

Modified 2D arc-star-shaped structure with negative Poisson's ratio

Vladimir Sindelić^{1*}, Aleksandar Nikolić¹, Nebojša Bogojević¹, Olivera Erić Cekić¹, Snežana Ćirić Kostić¹

¹ Faculty of Mechanical and Civil Engineering in Kraljevo, University of Kragujevac, Kraljevo (Serbia)

The metamaterials with negative Poisson's ratio have played an important role in engineering practice in the last decade. Their use in space, aviation, the automotive industry, and biomedicine increasing constantly. The basic idea of making these metamaterials is to use additive manufacturing to create a structure that has a negative Poisson's ratio, unlike conventional materials that are most often used in industry (steel, wood, rubber, etc.). The design process of these metamaterials takes place by first considering the properties of the elementary structure, and then by multiplying that structure, a metamaterial with the same Poisson's ratio as the basic structure will be obtained. The paper discusses a modified 2D arc-star-shaped structure in relation to the already existing variant in the literature. The influence of the parameters of the newly proposed structure on Poisson's ratio by using the finite element method was analyzed.

Keywords: Metamaterials, Negative Poisson's ratio, Auxetic structures, Arc-star-shaped structure

1. INTRODUCTION

Various branches of industry, such as aviation, space, or automotive, as well as biomedicine have needs for the application of ultralight materials which have the appropriate mechanical properties. Conventional materials (wood, metal, rubber, etc.) often cannot meet these two criteria simultaneously. For this reason, so-called mechanical metamaterials are increasingly used. The word "metamaterials" is a combination of the Greek word "meta" which means above and the Latin word "materia" which means material.

The most common requirement for these materials is that they have a negative Poisson's ratio (NPR in the further). Conventional materials have a positive Poisson's ratio. If this kind of material is pressed in one direction there will be an elongation in the direction perpendicular to the direction of pressure, as it is shown in Fig. 1 (a). On the contrary, in materials with a NPR due to pressure in one direction, the shortening occurs in the perpendicular direction, as shown in Fig. 1 (b).

The basic idea of making these metamaterials is to use additive manufacturing to create a structure that has a NPR. The design process of these metamaterials takes place by first considering the properties of the elementary structure, and then by multiplying that structure, a metamaterial with the same Poisson's ratio as the basic structure will be obtained.

Decades ago, a structure in the form of a honeycomb was used as the main mechanical metamaterial, the geometry of which corresponded to a structure with a positive Poisson's coefficient. However, in recent years, studies around the world have shown the possibility of designing mechanical metamaterials with a NPR.

Metamaterials with this property are called "Auxetic structures" in the literature. The word "Auxetic" comes from the Greek language, which in literal translation would represent "pulled in" [1-3]. The first structure with a NPR was created from a structure in the form of a honeycomb, in such a way that the two vertices of the structure of the honeycomb are drawn toward the

center of the structure, as shown in Fig. 2. The structure obtained in this way was named re-entrant Honeycomb.

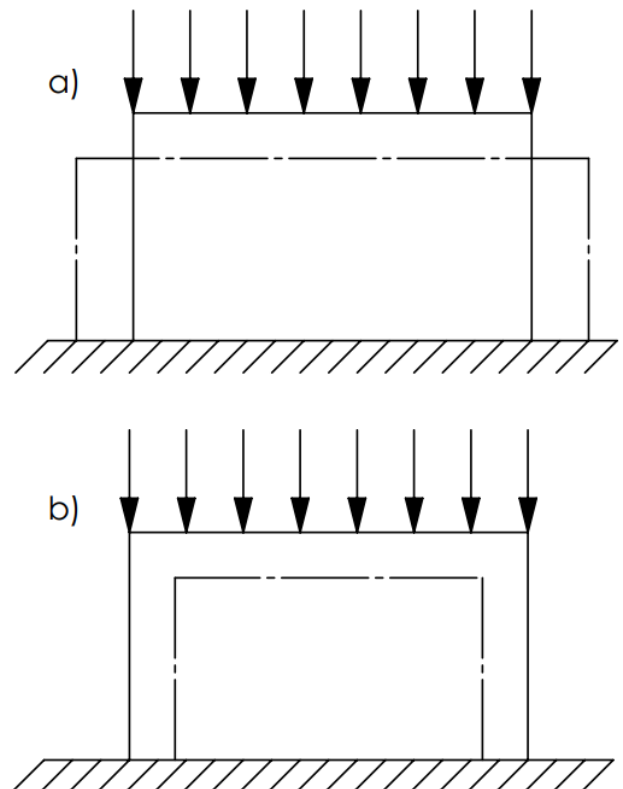


Figure 1: Deformations under pressure: (a) Conventional materials, (b) Metamaterials with negative Poisson's ratio

In the last decade, the re-entrant honeycomb structure has undergone a series of shape changes, with a tendency to retain its original behavior.

Thus, in reference [4] the new shape of the re-entrant honeycomb structure was presented, whereas an analytical and finite element method (FEM in the further) results were compared. Reference [5] provides an analytical model, the FEM model, as well as experimental verification on a model of a re-entrant honeycomb structure made by using additive manufacturing.

*Corresponding author: Dositejeva 19, 36000 Kraljevo, Serbia, e-mail: sindjelic.v@mfkv.kg.ac.rs

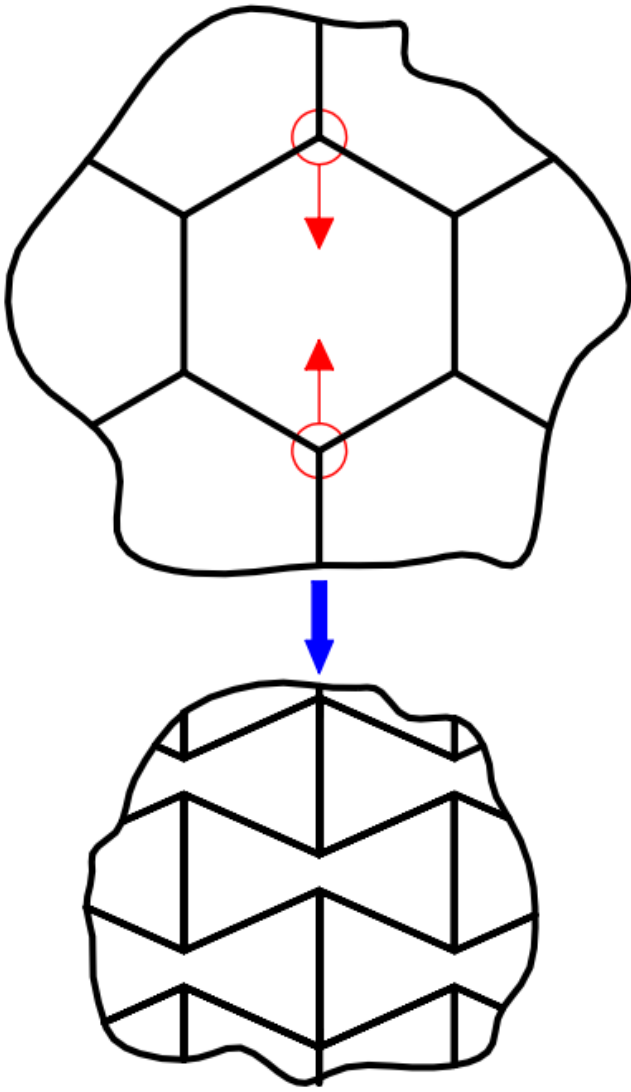


Figure 2: Honeycomb and Re-Entrant Honeycomb

The results of testing of energy absorption characteristics of the structure made by the combination of two structures with a NPR were given in [6]. In more detail, this structure was created by the fusion of the re-entrant honeycomb structure and a star-shaped structure. The star-shaped structure got its name because its geometry resembles a four-pointed star. Its shape guarantees that it will have the behavior of a structure with a NPR. Like the re-entrant honeycomb structures, it is also subject to changes in shape with the same goal of increasing the NPR and improving other mechanical properties. Changes in the shape of the star-shaped structure were experimentally examined in two papers [7-8]. The goal of these studies was to find an optimal shape that will improve the mechanical properties of the 2D star-shaped structure.

In the paper [9], a new form of 2D arc-star-shaped structure was analyzed, as well as two of its variants in 3D space. The shape of this structure is based on the classic star-shaped structure. This paper contains theoretical, numerical, and experimental analyses, to examine the influence of changes in the geometrical parameters of the structure on the NPR.

In this study, a modified 2D-AS structure is presented, based on the 2D-AS structure from the previously mentioned reference. The investigation of the modified 2D-AS structure was carried out by numerical analysis (FEM), the results of which were compared with the numerical results of the 2D-AS structure, to obtain higher values of NPR.

2. A MODIFIED 2D-AS STRUCTURE

2.1. Design of the modified 2D AS structure

Figure 3 shows the 2D-AS structure with all geometric parameters necessary for its construction: the length L , height h , thickness t , depth d , arc radius r , arc angle θ , angle coefficients a and b and total lengths L_x and L_y in the horizontal and vertical directions, respectively.

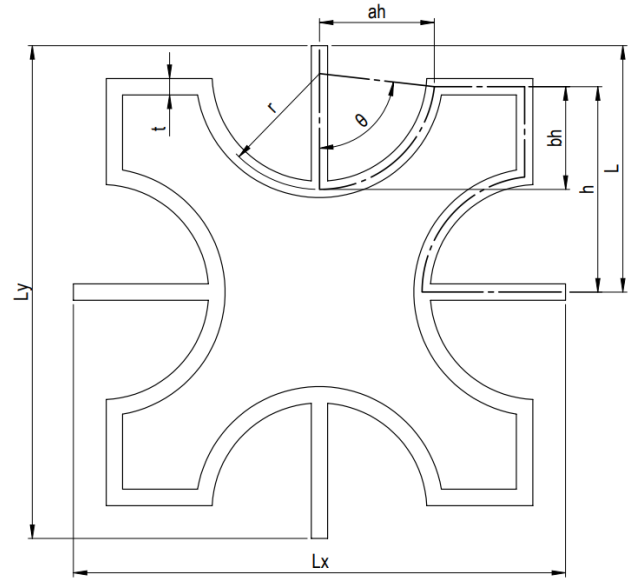


Figure 3: Geometric parameters of the 2D-AS structure [9]

The values of coefficients a and b are within the limits $0 < a < 1$ and $0 < b < 1$ [9]. Also, the parameters a , b , θ , and r are related in the following manner:

$$\theta = 2 \arctan \frac{bh}{ah}, \quad (1)$$

$$r = \frac{ah}{\sin \theta}. \quad (2)$$

The modified 2D-AS structure was created by redesigning the end branches of the 2D-AS structure, i.e. by adding another circular arc of radius R , as shown in Fig. 4.

Now, the relation between parameters a , h , and R holds:

$$R = \sqrt{2} \cdot (h - ah) / 2. \quad (3)$$

The relative density of the modified 2D-AS structure (RD) can be obtained as below:

$$RD = \frac{V_s}{V_c}, \quad (4)$$

where

$$V_s = 4dt \left(\frac{r \cdot \pi}{180} \cdot \theta + R \cdot \pi + L - h + bh \right) \quad (5)$$

represents the volume of solid model of the modified 2D-AS structure, and

$$V_C = L_x \cdot L_y \cdot d = 4dL^2 \quad (6)$$

represents the total volume that the solid model of the modified 2D-AS structure takes up in space.

By applying equations (4), (5) and (6) we obtain an expression for calculating the relative density of the modified 2D-AS structure:

$$RD = \frac{t \cdot \left(\frac{r \cdot \pi}{180} \cdot \theta + R \cdot \pi + L - h + bh \right)}{L^2} \quad (7)$$

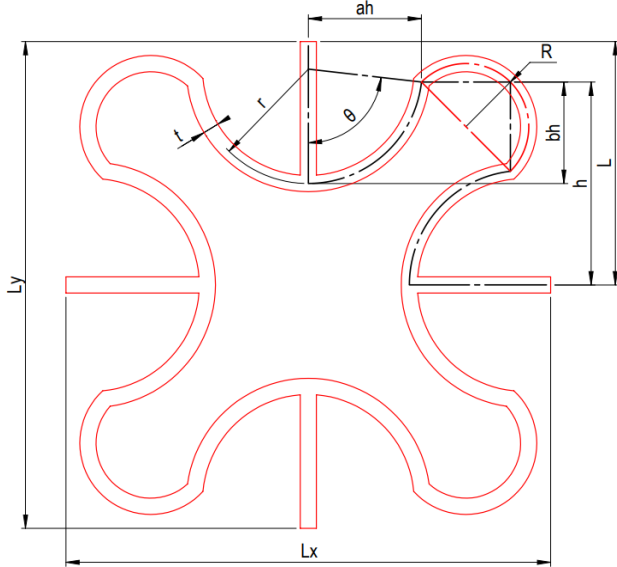


Figure 4: Geometric parameters of the modified 2D-AS structure

2.2. Deformation of the modified 2D-AS structure – numerical example

We report here the change of Poisson's ratio value of the modified 2D-AS structure relative to the initial shape of the structure by using the finite element method in the ANSYS software package.

The Fig. 5 shows how the elementary modified 2D-AS structure was extracted from the whole in the example of the 3x3 array. As it is shown in the Fig. 6, the bottom end A of the elementary structure is clamped. The vertical displacement of $\Delta yx=3\text{mm}$ was introduced at the upper end B of the structure. The examined structure had fixed dimensions: $L=30\text{ mm}$, $h=25\text{ mm}$, $t=2\text{ mm}$, $d=3\text{ mm}$, and $bh=12.5\text{ mm}$ except the value of the ah which takes the values from 8 mm to 22 mm with a step of 2 mm. The mesh size was 1x1 mm, and the number of finite elements varied depending on the parameter ah .

Based on the given vertical displacement Δyx , the horizontal displacements $\Delta xx/2$ of points C and D were measured in Ansys, as sketched in Fig. 6.

The value of the Poisson's ratio is determined according to the formula [10]:

$$\nu = -\frac{\varepsilon_x}{\varepsilon_y} = -\frac{\Delta_{yx}}{\Delta_{yy}} \cdot \frac{L_y}{L_x} \quad (8)$$

where ε_x and ε_y represents dilatation and Δyx and Δxx are displacements in horizontal and vertical directions, respectively.

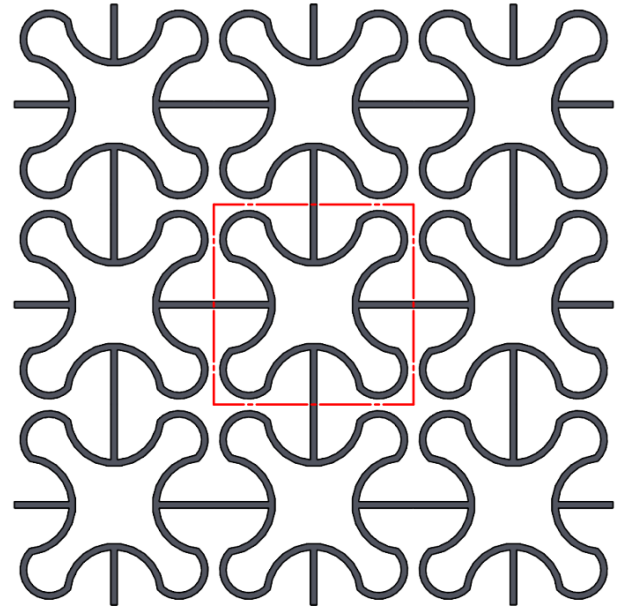


Figure 5: Schematic illustration of the modified 2D-AS structure in a 3x3 array

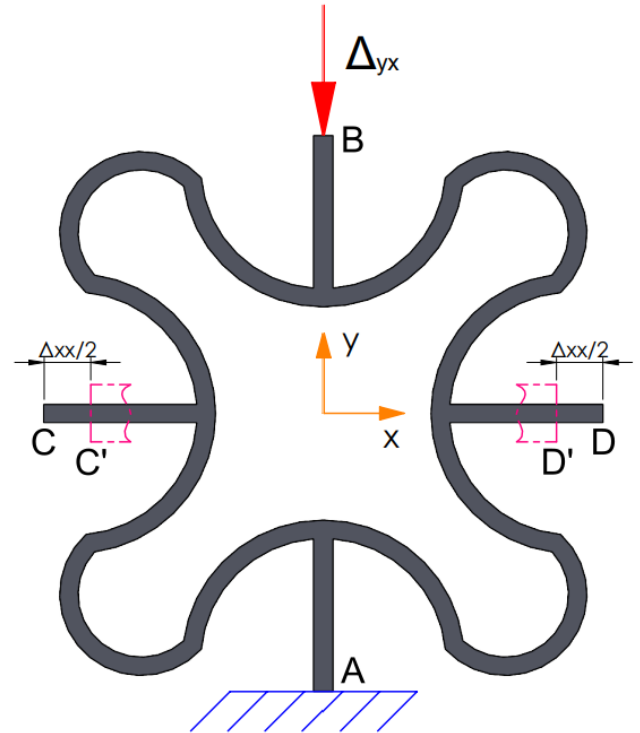
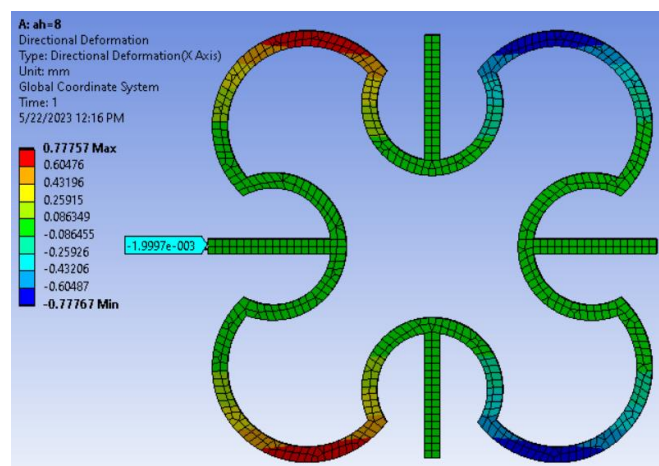
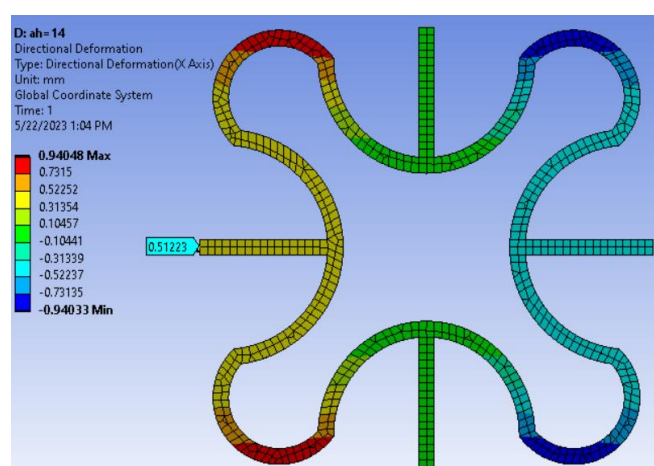
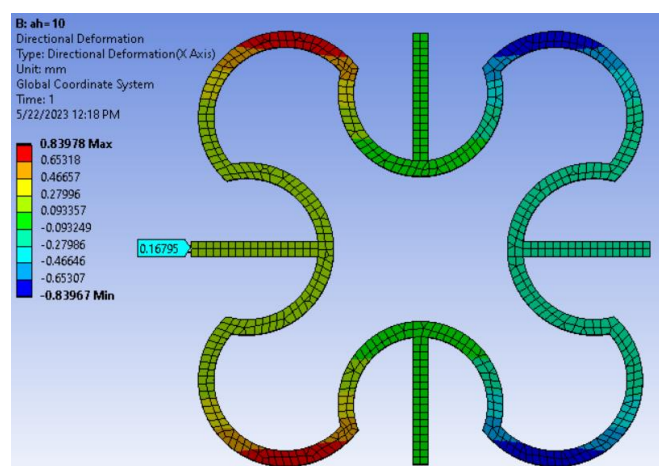
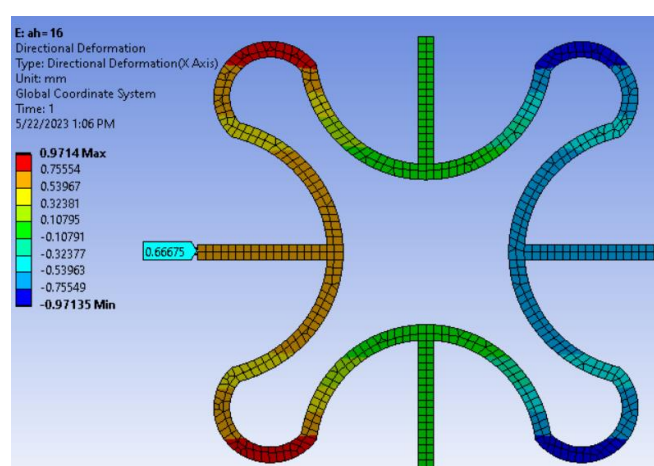
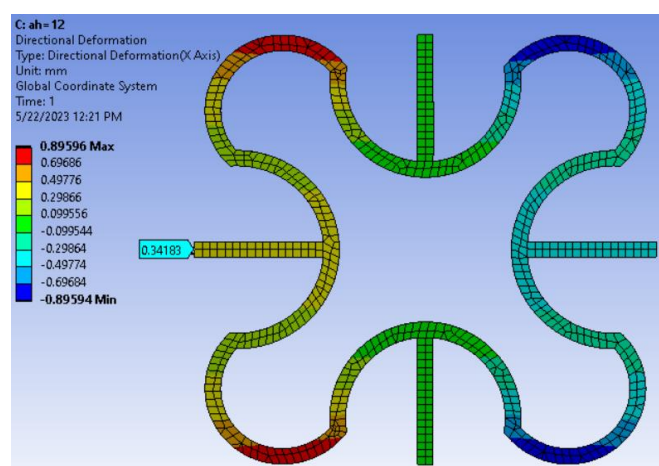
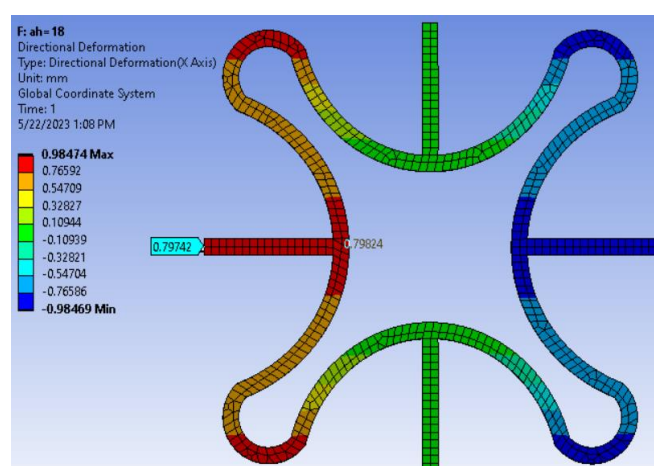
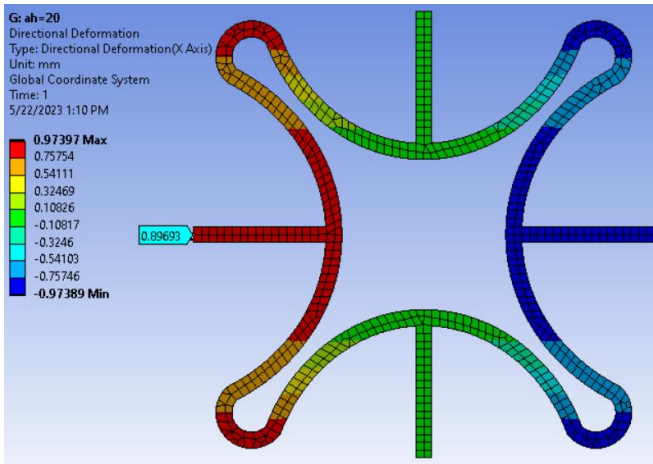
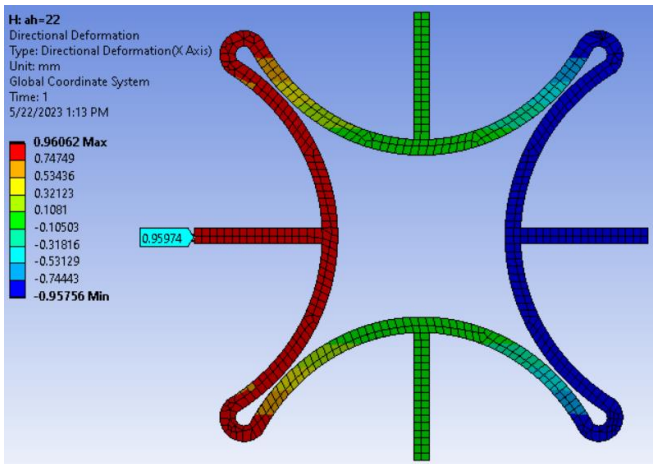


Figure 6: Deformation of the modified 2D-AS structure

The horizontal displacements Δxx for the various parameters ah of the modified 2D-AS obtained by Ansys are given at Figs. 1-14.

Now, based on the specified displacements in the vertical direction $\Delta yx=3\text{mm}$ and the measured displacements in the horizontal direction Δxx , the value of Poisson's ratio can be obtained for each of the specified values of the parameter ah by using formula (8).

Figure 7: Horizontal displacements Δ_{xx} for $ah=8$ mmFigure 10: Horizontal displacements Δ_{xx} for $ah=14$ mmFigure 8: Horizontal displacements Δ_{xx} for $ah=10$ mmFigure 11: Horizontal displacements Δ_{xx} for $ah=16$ mmFigure 9: Horizontal displacements Δ_{xx} for $ah=12$ mmFigure 12: Horizontal displacements Δ_{xx} for $ah=18$ mm


 Figure 13: Horizontal displacements Δ_{xx} for $ah=20$ mm

 Figure 14: Horizontal displacements Δ_{xx} for $ah=22$ mm

The comparison of the obtained values of the Poisson's ratio of the 2D-AS structure and the modified 2D-AS structure for the various values of the parameter ah is shown in Table 1. For the first three values of the parameter $ah=8, 10$, and 12 mm the decrease in the absolute values of the Poisson's ratio of the modified 2D-AS relative to 2S-AS is observed. For values of the parameter $ah>13$ mm, the absolute value of the Poisson's ratio in the modified 2D-AS is larger compared to the classical 2D-AS. It is interesting to note that for $ah=13$, an equal values of Poisson's ratio are obtained for both structures, modified 2D-AS and 2D-AS, which can be seen in the diagram shown in Fig. 15. Curves which describe the change of the Poisson's ratio of 2D-AS and the modified 2D-AS structure are thin solid line and slightly thicker dashed line, respectively.

Furthermore, the numerical values of the relative density RD of both above mentioned structures for the variable values of the parameter ah are given in Table 2. The relative density determined for the modified 2D-AS is higher than the corresponding density for the classical 2D-AS structure. The difference is the largest at $ah=8$ mm and amounts to 6.63%, and the smallest at $ah=22$ mm and amounts to 0.83%.

We can also conclude that the modified 2D-AS structure at very small parameters ah , in the examined case for $ah=8$ mm, does not behave like a structure with NPR. This behavior of the structure for $ah=8$ mm can be attributed to the fact that at the extreme limits of parameter

R has a very large value, where it tends to close the structure, and therefore the arch with the radius r does not have enough space to be able to retract the structure at a given vertical displacement.

 Table 1: The Poisson's ratio of 2D-AS and modified 2D-AS for various values of parameter ah

ah [mm]	Poisson's ratio ν		The relative difference
	2D-AS	Modified 2D-AS	
8	-0.027	0.001	-103.7%
10	-0.128	-0.112	-12.5%
12	-0.232	-0.228	-1.72%
14	-0.334	-0.341	2.1%
16	-0.427	-0.445	4.21%
18	-0.508	-0.532	4.72%
20	-0.571	-0.598	4.73%
22	-0.613	-0.640	4.40%

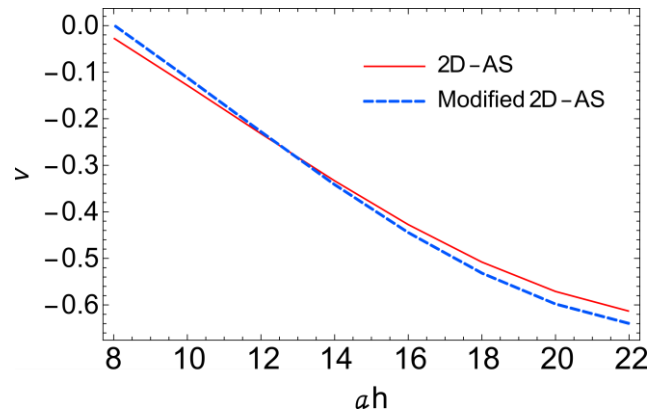

 Figure 15: Graphical representation of the Poisson's ratio of the 2D-AS and modified 2D-AS for various parameter ah

 Table 2: The relative density RD of 2D-AS and modified 2D-AS for various values of parameter ah

ah [mm]	Relative density RD		The relative difference
	2D-AS	Modified 2D-AS	
8	0.116	0.123	6.63%
10	0.107	0.114	6.23%
12	0.098	0.104	5.75%
14	0.089	0.094	5.17%
16	0.081	0.084	4.44%
18	0.072	0.074	3.54%
20	0.063	0.065	2.37%
22	0.054	0.055	0.83%

3. CONCLUSION

In this paper, a modified 2D-AS structure based on the 2D-AS structure [9] was designed. Numerical simulations, based on Ansys software, showed that the modified 2D-AS structure has higher values of NPR than the 2D-AS structure, only in the case when the values of the parameter ah are greater than 13mm. In fact, it has been shown that changing the shape of parts of the structure can affect the NPR value.

This paper has opened the door for new research on the further modifications of 2D-AS structure with the aim of achieving the maximum possible NPR value for the same or slightly different value of relative density RD.

ACKNOWLEDGEMENTS

This research was supported under grant no. 451-03-9/2021-14/200108 by the Ministry of Education, Science and Technological Development of the Republic of Serbia. This support is gratefully acknowledged.

REFERENCES

- [1] L.J. Gibson and M.F. Ashby, "Cellular solids, Structure and properties - Second edition", Cambridge Solid State Series, (1997)
- [2] T.C. Lim, "Auxetic Materials and Structures", Springer, Engineering Materials, School of Science and Tehnology, Singapore, (2022)
- [3] T.C. Lim, "Mechanics of Metamaterials with Negative Parameters", Springer, Engineering Materials, School of Science and Tehnology, Singapore, (2022)
- [4] E. Hakati, N. Daoubi, A. Bezazi and F. Scarpa, "In-plane elasticity of a multi re-entrant auxetic hooneycomb", Compos. Struct., vol. 180, pp. 130-139, (2017)
- [5] J. Shwn, K. Liu, Q. Zeng, J. Ge, Z. Dong and J. Liang, "Design and mechanical property studies of 3D re-entrant lattice auxetic structure", Aerosp. Sci. Technol., vol 118, pp. 106998, (2021)
- [6] H. Wang, Z. Lu, Z. Yang and X. Li, "A novel re-entrant auxetic honeycomb with enhanced in-plane impact resistance", vol. 208, pp. 758-770, (2019)
- [7] X. Li, L. Gao, W. Zhou, Y. Wang and Y. Lu, "Novel 2D metamaterials with negative Poisson's ratio and negative thermal expansion", Extreme Mech. Lett., vol. 30, pp. 100498, (2019)
- [8] N. Xu, H.T. Liu, M.R. An and L. Wang, "Novel 2D star-shaped honeycombs with enhanced effective Young's modulus and negative Poisson's ratio", Extreme Mech. Lett., vol. 43, pp. 101164, (2021)
- [9] Z.Y. Zhang, J. Li, H.T. Liu, Y.B. Wang, "Novel 2D arc-star-shaped structure with tunable Poisson's ratio and its 3D configurations", Mater. Today Commun., vol 30, pp.103016, (2022)