QUASI-STATIC RESPONSE OF PLANAR PARALLEL-CONNECTION FLEXURE HINGES MECHANISM

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Abstract

This paper deals with the quasi-static response analysis of planar parallel-connection flexure hinge mechanism. The flexure hinges are discretized by using the pseudo-rigid-body method. According to this approach, the elastic properties of the flexure hinges are modelled by introducing the springs of appropriate stiffnesses in the joint elements with three degrees of freedom which connect the light rigid segments. The mass of the flexure hinges is neglected due to their dimensions with respect to the rigid body. The small in-plane deformations of the flexure hinges was analyzed. The displacements of the arbitrarily selected point of the rigid body are obtained in the symbolic form. The proposed algorithm is universal, so the mechanism with the arbitrarily selected number, type or orientation of flexure hinges should be analyzed. The symmetric corner-filleted flexure hinge and circular flexure hinge were used in numerical example. The obtained results are more accurate than the results of similar methods from literature. Furthermore, the proposed method is numerically more efficient than the above mentioned similar methods.

Key words: flexure hinge mechanism, quasi-static response, pseudo-rigid-body method.

1. Introduction

The quasi-static analysis of the planar parallel-connection flexure hinge mechanism will be considered here. The concept of parallel-connection means that the rigid body is connected with the fixed end by n flexure hinges. Note that all of this hinges are rigidly connected on the left end with the fixed end and with the rigid body on the right hinge end. Also, it is supposed that the flexure hinge $\#i$ $\left(i=1,\ldots,n\right)$ is rotated by angle $\alpha_i\left(i=1,\ldots,n\right)$ relative to the horizontal axis, as shown in Fig. 1.

The pseudo-rigid-body method, whose basic assumptions are set in the references [1, 2], will be used. In the above mentioned references, the pseudo-rigid-body method was used for the quasi-static and modal analysis of a planar serial flexure-hinge mechanism. Here the using of this method will be extended to the planar parallel-connection flexure hinge mechanism.

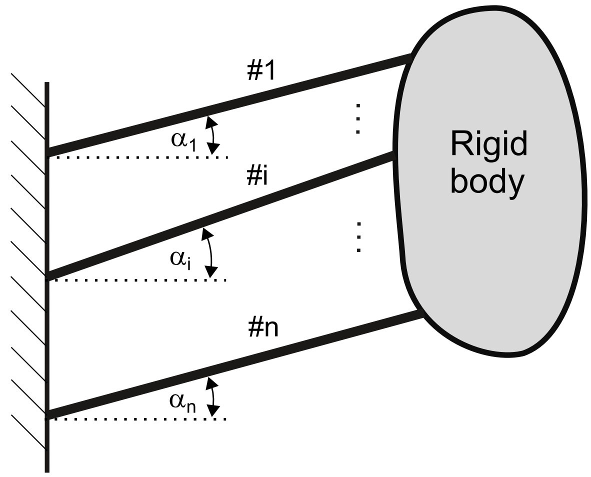


Fig. 2 Planar parallel-connection flexure hinges mechanisms

The goal of this paper is to make a universal algorithm for quasi-static analysis of the above mentioned mechanism in a symbolic form. The obtained numerical results will be compared with the results from the reference [3].

2. The discrete model of the flexure hinge

Here, the flexure hinge, as the base of the flexure hinge mechanism, will be discretized by using the pseudo-rigid-body method. The exact shape of the flexure hinge is shown in Fig. 2(a). The total length of the hinge is *L*, the width *b* is constant and the thickness of the hinge changes according to the law *t*(*x*), where *x* ($0 \leq x \leq L$) represent the axial coordinate.

The discretized model of the flexure hinge is shown in Fig. 2(b). This model consist of two light rigid segments of length *L*/*2* which are connected by a joint element with three degrees of freedom, two relative translations ($\Delta\xi$ and $\Delta\eta$) in the transverse and axial direction and one rotation ($\Delta\varphi$) about the axis perpendicular to the plane of hinge movement. The springs of the stiffnesses k_{L_1}, k_{L_2} and k_R which corresponds to the displacements $\Delta\xi$, $\Delta\eta$ and $\Delta\varphi$, respectively, are placed in joint element.

According to the references [1, 2], the above spring stiffnesses should be obtained as:

$$k_{L1} = \frac{1}{C_a}, \tag{1}$$

$$k_{L2} = \begin{cases} \dfrac{C_{b,r}}{[C_{b,t} + 2\alpha_f (1+\mu) C_a] C_{b,r} - L^2 C_{b,r}^2 / 4}, & \text{short hinges,} \\[2ex] \dfrac{C_{b,r}}{C_{b,t} C_{b,r} - L^2 C_{b,r}^2 / 4}, & \text{long hinges,} \end{cases} \tag{2}$$

$$k_R = \frac{1}{C_{b,r}}, \tag{3}$$

where α_f is the shear correction factor, μ is the Poisson's ratio and compliance coefficients C_a, $C_{b,t}$ and $C_{b,r}$ should be obtained by using the Castigliano's second displacement theorem [4] in the following manner:

$$C_a = \frac{1}{Eb}\int_0^L \frac{dx}{t(x)},\ C_{b,t} = \frac{12}{Eb}\int_0^L \frac{x^2 dx}{t(x)^3},\ C_{b,r} = \frac{12}{Eb}\int_0^L \frac{dx}{t(x)^3}, \tag{4}$$

whereas E is the Young's modulus.

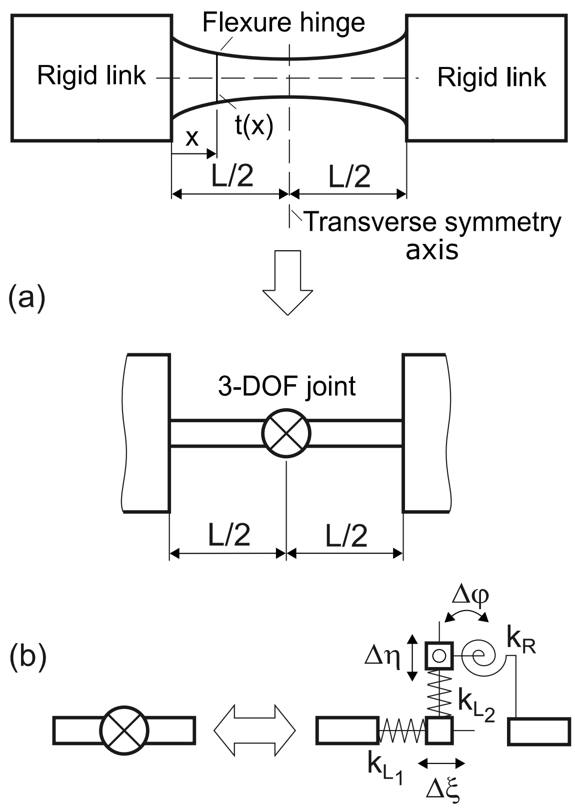


Fig. 2 Discretization of the flexure hinge: (a) exact model, (b) rigid segment model.

3. Planar parallel-connection flexure hinges mechanism

Fig. 1 shows the planar parallel-connection flexure hinge mechanism which consist of n flexure hinges $\#i$ ($i = 1,\ldots,n$) which connect rigid body with the fixed end. The pseudo-rigid-body model of the considered mechanism formed on the basis of the above described discretization procedure of the flexure hinge is shown in Fig. 3. Two kinds of coordinate frames are used here, the inertial coordinate frame $Oxyz$ and the coordinate frames $O_i\xi_i\eta_i\zeta_i$ ($i = 1,\ldots,n$) which are fixed to the flexure hinges in the undeformed state.
The vector of relative translations in the joint element Ji (i=1,…,n)

$$\mathbf{u}_i = \begin{bmatrix}\Delta\xi_i & \Delta\eta_i & 0\end{bmatrix}^T \tag{8}$$

should be obtained as:

$$\mathbf{u}_i = \mathbf{R}_{0,i}^{-1}\left(\mathbf{u}_p + \tilde{\boldsymbol{\delta}}\mathbf{d}_i\right), \tag{9}$$

where

$$\mathbf{u}_p = \begin{bmatrix} \Delta x & \Delta y & 0 \end{bmatrix}^T \tag{10}$$

is the vector of displacements of the arbitrarily selected point P relative to the inertial coordinate frame $Oxyz$, $\tilde{\boldsymbol{\delta}} \in R^{3\times 3}$ is the skew-symmetric matrix associated with the rotation vector

$$\boldsymbol{\delta} = \begin{bmatrix} 0 & 0 & \Delta\theta \end{bmatrix}^T, \tag{11}$$

$$\mathbf{d}_i = \begin{bmatrix} d_{i,x} & d_{i,y} & 0 \end{bmatrix}^T \tag{12}$$

represent the position vector of point J_i relative to the selected point P, and

$$\mathbf{R}_{0,i} = \begin{bmatrix} \cos\alpha_i & -\sin\alpha_i & 0 \\ \sin\alpha_i & \cos\alpha_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

is the coordinate transformation matrix from the inertial coordinate frame $Oxyz$ to the fixed coordinate frame $O_i\xi_i\eta_i\zeta_i$.

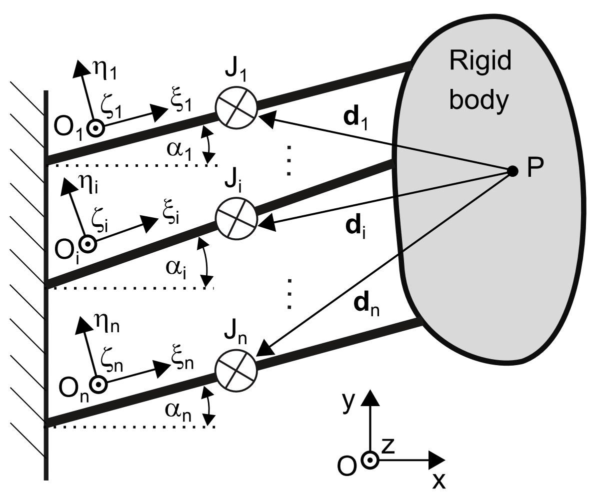


Fig. 3 Discretized planar parallel-connection flexure hinges mechanism

By using the equations (8-13), the relative translations in the joint element J_i reads:

$$\Delta\xi_i = \cos\alpha_i\Delta x + \sin\alpha_i\Delta y + \left(d_{i,x}\sin\alpha_i - d_{i,y}\cos\alpha_i\right)\Delta\theta, \tag{14}$$

$$\Delta\eta_i = -\sin\alpha_i\Delta x + \cos\alpha_i\Delta y + \left(d_{i,x}\cos\alpha_i + d_{i,y}\sin\alpha_i\right)\Delta\theta, \tag{15}$$

and for the relative rotation in the joint element J_i holds that:

$$\Delta\varphi_i = \Delta\theta. \tag{16}$$

The above three relations written in a vector form reads:

$$\mathbf{w}_i = \mathbf{P}_{w_i,v}\mathbf{v}, \tag{17}$$

where

$$\mathbf{w}_i = \begin{bmatrix}\Delta\xi_i & \Delta\eta_i & \Delta\varphi_i\end{bmatrix}^T,\ \mathbf{v} = \begin{bmatrix}\Delta x & \Delta y & \Delta\theta\end{bmatrix}^T, \tag{18}$$

and

$$\mathbf{P}_{w_i,v} = \begin{bmatrix}\cos\alpha_i & \sin\alpha_i & d_{i,x}\sin\alpha_i - d_{i,y}\cos\alpha_i \\ -\sin\alpha_i & \cos\alpha_i & d_{i,x}\cos\alpha_i + d_{i,y}\sin\alpha_i \\ 0 & 0 & 1\end{bmatrix}. \tag{19}$$

Now, the total potential energy of the considered planar parallel-connection flexure hinges mechanism is obtained by:

$$\Pi = \frac{1}{2}\sum_{i=1}^{n}\mathbf{w}_i^{\,T}\mathbf{K}_{w_i}\mathbf{w}_i. \tag{20}$$

Furthermore, after using the vector relation (17), the total potential energy becomes:

$$\Pi = \frac{1}{2}\mathbf{v}^T\mathbf{K}\mathbf{v}, \tag{21}$$

where

$$\mathbf{K} = \begin{bmatrix}k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33}\end{bmatrix} \tag{22}$$

is the stiffness matrix whose coefficients reads:

$$k_{11} = \sum_{i=1}^{n}\left(k_{L_1,i}\cos^2\alpha_i + k_{L_2,i}\sin^2\alpha_i\right), \tag{23}$$

$$k_{22} = \sum_{i=1}^{n}\left(k_{L_1,i}\sin^2\alpha_i + k_{L_2,i}\cos^2\alpha_i\right), \tag{24}$$

$$k_{33} = \sum_{i=1}^{n} \left(k_{R,i} + k_{L_1,i} \left(d_{i,x} \sin\alpha_i - d_{i,y} \cos\alpha_i \right)^2 + k_{L_2,i} \left(d_{i,x} \cos\alpha_i + d_{i,y} \sin\alpha_i \right)^2 \right), \tag{25}$$

$$k_{12} = k_{21} = \sum_{i=1}^{n} \left(k_{L_1,i} - k_{L_2,i} \right) \sin\alpha_i \cos\alpha_i, \tag{26}$$

$$k_{13} = k_{31} = \sum_{i=1}^{n} \left(k_{L_1,i} \cos\alpha_i \left(d_{i,x} \sin\alpha_i - d_{i,y} \cos\alpha_i \right) - k_{L_2,i} \sin\alpha_i \left(d_{i,x} \cos\alpha_i + d_{i,y} \sin\alpha_i \right) \right), \tag{27}$$

$$k_{23} = k_{32} = \sum_{i=1}^{n} \left(k_{L_1,i} \sin\alpha_i \left(d_{i,x} \sin\alpha_i - d_{i,y} \cos\alpha_i \right) + k_{L_2,i} \cos\alpha_i \left(d_{i,x} \cos\alpha_i + d_{i,y} \sin\alpha_i \right) \right). \tag{28}$$

If we assume that in the arbitrarily selected point P of the rigid body acts the horizontal and vertical forces F_x and F_y, respectively, as well as the bending moment M_z, the vector of generalized forces read:

$$\mathbf{Q} = \begin{bmatrix} F_x & F_y & M_z \end{bmatrix}^T. \tag{29}$$

By using the principle of virtual work [5], in the equilibrium position of the rigid body holds that:

$$-\mathbf{K}\mathbf{v} + \mathbf{Q} = \mathbf{0}_{3\times1}, \tag{30}$$

so the position vector of the rigid body can be determined as follows:

$$\mathbf{v} = \mathbf{K}^{-1}\mathbf{Q}. \tag{31}$$

4. Numerical example

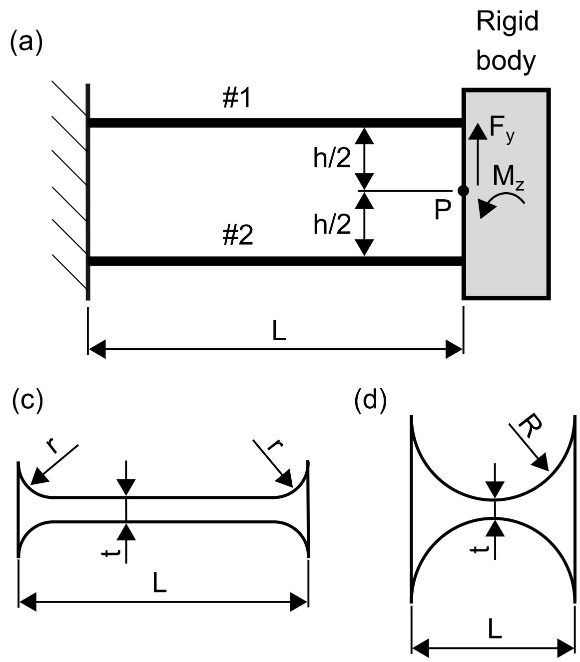
This numerical example deals with the planar mechanism with two identical parallel flexure hinges, as shown in Fig. 4(a). The numerical values of the mechanism parameters are shown in Fig. 4(b). Two types of flexure hinges will be considered, the symmetric corner-filleted flexure hinge and the circular flexure hinge which are shown in Fig. 4(c) and (d), respectively.

According to the reference [4] the thickness of the symmetric corner-filleted flexure hinge should be obtained by the following function:

$$t(x) = \begin{cases} t + 2\left[r - \sqrt{x(2r - x)} \right], x \in [0, r] \\ t, x \in [r, L - r] \\ t + 2\left\{ r - \sqrt{(L - x)\left[2r - (L - x)\right]} \right\}, x \in [L - r, r] \end{cases} \tag{32}$$

whereas the function of thickness change of the circular flexure hinge reads:

$$t(x) = t + 2\left[R - \sqrt{x(2R - x)} \right]. \tag{33}$$



(b) Flexure mechanism parameters:

$E = 2.1 \cdot 10^{11}$ [N/ m^2], $\mu = 0.33$,

$\alpha_f = 6/5$, $L = 0.01$ [m],

$h = 0.006$ [m], $t = 0.001$ [m],

$r = 0.002$ [m], $b = 0.006$ [m].

Fig. 4 Planar mechanism with two parallel flexure hinges

It is also supposed that the transversal force $F_y = 1$[N] and the bending moment $M_z = 0.02$[Nm] acts on the rigid body in the point *P*. The displacements Δy and $\Delta \theta$ in the point *P* of the rigid body are shown in Table 1. In the case when the symmetric corner-filleted flexure hinges was used, the obtained results are compared to those from reference [3], where the analytical method and finite element method (FEM) are used. Here, the results of the FEM analysis are considered to be the most accurate. It is obvious that our approach gives more precise results than analytical results from reference [3] in the case of determining transversal displacements Δy in point *P*. The obtained results for the angular displacement $\Delta \theta$ are the same for both used methods, the our and those analytical from reference [3]. The displacements in point *P* obtained by our approach, in the case when circular flexure hinge was used, are also given in Table 1.

	Symmetric corner-filleted flexure hinge			Circular flexure hinge
Displacements in point *P*	Our approach	Analytical [3]	FEM [3]	Our approach
Δy [m]	$2.3628 \cdot 10^{-7}$ (0.30%)	$2.359 \cdot 10^{-7}$ (0.46%)	$2.370 \cdot 10^{-7}$	$4.923 \cdot 10^{-8}$
$\Delta \theta$ [rad]	$9.373 \cdot 10^{-6}$ (5.75%)	$9.373 \cdot 10^{-6}$ (5.75%)	$9.945 \cdot 10^{-6}$	$5.349 \cdot 10^{-6}$

Table 1. Quasi-static response of planar mechanism with two parallel flexure hinges

5. Conclusions

The quasi-static response of planar parallel-connection flexure hinges mechanisms based on pseudo-rigid-body method approach was discussed in this paper. The parallel flexure hinges, which connect rigid body with the fixed end are discretized into the light rigid segments connected with the elastic joint elements with three degrees of freedom. The parameters of the elastic joint elements are taken from references [1,2]. The displacements of the arbitrarily selected point *P* of the rigid body are determined in the symbolic form. Therefore, the performed quasi-static analysis is numerically efficient, significantly more than the analytical approach from reference [3]. The obtained algorithm is universal, so the mechanism with the arbitrarily selected number, type and orientation of flexure hinges should be analyzed. The proposed approach should be extended to the planar flexure mechanisms with both parallel and serial connected flexure hinges. This will be the subject of further research by the authors.

Acknowledgments. This research was supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia (grant numbers TR35006 and ON174016).
This support is gratefully acknowledged.

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