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# Optimization of the parameters of PID controller on the model of inverted pendulum by using algorithm of particle swarm optimization

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*In the paper, two models for adjustment of the parameters of PID controller are analyzed. The first one is conventional relay method, which is based on artificially induced oscillations in the system. The second is unconventional, metaheuristics method of particle swarm optimization. Comparative results of the object simulation are managed by a controller, the parameters of which are adjusted by using these two methods. Inverted pendulum is used as controlled plant. The model of the object is determined by applying bond graph methodology. A response of non-linear and linear model for different initial conditions is analyzed.*

**Keywords: PID controller, inverted pendulum, Bond-graph, parameter optimization, particle swarm optimization.**

## 1. INTRODUCTION

PID controllers represent the most used type of controllers in the industry, but despite that fact, a significant part of controllers does not provide good control because of badly chosen parameters [1]. PID controller is relatively simple, but for its adequate behaviour a lot of experience is necessary. In 1942, Zigler and Nichols suggested experimental method of the adjustment of PID controllers, which gives rather good results. A problem with the Zigler-Nichols method occurs because it is necessary to bring the system to the limit of stability in order to find critical amplification and oscillation period, which are necessary in order to determine the parameters of the PID controller, which can be dangerous in some systems. In 1980s, the relay method appeared and it solved the problem. It implies the introduction of non-linearity (relay) into the main branch that causes system oscillation, and in that way, we can determine amplification and oscillation period.

Over the past few years, numerous metaheuristic optimization methods have appeared. These methods search randomly the solution space, looking for the solution that best satisfies the optimization criteria. The majority of these new algorithms are inspired by biological systems. In this paper, the method of particle swarm optimization is used. It is a relatively new method and it has given good results for different optimization problems. By using this method, parameters of the controller that provide a transitional process of much better quality than the parameters obtained by using the relay method.

## 2. ALGORITHM OF PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) represents metaheuristic method of optimization based on agents (particles) population, which was accidentally discovered by James Kennedy and Russell Eberhart in 1995, while studying the simulation of social behavior of bird flocking [2].

Figure 1 represents the algorithm of this method. Just as it is the case with all algorithms based on population, initial particle population is generated first. Position of the particle represents vector of parameters that are optimized: $\mathbf{x} = (x_1, x_2, ..., x_n)$, or a potential solution. Random position in space which is explored, as well as initial velocities, is given to each particle. After that, the value of the objective function of each particle is determined, and that value is added to it as the best value for the particle in question, while the initial position becomes the best position of the particle $\mathbf{p_{best}}$. When all the best values of particles are determined, the

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particle with the minimum value is searched, and its position becomes the best position for the entire swarm $\mathbf{p}_{\mathbf{gbest}}$. Afterwards, it is checked whether the criteria of optimization are satisfied, and if they are, the obtained results are displayed. If the criteria are not satisfied, new velocities and positions are to be calculated.

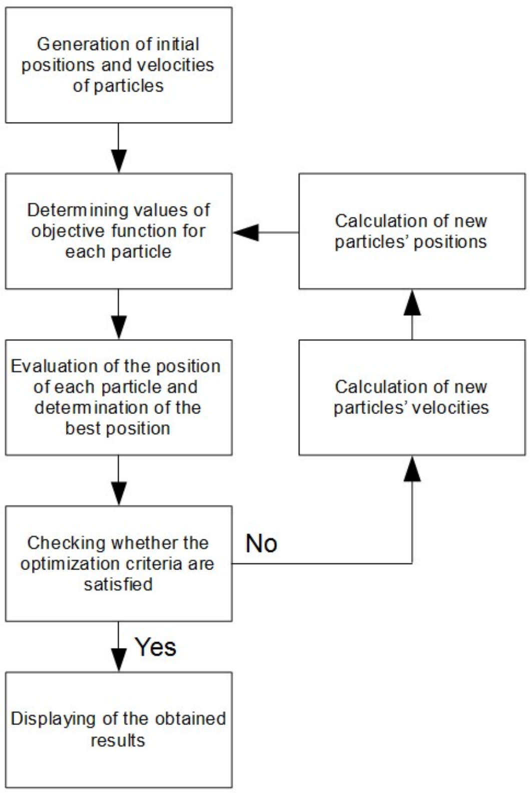


Fig. 1. *Algorithm of the method of particle swarm optimization*

Figure 2 graphically shows how to determine new velocities and positions in two-dimensional space of search.

New velocity of each particle consists of three components:

1. the component which depends on instantaneous particle velocity,
2. the component which is proportional to the distance of instantaneous position of the particle and its best value,
3. the component which is proportional to the distance of instantaneous position of the particle and its best position for the entire swarm.

$$\mathbf{v}_{i+1} = w \cdot \mathbf{v}_i + c_1 \cdot \mathbf{r}_1 \circ \left(\mathbf{p}_{\mathbf{best}i} - \mathbf{x}_i\right) + c_2 \cdot \mathbf{r}_2 \circ \left(\mathbf{p}_{\mathbf{gbest}i} - \mathbf{x}_i\right)$$

Where $w$ represents inertia weight, $c_1, c_2$ are acceleration coefficients or correction factors, $\mathbf{r}_1, \mathbf{r}_2$ represent two random vectors of the length $n$ within the limits [0,1]. The symbol $\circ$ represents Hadamard product:

$$\left(A \circ B\right)_{i,j} = \left(A\right)_{i,j} \cdot \left(B\right)_{i,j}$$

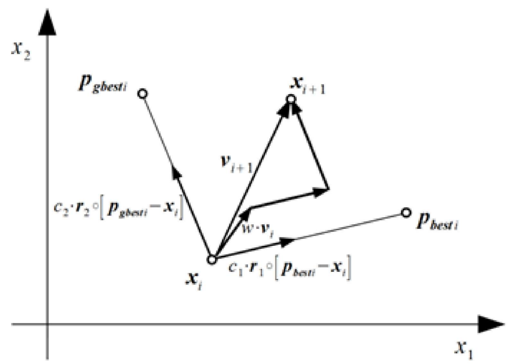

Fig. 2. *Updating of velocity and position of the i-th particle*

Inertia weight $w$ impacts the first component, and for the values in the range of 0,9 – 1,2 [4] it gives the best results, that is, the algorithm has greater chances of finding the global minimum for a reasonable number of iterations. For coefficient values which are smaller than 0,8, if algorithm finds global minimum it will find it fast. Particles in this case move quickly and it can happen that they "fly over" some area, so it can happen that they do not find global minimum. On the other side, if inertia weight has bigger value, then particles search the solution space more thoroughly and the chances of finding global minimum are greater.

New position of the particle is determined by simple adding of the current position $\mathbf{x}_i$ and new particle velocity $\mathbf{v}_{i+1}$.

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \mathbf{v}_{i+1}$$

The values of the objective function for new positions of the particle are determined again, and for each particle new and old values of the objective function are compared. If the new value is smaller, then it becomes new best value and the current position becomes the best position of that particle. The position of the particle with the smaller value becomes new best position for the entire swarm. Again, it needs to be checked whether the optimization criteria are satisfied; if they are, the results are shown, and if not, the entire procedure will be repeated until the criteria are satisfied.

## 3. PID CONTROLLER OPTIMIZATION

In this paper, parameter optimization of PID controller is performed, parameters of amplification are optimized ( $K_p$, $K_i$, and $K_d$ ). Algorithm of the PSO randomly chooses amplification, and after that, it calculates the system response, and determines the error. The given optimization criterion, which also presents objective function, is applied, and the values of amplification for which the function is minimum are chosen.

Four optimization criteria are used [6]:

1. IAE - integral absolute error ,
2. ISE - integral squared error,
3. ITAE - integral time multiplied absolute error,
4. ITSE - integral time multiplied squared error.

These criteria are defined in the following way:

$$J_1 = \int_0^T |e(t)|\, dt ,$$

$$J_2 = \int_0^T e(t)^2\, dt ,$$

$$J_3 = \int_0^T t|e(t)|\, dt ,$$

$$J_4 = \int_0^T te(t)^2\, dt .$$

Each of these criteria has its advantages and disadvantages. IAE provides good responses with relatively small overshoot, but with longer settling time. ISE decreases error very quickly, but oscillations occur. In order to get better performances and decrease the settling time, criteria ITAE and ITSE are used.

## 4. MATHEMATICAL MODEL OF INVERTED PENDULUM

Methodology used in modelling of physical systems primarily depends on the purpose of a model. If theoretic-analytical approach is used in studying of a system, mathematical models in form of differential and algebraic equations are usually used. However, optimization of controller parameters, which is used in this paper, is based on the results of simulation. For that reason, in this paper modelling is viewed from the simulation angle, that is, from performing of the "experiment" with the model. The final objective is not a mathematical model, but the model in the form that can be simulated easily on the computer.

For the inverted pendulum modelling we use bond graph (BG) [1, 2]. Bond graph is a graphical, object–oriented language, which is used for describing of energy processes in the system. The system is described as a network of domain-independent energy primitives each of which represents an ideal physical process. At the lowest level, the nodes in BG represent the main energy processes, and the edges among them represent the paths along which energy is exchanged. The main advantage is bigger flexibility of the model with the structure that follows the structure of a real system. Bond graph is translated easily into mathematical model by using the standard procedures [1, 2].

Figure 3 schematically shows inverted pendulum with the parameters. Pendulum is a uniform rigid rod of length $2l$, mass $m$, and moment of inertia $J$. The pendulum is coupled to the cart mass $M$, which moves under the influence of the force $F(t)$ along plane area.

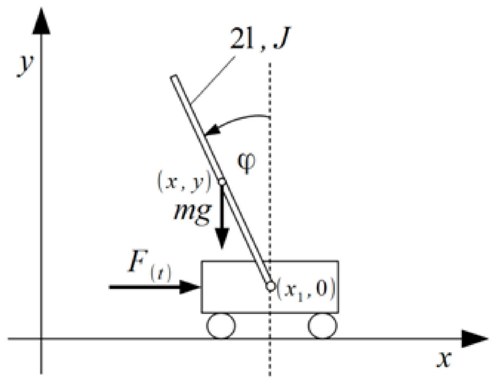


Fig. 3. *Schematic of an inverted pendulum*

In order to get to the model of the entire system, pendulum and cart models will be performed individually, and then they will be linked in a unique system. Figure (3) shows that the following geometrical relations are valid:

$$x(t)=x_1(t)-l\sin\varphi(t) \quad (1a)$$

$$y(t)=l\cos\varphi(t) \quad (1b)$$

Differentiation with respect to time gives:

$$\dot{x}(t)=\dot{x}_1(t)-l\omega(t)\cos\varphi(t) \quad (2a)$$

$$\dot{y}(t)=-l\omega(t)\sin\varphi(t) \quad (2b)$$

The velocity constraints (2) can be represented by the BG of Fig.4.

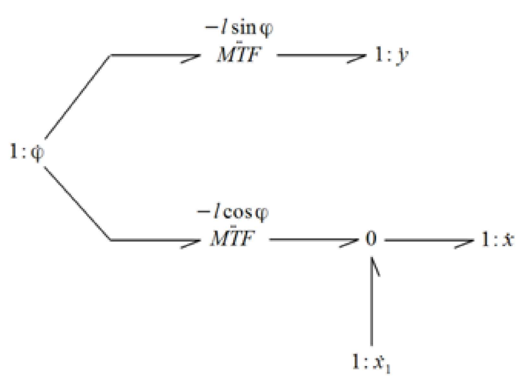


Fig. 4. *Bond graph representation of the velocity constraints (2)*

By adding the BG elements on the structure shown in fig.4, we come to the BG model of pendulum shown in fig 5.

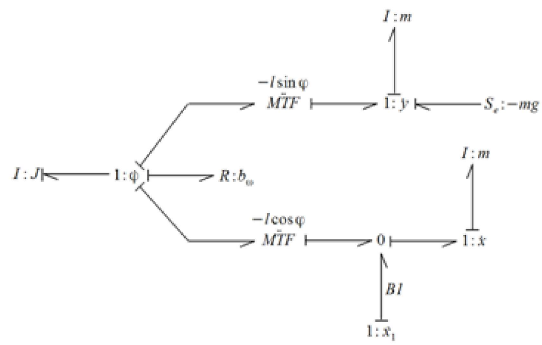


Fig. 5. *Bond graph of the pendulum*

The force of gravity is modeled by the effort source. Kinetic energy of translational and rotational movement is modeled by $I$ storage elements: $I:m$ and $I:J$. Resistor models the moment of viscous friction. For simplicity, a linear friction characteristic is assumed. Bond B1 represents the channel through which the energy from the cart towards the pendulum is transmitted. By using the procedure SCAP [1], causality is assigned to the model, as it is shown in the figure. It can be seen that $I$ elements which correspond to the translator movement have differential causality, that is, the pendulum has one degree of freedom. It can also be seen that the cart dictates the translational movement velocity along the axis $x$. The pendulum reacts with force, which acts upon the cart along the same direction.

Bond graph of the cart with causality is shown in Fig. 6.

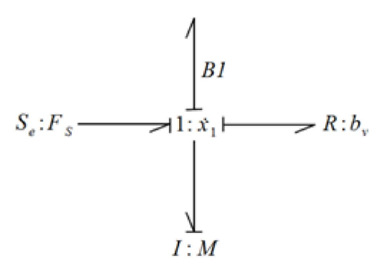


Fig. 6. *Bond graph of the cart*

The energy that comes from the external force is transformed into: kinetic energy of the cart, energy of losses due to viscous friction, and the part that is transmitted towards the pendulum.

All of these transformations have a common cart velocity.

By linking the model from Fig. 5, and the model from Fig. 6 by using bond B1, model of nonlinear bond graph is obtained. Nonlinearity comes from modulated transformers (MTF)

Linear model of the pendulum is obtained by introducing the assumption that rotations around the angle are small. In that case i is valid, and so, two transformers are obtained in the model (one modulated, and the other non-modulated) with coefficients i. Besides, based on the previous assumption, kinetic energy of translational moving along the axis is disregarded. This is how we get to the linearised model of the pendulum, which is shown in Figure 7.

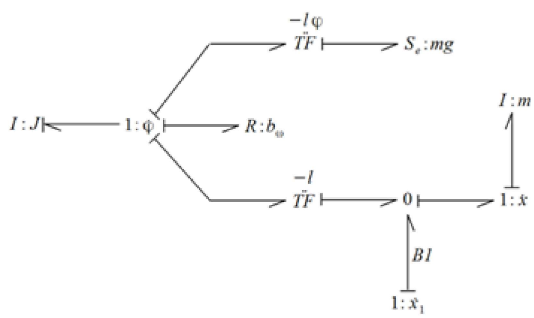


Fig. 7. *Linearised bond graph model of the pendulum*

By using the models shown in figure 5 and figure 6, that is, figure 6 and figure 7, in the continuation, we give the optimization results of parameters of controlling device of the inverted pendulum.

## 5. RESULTS

During the simulation of linear and non-linear model, the following parameters are used:

| $M$ | $m$ | $l$ | $b_v$ | $b_\omega$ | $g$ |
|---|---|---|---|---|---|
| 0,5 | 0,2 | 0,3 | 0,1 | 0,05 | 9,81 |

and the following parameters of the algorithm of particle swarm optimization (algorithm):

| | |
|---|---|
| Number of particles | 30 |
| Number of iterations | 20 |
| Inertia weight $w$ | 1 |
| Acceleration coefficient $c_1$ | 2 |
| Acceleration coefficient $c_2$ | 2 |
| Boundaries for $K_p$ | $0-1000$ |
| Boundaries for $K_i$ | $0-1000$ |
| Boundaries for $K_d$ | $0-1000$ |

The controller parameters obtained by using the relay method for linear model are:

| Controller | $K_p$ | $K_i$ | $K_d$ |
|---|---|---|---|
| P | $1{,}8791\cdot10^5$ | $-$ | $-$ |
| PI | $1{,}5032\cdot10^5$ | $2{,}5917\cdot10^5$ | $-$ |
| PID | $1{,}5032\cdot10^5$ | $1{,}9272\cdot10^7$ | 137,167 |

Optimization resulted in the following values for amplification of P controller for linear model of inverted pendulum:

| Optimization criterion | $K_p$ |
|---|---|
| IAE | 52,0890 |
| ISE | 52,0887 |
| ITAE | 98,0892 |
| ITSE | 52,0879 |

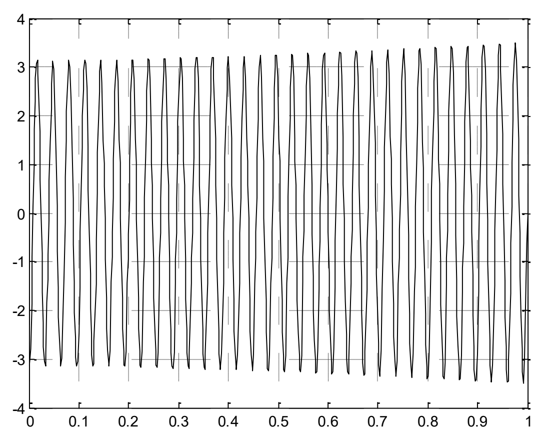


Fig. 8. *System response for the P controller that is adopted by relay method*

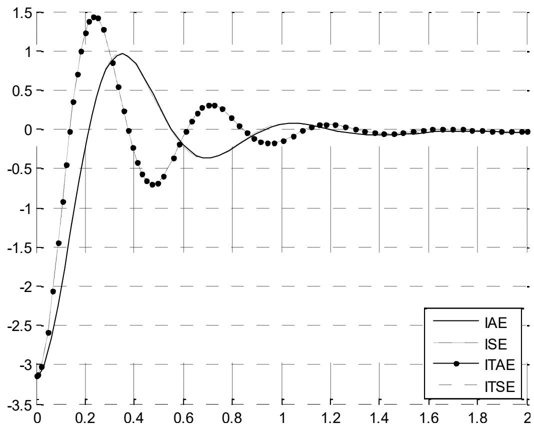

Fig. 9. *System responses with P controllers that are obtained by using PSO optimization*

Optimization resulted in the following values for amplification of PI controller for a linear model of inverted pendulum:

| Optimization criterion | $K_p$ | $K_i$ |
|---|---|---|
| IAE | 36,3938 | 5,0456 |
| ISE | 52,0914 | 0 |
| ITAE | 37,3143 | 5,4657 |
| ITSE | 52,1357 | 0,9683 |

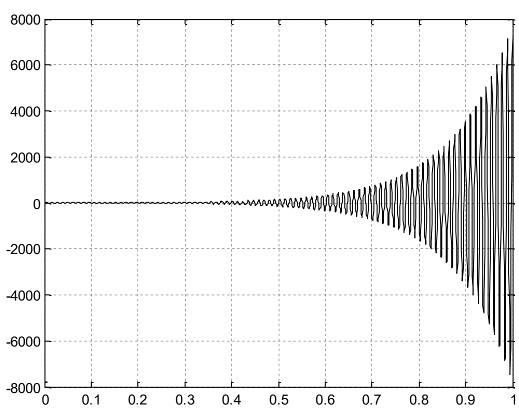
Fig. 10. *System responses with PI controllers that are obtained by using relay method*

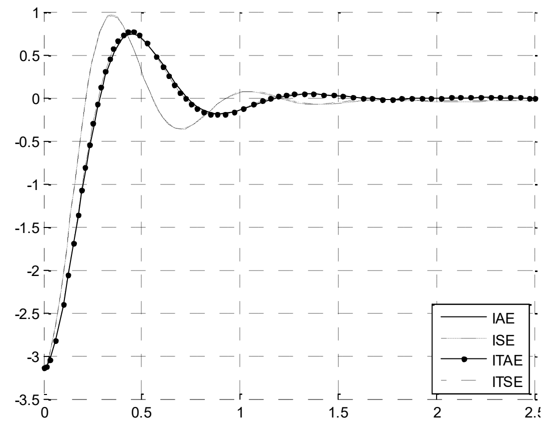

Fig. 11. *Linear system responses with PI controllers that are obtained by using PSO optimization*

Optimization resulted in the following values for amplification of PID controller for a linear model of inverted pendulum:

| Optimization criterion | $K_p$ | $K_i$ | $K_d$ |
|---|---|---|---|
| IAE | 557,4205 | 0 | 12,1933 |
| ISE | 665,4343 | 0 | 8,4662 |
| ITAE | 1000 | 15,5363 | 24,7509 |
| ITSE | 1000 | 0 | 20,3392 |

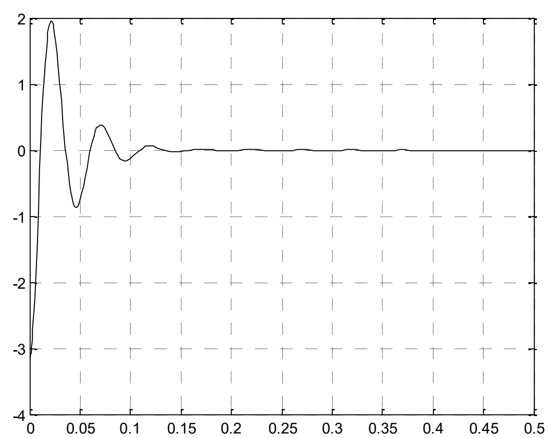
Fig. 12. *System responses with PID controllers which are obtained by using relay method*

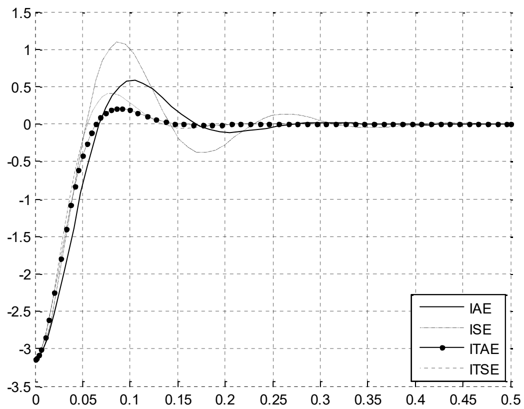


Fig. 13. *System responses with PID controllers that are obtained by using PSO optimization*

Optimization resulted in the following values for amplification of PID controller for a nonlinear model of inverted pendulum:

| Kriterijum optimizacije | $K_p$ | $K_i$ | $K_d$ |
|---|---|---|---|
| IAE | 1000 | 25,5908 | 67,2648 |
| ISE | 1000 | 0 | 67,0131 |
| ITAE | 416,6178 | 1000 | 48,2147 |
| ITSE | 1000 | 12,5959 | 71,6567 |

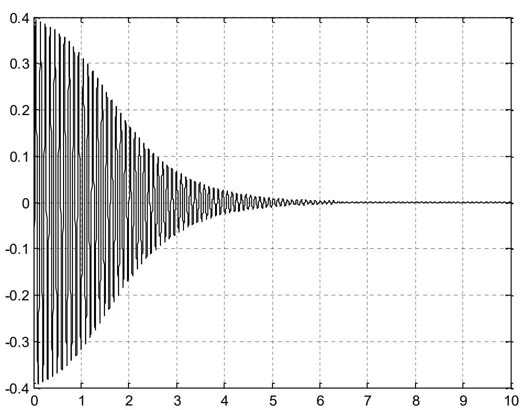

Fig. 13. *Nonlinear system responses with PID controllers that are obtained by using relay method*

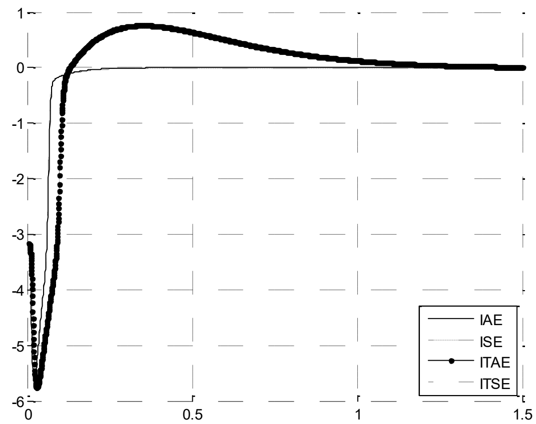


Fig. 14. *Nonlinear system responses with PID controllers that are obtained by using PSO optimization*

## 6. CONCLUSION

By using the relay method for obtaining controller parameters, we only obtained stable response for the case of PID regulator for linear and nonlinear model, while unstable response is present when P I PI controllers are used on the linear model. Likewise, PID controller at nonlinear model has a narrow region in which it works well, and that is the region around the vertical axis in which nonlinearity is expressed the least.

On the other side, controllers with the parameters obtained by using particle swarm optimization provided stable responses for both linear and nonlinear model of inverted pendulum. Optimized parameters are several times smaller than the parameters obtained by using the relay method. For both linear and nonlinear model, optimized controllers reduced the angle of the rod to zero, with the initial angle of 180˚; in other words, the rod was in the lower position at the initial moment.

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