \tag{34}$$

$$g_7(\boldsymbol{x}) = \frac{x_2x_3}{40} - 1 \le 0\,, \tag{35}$$

$$g_8(\boldsymbol{x}) = \frac{5x_2}{x_1} - 1 \le 0\,, \tag{36}$$

$$g_9(\boldsymbol{x}) = \frac{x_1}{12x_2} - 1 \le 0\,, \tag{37}$$

$$g_{10}(\boldsymbol{x}) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0\,, \tag{38}$$

$$g_{11}(\boldsymbol{x}) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0. \tag{39}$$

The ranges of the design parameters are: $2.6 \le x_1 \le 3.6$, $0.7 \le x_2 \le 0.8$, $17 \le x_3 \le 28$, $7.3 \le x_4 \le 8.3$, $7.3 \le x_5 \le 8.3$, $2.9 \le x_6 \le 3.9$ and $5.0 \le x_7 \le 5.5$.

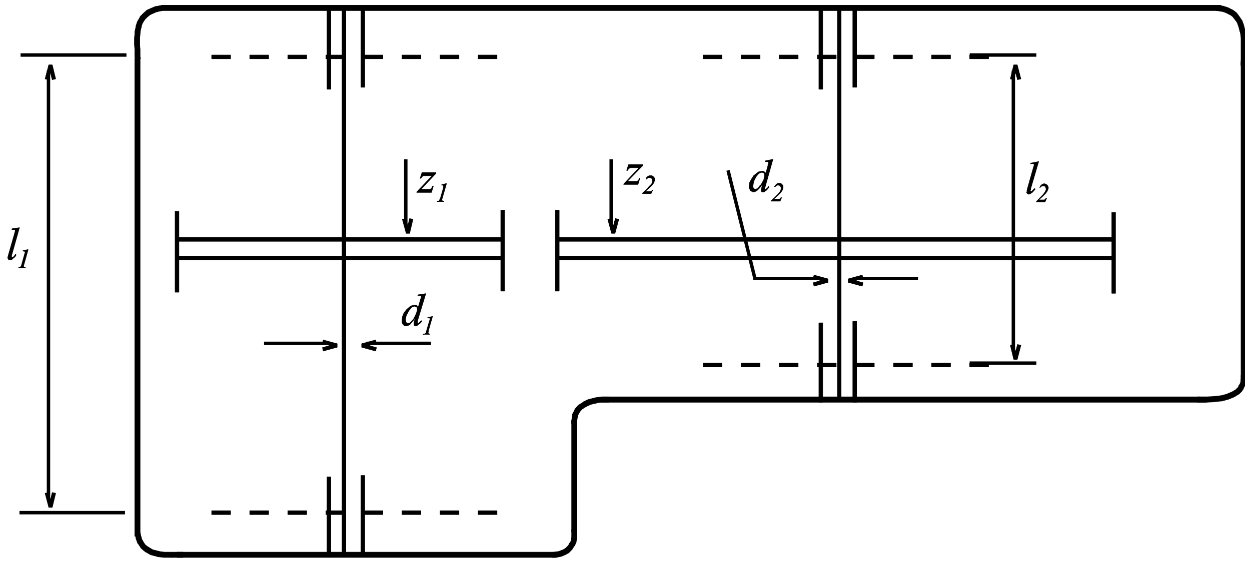

Figure 4. Shematicof the speed reducer

Table 5. Optimal results for speed reducer design

	Akay and Karaboga (2010) (ABC)	Gandomi et al. (2013) (CS)	Jaberipour and Khorram (2010) (HS)	Zhao et al. (2012) (HGA)	Proposed method(ICS)
x_1	3.499999	3.5015	3.500	3.5000	3.50000243
x_2	0.7000	0.7000	0.700	0.7000	0.70000000
x_3	17	17	17	17	17.00000000
x_4	7.3000	7.6050	7.3	7.3000	7.30024549
x_5	7.8000	7.8181	7.71533234	7.71531	7.80004077
x_6	3.350215	3.3520	3.35021511	3.350215	3.35025112
x_7	5.287800	5.2875	5.28666404	5.286654	5.28652023
$g_1(\mathbf{x})$	-0.073915	-0.0743	N/A	N/A	-0.0739159
$g_2(\mathbf{x})$	-0.0197999	-0.1983	N/A	N/A	-0.1979991
$g_3(\mathbf{x})$	-0.499172	-0.4349	N/A	N/A	-0.4991435
$g_4(\mathbf{x})$	-0.901555	-0.9008	N/A	N/A	-0.9014580
$g_5(\mathbf{x})$	0.00000	-0.0011	N/A	N/A	-3.223e-005
$g_6(\mathbf{x})$	-0.00000	-0.0004	N/A	N/A	-1.265e-006
$g_7(\mathbf{x})$	-0.7025	-0.7025	N/A	N/A	-0.70250000
$g_8(\mathbf{x})$	-0.00000	-0.0004	N/A	N/A	-6.943e-007
$g_9(\mathbf{x})$	-0.583333	-0.5832	N/A	N/A	-0.5833330
$g_{10}(\mathbf{x})$	-0.51326	-0.0890	N/A	N/A	-0.0513502
$g_{11}(\mathbf{x})$	-0.010695	-0.0130	N/A	N/A	-0.0108805
$f(\mathbf{x})$	2997.058412	3000.9810	2994.4	2994.4710	2996.2578

The results obtained using different methods are shown in Table 5. It can be seen from the table that the results obtained by ICS algorithm are better than the results which are presented in paper Gandomi et al., (2013), which were obtained by using CS algorithm. Also, the results in this paper are better than the results in paper Akay and Karaboga (2010), but they are worse than the results in Jaberipour and Khorram (2010) and Zhao et al., (2012). However, in these papers the values of constraints are not displayed, in other words cannot be seen whether the constraints are satisfied.

6 CONCLUSIONS

The Improved Cuckoo Search algorithm has been validated first using several benchmark engineering problems and found to be very efficient. Proposed ICS algorithm provides better performance than standard CS algorithm, and also from other methods proposed in the literature for solving four constrained problems from the practice. ICS algorithm is suitable for finding global optimum in solving problem of constrained optimization. Proposed algorithm is very flexible and aplicable for solving engineering problems and also promising for further research in this area.

The future works should focus on expanding the ICS algorithm to other fields of optimization due to high potential of the algorithm in solving difficult optimization problems.

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Serbian Ministry of Science and Technology for supporting this paper through project TR35038.

References

Akay, B., Karaboga, D., (2012). Artificial bee colony algorithm for large-scale problems and engineering design optimization. Journal of Intelligent Manufacturing 23(4),1001-1014.

Arora, J. S., (1989). Introduction to optimum design,New York; McGraw-Hill.

Ashtari, P., Barzegar, F., (2012).Accelerating fuzzy genetic algorithm for the optimization of steel structures, Structural and Multidisciplinary Opotimization 45(2), 275-285.

Bulatović, R.R., Đorđević R. S., Đorđević S. V., (2013). Cuckoo Search algorithm: A metaheuristic approach to solving the problem of opitmum synthesis of a six-bar double dwell linkage. Mechanism and Machine Theory 61, 1-13.

Coello, C. A. C., (2000(a)). Use of a self-adaptive penalty approach for engineering optimization problems. Computer in Industry 41(2), 113-127.

Coello, C. A. C., (2000(b)). Constraint-handling using an evolutionary multiobjective optimization technique. Civil Engineering and Environmental Systems 17, 319-346.

Datta, D., Figueira, J. R., (2011). A real-integer-discrete-coded particle swarm optimization for design problems. Applied Soft Computing 11(4), 3625-3633.

Gandomi, A. H., Yang Xin-She, Alavi A. H., (2011(a)). Mixed variable strucutural optimization using Firefly Algorithm. Computers and Structures 89(23-24), 2325-2336.

Gandomi, A. H., Yang Xin-She, Alavi A. H., (2013). Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems. Engineering with Computers, 21(1), 17-35.

Grković, V., Bulatović, R. R., (2013). Modified Ant Colony Algorithm for Solving Engineering Optimization Problems, IMK-14 – Research & Development, 18, 115-122.

Lee, K. S., Geem Z. W., (2004). A new meta-heuristic algorithm for continues engineering optimization: harmony search theory and practice. Computer methods in Applied Mechanics and Engineering 194(36-38), 3902-3933.

Jaberipour, M., Khorram, E., (2010). Two improved harmony search algorithms for solving engineering optimization problems. Commun Nonlinear SciNumerSimulat 15(11), 3316-3331.

Karaboga, D., Akay, B., (2011). A modified Artificial Bee Colony (ABC) algorithm for constrained optimization problems.Applied Soft Computing 11(3), 3021-3031.

Kayhan, A. H., Ceylan, H., Ayvaz M. T., Gurarslan, G., (2010). PSOLVER: A new hybrid particle swarm optimization algorithm for solving cintinuous optimization problems. Expert Systems with Applications 37(10), 6798-6808.

Lobato S. F., Valder Steffen Jr., (2014). Fish Swarm Optimization Algorithm Applied to Engineering System Dewsign, Latin American Journal of Solids and Structures, 11, 143-156.

Mahdavi, M., Fesanghary, M., Damangir, E., (2007). An improved harmony serch algorithm for solving optimization problems. Applied Mathematic and Computation 188(2), 1567-1579.

Mantegna, R.N., (1994). Fast, accurate algorithm for numerical simulation of Levy stable stochastic processes. Physical Review E 49, 4677-4683.

Moumen, S. E., Ellaia, R., Aboulaich, R., (2011). A new hybrid method for solving global optimization problem.Applied Mathematics and Computation 218(7), 3265-3276.

Mun, S., Cho, Y.Ho.,(2012). Modified harmony search optimization for constrained design problems. Expert Systems with Applications, 39(1), 419-423.

Yang, X.S., Deb, S., (2009). Cuckoo search via Levy flights, World Congres on Nature & Biologically Inspired Computing (NaBIC). IEEE Publications, 210-214.

Yang, X.S., (2010(a)).Nature-Inspired Metaheuristicalgorthms, Second edition Luniver Press, United Kingdom.

Yang, X.S., Deb, S., (2010(b)). Engineering Optimisation by Cuckoo Search.International Journal of Mathematical Modelling and Numerical Optimisation 2(4), 330-343.

Valian, E., Tavakoli, S., Mohanna, S., Haghi, A., (2013). Improved cuckoo search for reliability optimization problems, Computers & Industrial Engineering, 64, 1, 459-468.

Zahara, E., Kao Y. T., (2009). Hybrid Nelder-Mead simplex search and particle swarm optimization for constrained engineering design problems. Expert Systems with Applications 36(2), 3880-3886.Part 2.

Zou, D., Liu, H., Gao, L., Li., S., (2011). Directed searching optimization algorithm for constrained optimization problems.Expert Systems with Applications 38(7), 8716-8723.

Zhao, Jia-ging., Wang, L., Zeng, P., Fan, Wen-hui, (2012). An effective hybrid genetic algorithm with flexible allowance technique for constrained engineering design optimization. Expert Systems with Applications 39(5), 6041-6051.