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Optimization of the plane truss by using the Method of Particle Swarm Optimization (PSO)

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This paper analyses optimization of the tubular plane truss with two case of loads. The aim of the analyses was to find out minimal weight of plane truss using PSO. This optimum design also has to satisfy the stress and the displacement constraints, and the elastic stability too.

Keywords: Particle Swarm Optimization, structural optimization, truss

0 INTRODUCTION

Determining the optimal construction dimensions is one of the major demands in the process of construction.
Their determination importantly influences the reduction of construction overall mass and costs too. According this fact, the construction solution becomes competitive. In the analysis of the metal construction cost, Farkas deduce that the price is primarily influenced by the price of the material (30–73)%, while the other costs are lower: manufacture (16–22)%, assembling (5–20)%, transportation (3–7)%, design (2–3)% [2]. By this data, reducing of material, i.e., weigth, is major tasks in optimization process.
Numerous researchers have dealt with the construction optimization using different methods of optimization[5], [6]. Olsen and Vanderplaats, and Jalkanen too, have treated the problem of ten-bar tubular plane truss [9].
Kennedy and Eberhart[7], studing social behavior of bird flocking, developed one new heuristic optimization method.
Ostrić and Petković [1] have treated the problem of steel truss construction.

In this paper we analyze eigth-bar steel plane truss example. Yet, ten-bars aluminium plane truss is benchmark in the research papers and it can be seen that two additional diagonal bars are needless too, because the kinematical stability is satisfied without its.

1 PARTICLE SWARM OPTIMIZATION ALGORITHM

Particle swarm optimization (PSO) is a population based metaheuristic optimization technique developed by Eberhart and Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling. It is based on the evolutionary cultural model of Boyd and Richerson which states that in social environments individuals have two learning sources: individual learning and cultural transmission. PSO and evolutionary algorithms (EA) such as Genetic Algorithms (GA) and Simulated Annealing (SA) have many similarities, however, some literature suggests they should be treated separately. These methods use a stochastic search process. PSO does not use the concept of survival of the fittest. In the PSO unfit individuals do not die. The system is initialized with a population of random solutions and searches for optima by updating generations. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. It is demonstrated that PSO gets better results in a faster, cheaper way compared with other methods. Another reason that PSO is attractive is that there are few parameters to adjust. One version, with slight variations, works well in a wide variety of applications. If one sees a desirable path to go (e.g., for food, protection, etc.) the rest of the swarm will be able to follow quickly even if they are on the opposite side of the swarm. This is performed by particles in multidimensional space that have a position and a velocity. These particles are flying through hyperspace (i.e., _n) and have two essential reasoning capabilities: their memory of their own best position and knowledge of the swarm’s best („best“ - the position with the smallest objective value). Consider Swarm of particles is flying through the parameter space and searching for optimum. Each particle is characterized by,

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Position vector . . . . $\boldsymbol{x}_i$ *(t)*
Velocity vector . . . . $\boldsymbol{v}_i$ *(t)*
as shown in Fig. 1.

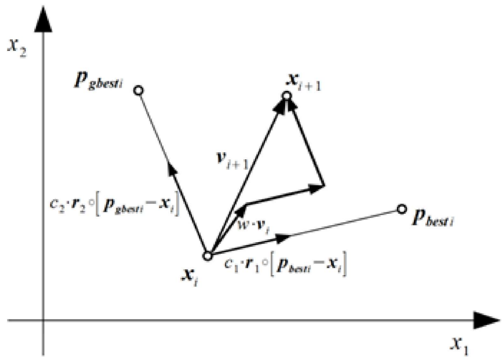

Fig. 1. Updating of position of the i-th particle

During the process, each particle will have its individual knowledge *pbest* and its own best-so-far in the position and social knowledge *gbest.*
Performing the velocity update, using the formula (1) given below,

$$v_i\,(t+1) = \omega\, v_i + c_1 \times rand \times pbest((t)\text{-}x_i\,(t)) + $$
$$+\, c_2 \times rand \times gbest((t)\text{-}x_i\,(t)) \qquad \ldots (1)$$

where ω is the inertia weight that controls the exploration and exploitation of the search space. Inertia weight w impacts the first component, and for the values in the range of 0,9 – 1,2 it gives the best results, that is, the algorithm has greater chances of finding the global minimum for a reasonable number of iterations. For coefficient values which are smaller than 0,8, if algorithm finds global minimum it will find it fast. Particles in this case move quickly and it can happen that they "fly over" some area, so it can happen that they do not find global minimum. On the other side, if inertia weight is bigger value, then particles search the solution space more thoroughly and the chances of finding global minimum are greater.
Coefficients c_1 and c_2, the cognition and social components respectively are the acceleration constants which changes the velocity of a particle towards the *pbest* and *gbest*, *rand* is a random number between 0 and 1. Usually c_1 and c_2 values are set to 2.
The particle swarm optimization concept consists of, at each time step, changing the velocity of (accelerating) each particle toward its p*best* and *lbest* locations (local version of PSO). Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward *pbest* and *lbest* locations.
Now, performing the position update(as shown in Fig. 1.),

$$Xi(t+1) = Xi(t) + Vi(t+1) \qquad \ldots (2)$$

This process is repeated for each and every particle considered in the solution space and the best optimal solution is obtained.

Basic Flow of Particle Swarm Optimization

The basic operation of PSO is given by,
Step1: Initialize the *swarm* from the solution space
Step 2: Evaluate *fitness* of individual particles
Step 3: Modify *gbest, pbest* and *velocity*
Step 4: Move each *particle* to a new *position*
Step 5: Go to step 2, and repeat until convergence or stopping condition is satisfied.

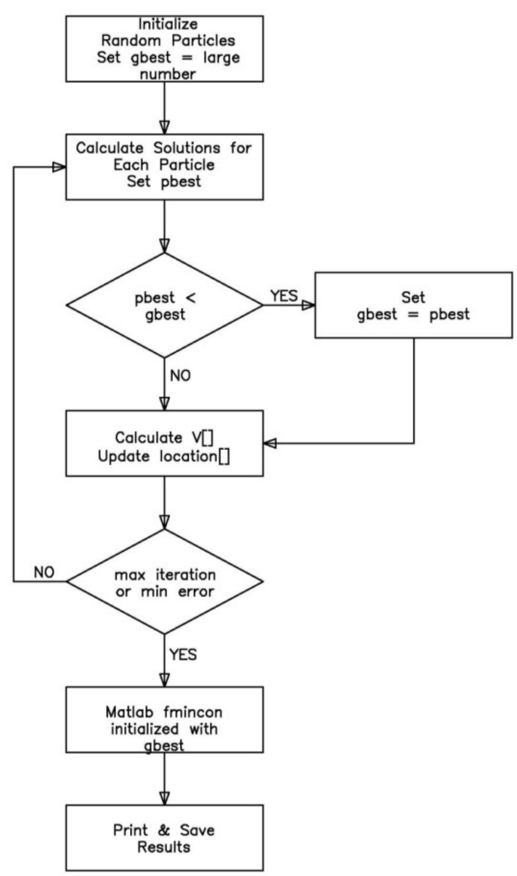

Fig. 2. Particle swarm optimizer flowchart

While maximum iterations or minimum error criteria is not attained Particles' velocities on each dimension are clamped to a maximum velocity Vmax. If the sum of accelerations would cause the velocity on that dimension to exceed Vmax, which is a parameter specified by the user, then the velocity on that dimension is limited to Vmax.

In recent years the concept of PSO has been applied to various engineering problems. Specifically, it has been applied to structural design optimization problems. Ant colony optimization (ACO), a type of PSO, was tested on steel frame optimization problems with discrete variables by Camp et al. The PSO method found better results on these test problems than any of the other optimization algorithms used in previous research.

2 STRUCTUAL OPTIMIZATION

The term *optimal structure* is very vague because a structure can be optimal in different aspects. These different aspects are called objectives, and may for instance be the weight, stiffness or cost of the structure. A numerical evaluation of a certain objective is possible through an *objective function, f,* which determines the goodness of the structure in terms of weight, stiffness or cost [4]. To be well defined solution, the optimization has to be done within some constraints. Firstly, there are *design constraints*, like a limited geometrical extension or limited availability of different structural parts. Secondly, there are *behavioral constraints* on the structure that denotes the structural response under a certain load condition, for instance, limits on displacements, stresses, forces and dynamic response. Finally, there is *kinematical stability* that is valid for all structures, otherwise they are mechanisms. This can be seen as a behavioral constraint. Structures that lie within the constraints are called *feasible* solutions to the optimization problem.

A general expression for structural optimization is given for instance by Christensen &Klarbring [4]:

$$SO \begin{cases} \text{minimize } f(x, y) \text{ with respect to } x \text{ and } y \\ \text{subject to } \begin{cases} \text{design constrains on } x \\ \text{behavioral constrains on } y \\ \text{stability constraint} \end{cases} \end{cases}$$

where f is the objective function;

x is a function or vector representing the design variables, and;

y is a function or vector representing the state variables, i.e. the response of the structure.

Multi-objective optimization (also called multi-criterion or vector optimization) can be done with respect to two or more different objective functions.

When it applies to trusses, the optimization can be divided into sizing, shape and topology optimization.

Sizing optimization refers to finding the optimal cross section area of each member of the structure; *shape optimization* means optimizing the outer shape of the structure; and *topology optimization* describes the search for the best inner connectivity of the members.

One way of optimizing these three parameters is to take them into consideration one at a time, starting with the topology optimization, a so called multi-level optimization technique (also called layered optimization). One of the strengths and advantages of a genetic algorithm is that a simultaneous optimization of all three parameters can be done.

Structural optimization, especially discrete structural optimization of practical problems, requires low computational cost and accuracy for all of the processes. By far the most computationally costly process is the FEA. The FEA for large-scale three-dimensional problems and eigenfrequency problems becomes difficult to optimize practically.

2.1 Trusses

As long as the load is applied in some of the nodes, the bars will only be subjected to compressive or tensile normal forces. This is one part of the explanation to why trusses are so light compared to their load capacity; bar effect is more efficient than beam effect. The other part is that the triangle is the simplest *stable* structure that extends in two dimensions. Due to their efficiency, trusses are desirable in long span structures with high demands in stiffness and strength.

There are benefits of weight optimized structures in many engineering fields. In engineering it can for instance be associated with cheaper structural parts and easier transportation. In this paper a

PSO algorithm for weight minimization of steel trusses has been developed in MATLAB.

The objective is to minimize the weight of the plane truss. This optimum design also has to satisfy the stress and the displacement constraints, and the elastic stability too.

2.2 Eight-bar truss

The first numerical example problem deals with the discrete optimization of a eight-bar steel tubular plane truss presented in Fig. 3.

$L = 9{,}1$ m, $F = 444{,}8$ kN, E=210000Mpa,

$\rho = 7800\text{kg/m}^3$, $\sigma_d = 160$Mpa, $f_d^{max} = 5$cm

Mass of the truss should be minimized so that the normal stress is less than σ_d in all the bars and the maximum deflection in nodes 3, 4, 5 and 6 is less than the maximum allowed value f_d^{max}. The cross section areas Ai are the design variables and their values should be chosen from a set which includes 50 evenly distributed values. The cross section of bars is pipe-shape and its area is:

$$A_i = \frac{\pi d_i^2}{4} - \frac{\pi (d_i - 2t)^2}{4}$$

If the thickness t is constant and it is assumed that t = d/10, then: $A_i = 0{,}09 \pi d_i^2$

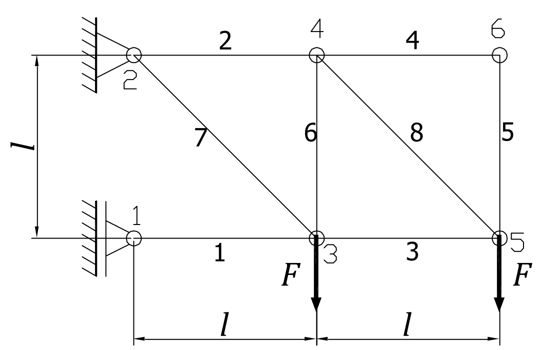

Fig. 3. The eight-bar steel plane truss

The set of discrete variables (diametars) is:

$x \in \{x_1, x_2, ..., x_8\}$

Therefore, the problem can be stated as:

The objective function is:

min $m(\mathbf{x}) = \sum \rho\, L\, A_i$ $\quad$ (i =1,2,...,8)

Design constraint functions:

$\sigma(\{x\}) - \sigma_d \le 0$, $f(\{x\}) - f\,max \le 0$

Variable regions:

$1 \le d \le 60$,

where the maximum allowable stress (σ_d) is 160 MPa and the only displacement constraint is the maximum (f max) limited to 5 cm.

The structural analysis is done using analitic method. Normal force is the only internal force in bars and results of analyzis are presented in Table1:

N_i	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
$N_{i(1)}$	-2F	F	-F	0	0	-F	√2F	√2F
$N_{i(2)}$	-F	0	0	0	0	0	√2F	0
l_i	l	l	l	l	l	l	$\sqrt{2}l$	$\sqrt{2}l$

In this paper we analyze eigth-bar plane truss example. Although, ten-bars plane truss is benchmark in the research papers and we can see that two additional diagonal bars are needless too, because the kinematical stability is satisfied without its. Furthermore, due to analysis of normal forces, it is obvious that bars 4 and 5 (Fig. 3) are needless and it can be reduced till only the necessary elements are remaining. This shape of 8-bar plane truss example is more practical and very often in constructions.

The deflection analysis is done using method of deformation energy and maximal displacement for load case 1 - force F at the end (node 5) of the truss is calculated as:

$$f_5 = \frac{Fl}{E}\left[\frac{4}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_6} + \frac{2\sqrt{2}}{A_7} + \frac{2\sqrt{2}}{A_8}\right]$$

Vertical displacement of node 3 for load case 1 is calculated as:

$$f_3^{(1)} = \frac{2Fl}{E}\left[\frac{1}{A_1} + \frac{\sqrt{2}}{A_7}\right]$$

Maximal displacement for load case 2 - force F at the node 3 of the truss is calculated as:

$$f_5^{(2)} = \frac{2Fl}{E}\left[\frac{1}{A_1} + \frac{\sqrt{2}}{A_7}\right]$$

Vertical displacement of node 3 for load case 2 is calculated as:

$$f_3^{(2)} = \frac{Fl}{E}\left[\frac{1}{A_1} + \frac{2\sqrt{2}}{A_7}\right]$$

Analyzing elastic stability of the truss, we have to test only compressed bars which marked 1, 3 and 6.

Firstly, we calculate effective slendernees ratio:

$$\lambda_r = \frac{l_r}{i_{\min}}$$

where is:

minimal radius of qyration i_{min} is calculate as:

$$i_{\min} = \sqrt{\frac{I_{\min}}{A_i}} = 0{,}0597 d_i$$

I_{min} – minimal moment of inertia of tubular cross-section and it is:

$$I_{\min}^i = 0{,}00537 \pi d_i^2$$

l_r – effective length of the bar

λ_r – effective slendernees ratio is quotient of effective length of the bar l_r and minimal radius of qyration i_{min}

Now, we compare effective slendernees ratio λ_r with $\lambda_P = 108$ (Č.0361).

If the effective slendernees ratio λ_r has bigger valeu than slendernees at proportional limit λ_P

Eulers' critical force for a bar is calculated as:

$$F_{kr} = \frac{\pi^2 E I_{\min}}{l_r^2} = \frac{0{,}0537 \pi^3 E \cdot d^2}{l^2}$$

$$N_d = \frac{F_{kr}}{\nu} = \frac{0{,}0537 \pi^3 E \cdot d^2}{\nu \cdot l^2}$$

where υ=1,5 – coefficient of safety

On the contrary, critical buckling force is:

$$F_{kr}^i = (289 - 0{,}82\lambda_r) \cdot A_i =$$
$$= (289 - 0{,}82\lambda_r) \cdot 0{,}09 \pi d^2$$

3 OPTIMIZATION RESULTS

Following parameters of the algorithm of particle swarm optimization (algorithm):

Number of particles	30
Number of iterations	10000
Inertia weight w	1
Acceleration coefficient c_1	2
Acceleration coefficient c_2	2
Boundaries for d	1-60

The truss has to be analyzed ten thousands times during the optimization. The obtained results for the both load cases is represented in table:

d_i (cm)	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
$d_{(1)case}$	25	13	20	1	1	20	16	16
$d_{(2)case}$	20	1	1	1	1	1	12	1

The objective function, i.e., overall mass of the truss is:

-load case 1: m = 3645kg

-load case 2: m = 1038kg

By comparing this results with the other researches, Jalkanen has obtained for total weigth of ten-bar plane truss m = 2303kg, and we can see similarities, but this eigth-bar truss is made of *steel.*

4 CONCLUSION

This paper analyses optimization of the tubular plane truss with two case of loads. The aim of the analyses was to find out minimal weight of plane truss using PSO.

According to results we can see that the value of totalmass for the first load case is very close to the overall value 3645 kg, and the sum of mass all bars for the second load case is 1038 kg, which is expeced due to the position of forces. By comparing this results with the other researches, for example Jalkanen has obtained for total weigth of ten-bar *aluminium* plane truss m = 2303kg, we can see that, regardless three times bigger steel density, this eigth-bar *steel* truss is more practical in engineering problems. The method of PSO is suitable for this type of constructions.

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