The analysis of a sustained oscillation in the heat pump system with buffer tank

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The most of domestic scale heat pumps with fixed speed compressors are thermostatic controlled. Control units with a relay characteristic are used to control these pumps. Such pumps operate in the mode of sustainable oscillations. To overcome short cycling, i.e. switching on and off of heat pumps, as this reduce energy efficiency and pump life cycle, buffer tanks are installed with the task of absorbing the excess heat delivered by the pump. The paper analyzes how the properties of a heat accumulator and a heat pump affect the oscillations parameters. The buffer tank is described by the FOPDTmodel while the pump is approximated by relay nonlinearity with symetric hysteresis. A relay feedback technique with a describing function was used for the analysis. The results were confirmed by simulation in Simulink / Matlab environment.

Keywords: Heat pump, Relay with hysteresis, Sustainable oscillations, Describing function

1. INTRODUCTION

In the European domestic sector, water-to-water heat pumps (*HP*) with fixed-speed compressors are most commonly used [1,2]. Control units with a relay characteristic are used to control these pumps [3]. To overcome short cycling, i.e. fast switching on and off, as this reduce energy efficiency and pump life cycle, buffer tanks (*BT*) are installed with the task of absorbing the excess heat delivered by the pump. The use of heat accumulators affects both the dynamics of the pump itself and the accuracy of the controlled temperature. Energy flow through heat pump with *BT* is shown in Fig. 1.

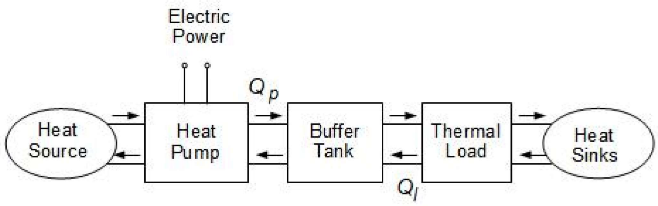

Figure 1: Heat pump system with buffer tank

In this paper we are primarily interested in buffer dynamics. The environment influence on *BT* is considered through heat flow $\dot{Q}_p$ (between *HP* and *BT*) and heat flow $\dot{Q}_l$ (between *BT* and thermal load).
From control point of view the system can be described by block diagram shown in Fig. 2. The dynamics of the heat pump is neglected in relation to the dynamics of the buffer. The combined impact of the *HP* and the thermal load on the buffer is modeled as nonlinearity (*NL*) of relay type with non symetric hysteresis (Fig. 3). Manipulated variable u represent resulting heat flow $(\dot{Q}_p - \dot{Q}_l)$ in to the *TB*.
It should be noted that this characteristic corresponds to the heating mode of of a pump. In cooling mode, the nonlinearity changes phase by π.

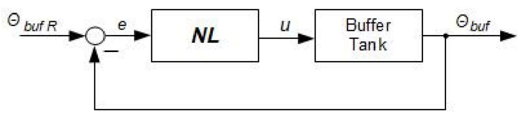

Figure 2: Block diagram of a heat pump + buffer tank

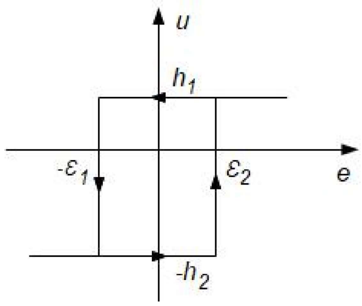

Figure 3: Asymmetric relay with hysteresis

In steady state there are sustainable oscillatios at the output of *TB*. Our intention is to establish a quality relationship between the period of these oscillations and the parameters of the NL and the buffer tank itself..

2. DESCRIBING FUNCTION OF ASYMMETRIC RELAY WITH HYSTERESIS

In order for the describing function to accurately approximate the periodic function at the output of the nonlinearity in the relay feedback loop (Fig.2) it is necessary that BT has the characteristics of the low pass filter [4]. Actual changes in real buffer temperature is shown in Fig.4 [3].

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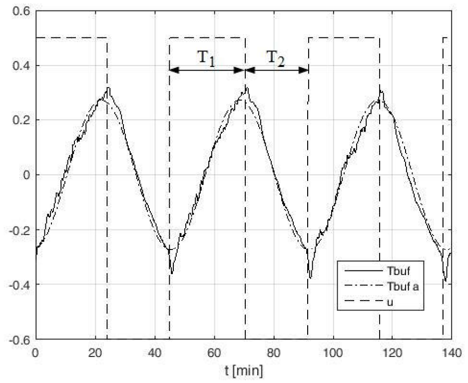

Figure 4: Buffer tank temperature

If the Fourier analysis of the temperature signal is performed with a series of order 10, the values of the coefficients shown in Fig.5 are obtained.

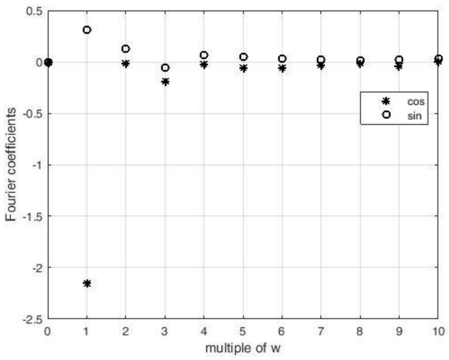

Figure 5: Fourier coefficients of buffer tank temperature

It can be seen that the first harmonic is dominant. For example, the ratio of the amplitudes of the first and second harmonics is 18.16. This means that the buffer can be viewed as a low pass filter. Fig.4 shows actual temperature approximation with first harmonic (*Tbuf a*).

If the analysis of the manipulated variable (Fig.4-u) is performed with a Fourier series of order 10, the values of the coefficients is shown in Fig. 6.

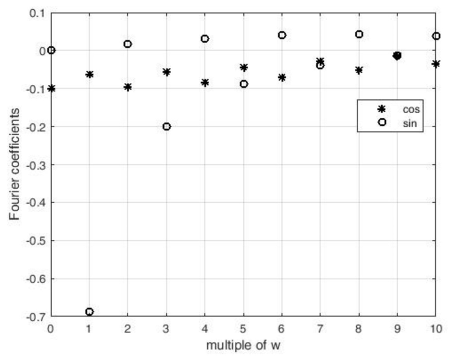

Figure 6: Fourier coefficients of manipulated variable

It can be noticed that the complex member of the first harmonic has a dominant importance. However, higher harmonics also have an effect on the output signal of nonlinearity. Ignoring higher harmonics can lead to a significant error if the approximation is performed only with a first-order harmonic. Figure 7 shows the actual value of the output magnitude of the nonlinearity (u) and its approximation (u_a).

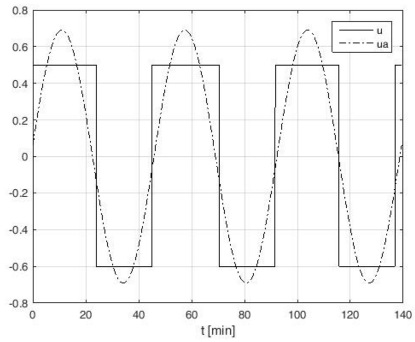

Figure 7: Manipulated variable and its approximation

The justification for approximating the output signal using the first harmonic lies in the fact that the buffer tank behaves like a low pass filter. In other words, it nullifies the influence of higher harmonics.

Due to this approximation, an error may occur in determining the position of the critical point on the Nyquist curve of a plant from a realay feedback experiment [5]. Nevertheless, such an approximation can help us in a qualitative analysis of the process.

The nonlinearity from Fig. 3 can be described by equation (1):

$$u = \begin{cases} h_1 & e > \varepsilon_2 \ \vee \ e \leq \varepsilon_2 \wedge e^- = h_1 \\ -h_2 & e < -\varepsilon_1 \ \vee \ e \geq -\varepsilon_1 \wedge e^- = -h_2 \end{cases} \tag{1}$$

If the error is of the form:

$$e(t) = e_0 + e_A \sin(\omega t) \tag{2}$$

where e_0 is DC component, e_A magnitude and ω error radial frequency (e), then output signal of the nonlinearity can be approximated by harmonic of the first order. If the relation:

$$e_A \geq \max(\varepsilon_1, \varepsilon_2) \tag{3}$$

is fullfiled, then it is valid:

$$u = \frac{a_0}{2} + a_1 \cos(\omega t) + b_1 \sin(\omega t) \tag{4a}$$

where is

$$a_0 = (h_1 - h_2) + \frac{1}{\pi}(h_1 + h_2)(\theta_2 - \theta_1) \tag{4b}$$

$$a_1 = -\frac{(h_1+h_2)}{\pi}\frac{(\varepsilon_1+\varepsilon_2)}{e_A} \tag{4c}$$

$$b_1 = \frac{(h_1+h_2)}{\pi}(\cos(\theta_1) + cos(\theta_2)) \tag{4d}$$

$$\theta_1 = \operatorname{asin}\left(\frac{\varepsilon_1+e_0}{e_A}\right) \tag{4e}$$

$$\theta_2 = \operatorname{asin}\left(\frac{\varepsilon_2-e_0}{e_A}\right) \tag{4f}$$

For a describing function we can write:

$$DF = \frac{b_1}{e_A} + j\frac{a_1}{e_A} \tag{5}$$

If the condition:

$$\dot{Q}_P = 2\,\dot{Q}_l \tag{6}$$

is met, i.e. if the heat flow from the pump is equal to twice the value of the heat flow going out to the thermal load, then:

$$h_1 = h_2 = h \quad (7a)$$
$$\varepsilon_1 = \varepsilon_2 = \varepsilon \quad (7b)$$
$$e_0 = 0 \quad (7c)$$

we get:

$$DF = \frac{4h}{\pi e_A}\left\{\sqrt{1-\left(\frac{\varepsilon}{e_A}\right)^2} - j\frac{\varepsilon}{e_A}\right\} \quad (8a)$$

or

$$DF = \frac{4h}{\pi e_A} \measuredangle - \operatorname{asin}\left(\frac{\varepsilon}{e_A}\right) \quad (8b)$$

If $\varepsilon = 0$ then equations (8) represents DF of realy without hysteresis with magnitude h:

$$DF = \frac{4h}{\pi e_A} \quad (9)$$

It should be noted that DF does not depend on the frequency of the output signal but only on its amplitude. The hysteresis in the relay characteristic in Fig. 3 does not affect the change in amplitude but introduces the phase shift of the signal u in relation to e. The phase lag depends on the width of the hysteresis and the amplitude of the output signal.

From (8a) we can find the negative reciprocal of the DF:

$$-\frac{1}{DF} = -\frac{\pi e_A}{4h}\sqrt{1-\left(\frac{\varepsilon}{e_A}\right)^2} - j\frac{\pi\varepsilon}{4h} \quad (10a)$$

where is:

$$Re(-1/DF) = -\frac{\pi e_A}{4h}\sqrt{1-\left(\frac{\varepsilon}{e_A}\right)^2} \quad (10b)$$

$$Im(-1/DF) = -\frac{\pi\varepsilon}{4h} \quad (10b)$$

The complex functions (10) determine the position of the critical points of Nyquist plot for the buffer tank. It can be noticed that for a given nonlinearity with a fixed (ε, h) the imaginary part $Im(-1/DF)$ has a constant value and does not depend on the properties of the plant. On the other hand, the absolute value of real part grows with increasing amplitude of the output signal.

3. THE PERIOD OF SUSTAINED OSCILLATION

We assume that the buffer dynamics can be described by first-order-plus-dead-time (FOPDT) model [3]:

$$G_P(s) = \frac{K_P}{\tau_P s+1} e^{-T_d s} \quad (11)$$

where is: K_P - plant static gain, τ_P- plant time constant, T_d- plant dead time.

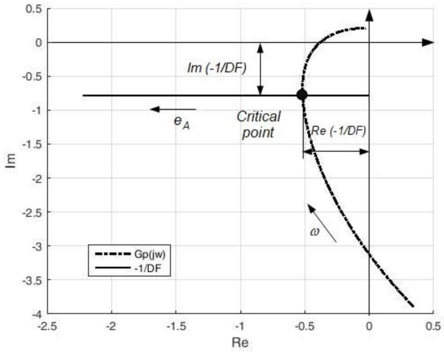

Figure 8: DF analysis of heat pump+buffer tank

Then the critical point in the frequency plane can be found in the cross section of $G_P(j\omega)$ and $-1/DF$ as showen in Fig. 8.

Bearing in mind (10), for a given buffer tank, the period of sustained oscillation can be influenced in the following way:

- with increasing hysteresis width (ε) we increase the period of oscillation,
- by changing the width of the hysteresis (ε) we can influence the amplitude of the oscillations. For example. by increasing the hysteresis we can reduce the amplitude of the oscillations. But $e_A \geq \varepsilon$ must always be valid,
- a change in thermal load can affect the period of oscillation. The increase in load reduces the period of oscillation (provided that it is valid (6)). If the load decreases (less h) the period of oscillation increases.

If the width of the hysteresis increases beyond some limit value, at some point the system can go out of the mode of its own oscillations. For example. if the heat flow from the pump is not sufficient to exceed the shut-off threshold then the pump runs continuously.

In addition to the nonlinearity, the parameters of the buffer itself affect the period of oscillation of self-sustaining oscillations. Fig. 9 shows the dependence of the amplitude and the period of the limit cycle for different values of plant static gain (K_P).

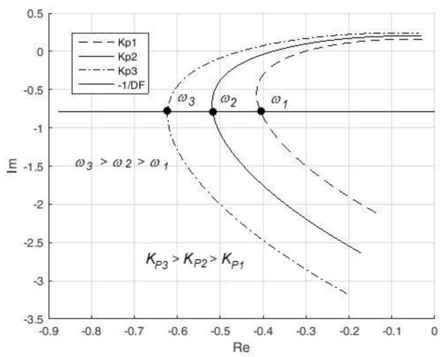

Figure 9: DF analysis for diferent values of K_P

As the gain increases, the amplitude of oscillation increases and the period decreases.

Figure 10 shows the dependence of the amplitude and period of the limit cycle for different plant time constant values (τ_P).

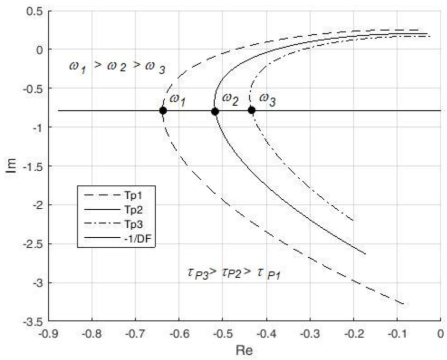

Figure 10: DF analysis for diferent values of τ_P

As the time constant increases, the oscillation amplitude decreases and the period increases.

Figure 11 shows dependence of the amplitude and period of limit cycle for different plant dead time (T_d).

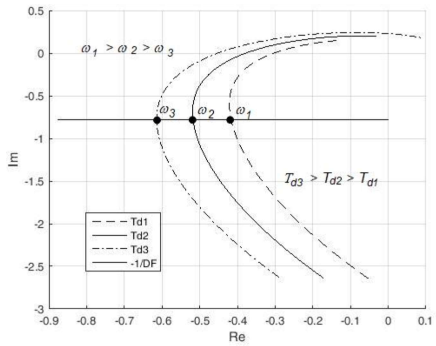

Figure 11: DF analysis for diferent values of T_d

As the dead time increases, both the oscillation amplitude and the period increases.

Prethodna analiza je sprovedena uz pretpostavku da važi uslov (6). U tom slucaju nelinearnost se modeluje pomocu simetricnog histerezisa. Kao rezultat vaze relacije (7). U odzivu sistema ne postoji DC komponenta i $T_1 = T_2$ (Fig. 12).

The previous analysis was performed assuming that condition (6) holds. In this case, the nonlinearity is modeled using symmetric hysteresis. As a result, relationships (7) apply. There are no DC components in the system response and $T_1 = T_2$ (Fig. 12).

We have introduced this simplification to make it easier to conduct a qualitative analysis. In practice, this is a rare case and usually applies:

$$\dot{Q}_P \neq 2\,\dot{Q}_l \tag{12a}$$

As a consequence, we have:

$$T_1 \neq T_2 \tag{12b}$$

and a DC component appears in the system response.

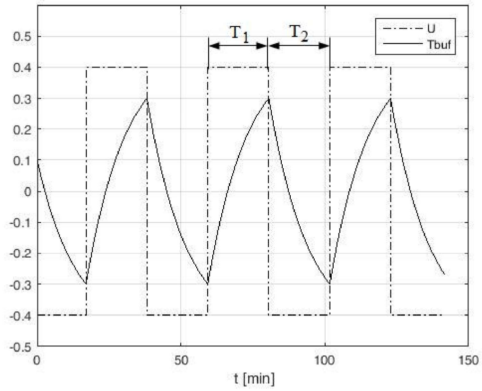

Figure 12: Output for symetric hysteresis (Simulation)

4. CONCLUSION

Heat pump with on/off controll operate in the mode of sustainable oscillations. Rrelay feedback technik with describing function is a natural way to analyze such systems. Fourier analysis of the experimental data shows that the buffer tank can be viewed as a low pass filter in relation to the remaining parts of the heat pump system. Thanks to this, hard nonlinearity such as the relay with hysteresis can be approximated using the describing function. The paper uses symmetric hysteresis to describe relay nonlinearity. Even such a simplified model analytically confirms our intuitive knowledge about oscillations in the system.

ACKNOWLEDGEMENTS

This research has been supported by the Serbian Ministry of Education, Science and Technological Development under grant No. 451-03-9/2021-14/200108

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