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Switching Predictive Control: Controller Design and Simulations

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Abstract: *In this paper we will consider one class of switching controllers. Such control strategy is a mix of continuous dynamics and discrete events philosophy. Here we consider a finite set of the model predictive controllers (MPC) which are the only advanced control technique to have had a significant and wide spread impact on industrial process control. There are several advantages for wide acceptance of MPC: guaranteed stability, constraints handling and easy extension to multivariable and nonlinear systems. In this paper we add else one important property: significantly increasing of the transient performance using switching control strategy. Also, illustrative example is presented.*
Keywords: *Railway vehicle, independently rotating wheelset, active steering*

1 INTRODUCTION

The model predictive control (MPC) is the only advanced control methodology which has made a significant impact in industrial control engineering. We will mention that the main features of MPC are

(i) The extension to multivariable case is easy

(ii) It handles constraints. The higher performance levels are associated with pushing the limits. That frequently leads to more profitable operation

(iii) In industrial applications control update rate are relatively low and there is enough time for on-line computation.

Several important publications, in the form of survey papers and books, provide introduction to theoretical and practical issues associated with the MPC philosophy [1] and [2].

They noticed that most control laws, for example PID, do not explicitly consider the future implication of the current control actions. MPC, on the other hand, explicitly computes the predicted behavior over some horizon. One can therefore restrict the choice of current proposed input trajectories to those that do not lead to difficulties in the future.

Originally developed to meet the specialized control needs of the power plants and petroleum industry, MPC strategies can now be found in a wide variety of application areas such as discrete-event systems [3], cooperative control [4], digital electronic [5] and financial engineering [6].

For control of complex systems very important is the field of hybrid control. The hybrid systems describe the interaction of software, modeled by finite state systems such as finite state machines, with the physical world, described by differential or difference equations [7]. Specific problems in this field are presented in references [8] and [9]. The paper [10] presents a hybrid MPC. Authors propose frame for modeling and controlling models of the systems described by interacting physical laws, logical rules, and operating constraints.

As pointed out in [1] the consideration of hybrid systems opens up a rich area of research. Interesting application is presented in the field of power electronics (design of DC-DC converters). The application of hybrid model predictive control for step-down DC-DC converter is described in [11].

In this paper we introduce different strategy for switching predictive control in comparison with above mentioned papers. The controller is based on conventional optimal control that is obtained by minimization of some performance criteria. To be more specific, in the paper is considered the switching receding horizon control with the quadratic performance criterion. The performance criterion includes the prescribed degree of stability. The switching rule is based on the selection of the best performance from the finite set of the closed-loop systems. The main ingredient of the switching predictive controllers is the solution of the finite set of Riccati equations. Here is considered control of stable unconstrained systems.

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2 MULTIPLE MODELS

In this part of paper we consider multiple models description of processes. It will be assumed that the process model is a member of admissible process models

$$F = \underset{p \in P}{U} F_p \tag{1}$$

where P is index set which represents the range of parametric uncertainty so that for each fixed $p \in P$ the subfamily F_p accounts for unmodeled dynamics. Usually, P is compact subset of the finite-dimensional, normed linear vector space [12].

Here we will suppose that system for the large class of structured uncertainty can be described with collection of linear time invariant systems

$$x(k+1) = A_p x(k) + B_p u(k),\ p = 1,2,\ldots,s \tag{2}$$

where $x \in R^n$ and $u \in R^m$ are state and control signal of the system respectively. Relation (2) describes the continuous part of the system. The event driven part can be described in next general form

$$p(k+1) = \phi(k, p(k), x(k), z(k)) \tag{3}$$

where $p(k)$ is discrete event variable, $z(k)$ is external signal produced by other devices and $\phi(\cdot,\cdot,\cdot,\cdot)$ is a function which describes behavior of $p(k)$. In our case the switching signal is given as

$$p(k+1) = f(J_p),\ p = 1,2,\ldots,s \tag{4}$$

where J_p, $p = 1,2,...s$ corresponding performance index for subsystem collection.
The form of a function $f(\cdot)$ will be described later (see relation (19)).

3 THE SWITCHING MODEL PREDICTIVE CONTROL

Generally, no single controller is capable of solving the regulation problem for the entire set of process models (1). Owing that we will use the family of controllers [13]

$$\{C_q : q \in D\} \tag{5}$$

where D is index set. It is supposed that this family is sufficiently rich so that admissible process model can be stabilized by controller C_q for some index $q \in D$. In this paper we will consider the case

$$F = D. \tag{6}$$

According with [14], for non-switching stable systems, an orthonormal basis for discrete time system is

$$V_k = \begin{bmatrix} 0 & 0 & .. & 0 & I_{mN_u} & 0 & 0 & ... \end{bmatrix}^T \tag{7}$$

where I_{mN_u} is on the $k-th$ location. Function V_k is complete in the space of square summable inputs. Also N_u - dimensional projection of the input into the basis is

$$u^N = [I_N, 0] \sum_{k=k_0}^{N-1} V_k u(k) \tag{8}$$

where $u(k)$ is the control move at sample time $k, u(k) = 0$ for all $k \geq N_u$ and u^{N_u} is the mN_u vector with the nonzero inputs in the horizon N_u.

The control signal is given by minimization

$$\min_{u^{N_u}} \sum_{k=k_0}^{\infty} \lambda^{-2k} \left[x^T(k) Q x(k) + u^T(k) R u(k) \right] \tag{9}$$

where $\lambda \in (0,1]$.

A receding horizon regulator is based on minimization of the next criterion [14]

$$\min_{u^{N_u}} \left\{ \sum_{k=k_0}^{N_u-1} \lambda^{-2k} \left[x^T(k) Q x(k) + u^T(k) R u(k) \right] + \sum_{k=N_u}^{\infty} \lambda^{-2k} x^T(k) Q x(k) \right\} \tag{10}$$

When the matrices $A_p\ (p = 1,2,...s)$ are stable the last term in the relation (10) can be transformed into a penalty on the terminal state

$$\sum_{k=N_u}^{\infty} \lambda^{-2k} x^T(k) Q x(k) = \lambda^{-2N_u} x^T(N_u) Q_{N_u p} x(N_u)$$

$$p = 1,2,...s;\ j = N_u, N_u+1,... \tag{11}$$

The problem (10) now is

$$\min_{u^{N_u}} \left\{ \sum_{k=k_0}^{N_u-1} \lambda^{-2k} \left[x^T(k) Q x(k) + u^T(k) R u(k) \right] + \lambda^{-2N_u} x^T(N_u) Q_{N_u p} x(N_u) \right\} \tag{12}$$

$$p = 1,2,...s;\ j = 0,1...,N_u - 1$$

The terminal state penalty matrices are computed from the next discrete Lyapunov equations

$$Q_{N_u p} = \lambda^{-2} A_p^T Q_{N_u p} A_p + Q\ ,\ \ p = 1,2,...s \qquad (13)$$

According to the theory of the finite-time regulator problem [15] it is possible to get the feedback gain of switching predictive controller

$$K_{(N_u-1)p} = \lambda^{-1}\left(R + B_p^T P_{(N_u-1)p} B_p\right)^{-1} B_p^T P_{(N_u-1)p} A_p \quad (14)$$

$p = 1,2,...s$

where Riccati difference equations have a form

$$P_{(j+1)p} = Q + \lambda^{-2} A_p^T \left[P_{jp} - P_{jp} B_p \left(B_p^T P_{jp} B_p + R\right)^{-1} B_p^T P_{jp} \right] A_p \quad (15)$$

$$P_{0p} = Q_{N_u p}\ ,\ N_u > 1 \qquad (16)$$

$p = 1,2,...s$; $\quad j = 0,1...,N_u - 1$

The control law is

$$u(k) = -K_{(N_u-1)p} x(k) \qquad (17)$$

$p = 1,2,...s$, $k = 0,1,2,...$

The basic concept of receding horizon control is as follows. The optimal control, at the current time k, is obtained on a fixed horizon $[k, k+N_u]$. Among the optimal controls on the horizon $[k, k+N_u]$ only the first one is used as the current control law. The procedure is then repeated at the horizon $[k+1, k+1+N_u]$.

Finally we will determine the function $f(\cdot)$ in relation (4). The optimal value of the objective function (13), for fixed p, is given as in [15]

$$-\lambda^{-2k} x^T(k) P_{N_u p} x(k)\ ,\ k = 0,1.... \qquad (18)$$

The discrete feedback (function $f(\cdot)$) is

$$p(k+1) = \arg\min\left\{\lambda^{-2k} x^T(k) P_{N_u p} x(k)\right\} \qquad (19)$$

$p = 1,2,...s$, $k = 0,1,2,...$

A last relation is a specific form of supervisor in switching control systems.

Remark 1. The hybrid LQ control in continuous-time domain, based on performance guarded principle, is considered in [16].

Remark 2. The system (2) can be written in the referenced predictive form [2]

$$x(k+j+1/k) = A_p x(k+j/k) + B_p u(k+j/k)$$

In that case for MPC it is possible to introduce two performance criterias

A) with free terminal cost

$$J_{FTC} = \sum_{j=0}^{N-1}\Big[x^T(k+j/k)\ Qx(k+j/k) + u^T(k+j/k) Ru(k+j/k)\Big] + x^T(k+N/k)\ Qx(k+N/k)$$

B) with terminal equality constraint

$$J_{TEC} = \sum_{j=0}^{N-1}\Big[x^T(k+j/k)\ Qx(k+j/k) + u^T(k+j/k) Ru(k+j/k)\Big],\ \ x(k+N/k) = 0$$

But proof for stability is different and not presented in literature.

Remark 3. Suppose that the input and state have constraints

$D_p u(k) \le d_p$, $D_p \in R^{m_1 x m}$, $d_p \in R^{m_1}$

$p = 1,2,...s$, $k = 0,1,...,N_u - 1$

and

$H_p x(k) \le h_p$, $H_p \in R^{n_1 x n}$, $h_p \in R^{n_1}$

$p = 1,2,...s$, $\forall k > k_2$

where k_2 is determined in [14].

The solution of receding horizon problem can be found by quadratic programming. The closed-loop system can be expressed as follows [17]

$$x(k+1) = A_p x(k) + B_p \psi(x(k))$$

$p = 1,2,...s$, $k = 0,1,...$

where $\psi(x(k))$ is control input $u(k)$ determined as the solution of quadratic program. Owing the constraints presence $\psi(x(k))$ is nonlinear function of the state $x(k)$. Reference [18] discusses the nonlinearity properties of the solution of the linear model predictive control quadratic program. For the constrained receding horizon regulator the *Theorem 1* is not applicable.

Remark 4. Very new investigations which can be interesting for further development of switching receding horizon control are

a) Robustness of MPC is a very important property of MPC [19], [20]. The robust MPC utilizes a description of the model uncertainty and is aimed at guaranteeing both constraints satisfaction and closed-loop stability. In [20] is proposed a robust output feedback MPC design for a class of open-loop stable systems having non-vanishing output disturbances, hard constraints and linear time invariant model uncertainty.

b) Early implementations of MPC where constrained in the process industry. In such applications the sample periods is long and set-points are constant. But, new techniques and faster sampling rates include the new applications:electromechanical, power electronics and telecommunications problems. In these areas the reference signal is not constant or even piecewise constant. In [21] is described a novel strategy for MPC design which incorporates feedback, reference feed-forward and preview.

c) Stochastic MPC is, also, important field of investigations. In [22] is proposed the MPC strategy which handles probabilistic constraints with acceptable computational load. This is achieved by fixing the cross-sectional shapes of tubes containing predicted states and allowing their centers and scaling to vary with time.

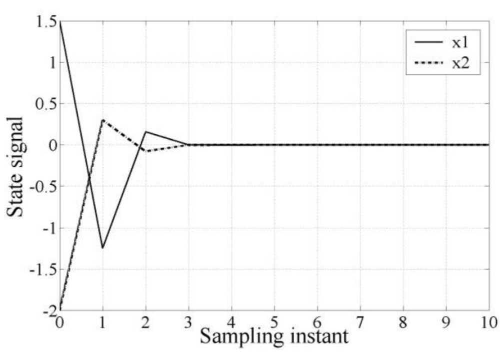
4 ILUSTRATIVE EXAMPLE

Consider the following collection of stable plants

$$A_1 = \begin{bmatrix} 0.2 & 2 \\ 0 & 0.8 \end{bmatrix},\ b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.5 & 1 \\ 0 & 0.7 \end{bmatrix},\ b_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix},\ x(0) = \begin{bmatrix} 1.5 \\ -2 \end{bmatrix}$$



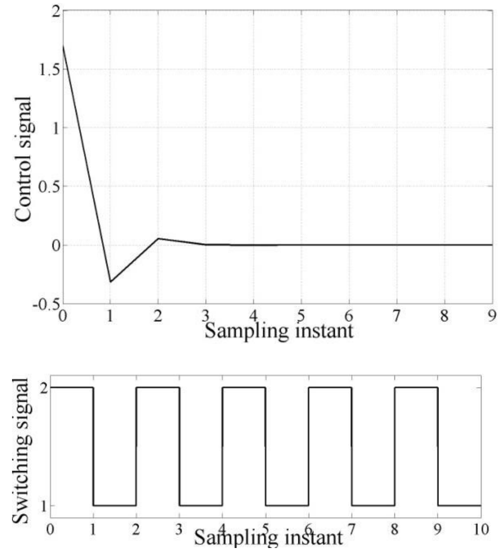


Fig. 1. *States, control signal and switching signal for* $r = 0.05$, $N_u = 4$ *and* $\lambda = 1$

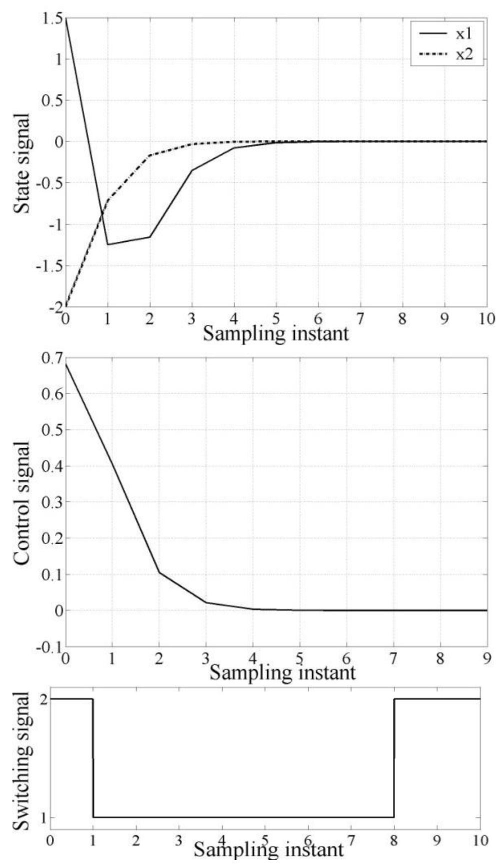


Fig. 2. *States, control signal and switching signal for* $r = 10$, $N_u = 4$ *and* $\lambda = 1$

With tuning parameters R (in quadratic criterion), N_u (control horizon) and λ (degree of the stability). In our case R is scalar and will be denoted with r.

In what follows we consider tuning parameter r (in the Fig.1. and Fig.2).

From above figures we see that if it is more important that the control energy be small, then we should select a large value of r (see Fig. 2). Also, it is possible to notice that exists correlation between choice of r and a form of control signal and switching signal. For small r the control u is a large and switching between subsystems is fast but the state trajectory convergence is, also, faster. (See Fig. 1)

In the next figure we consider the case when control horizon is $N_u = 10$

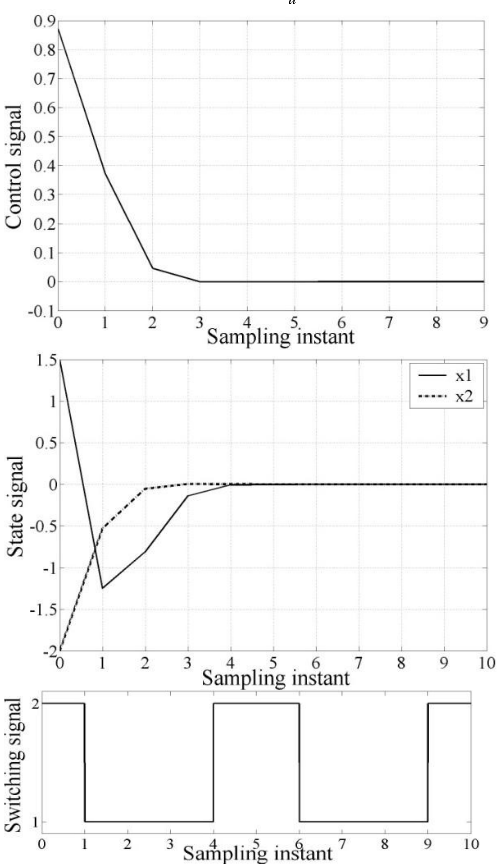


Fig. 3. *States, control signal and switching signal for* $r = 10$, $N_u = 10$ *and* $\lambda = 1$

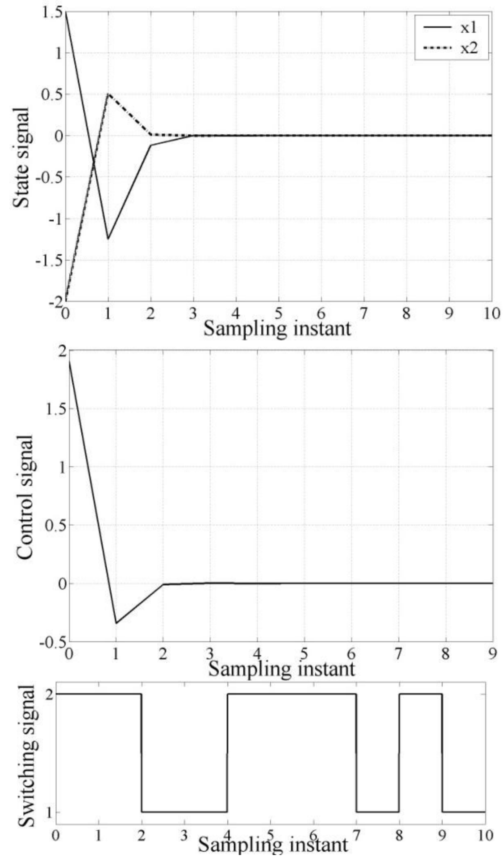


Fig. 4. *States, control signal and switching signal for* $r = 0.5$, $N_u = 4$ *and* $\lambda = 0.8$

From comparison of Fig.2 and Fig.3, it is possible to conclude that for last case the transient behavior is better whereby the control signal is slightly larger.

From Fig. 4 it is follows that closed-loop system has a good behavior (only 3 sampling instants is enough for practical state convergence).

Finally, from intensive simulations the acceptable set of tuning parameters is: $r = 0.5$, $N_u = 4$ and $\lambda = 0.8$.

5 CONCLUSION

In this paper the problem of design of switching model predictive controllers is considered. The main motivation for such type of controllers is performance improvement of feedback system. Also, very important fact is that in practice exist systems which impossible to control using classical control strategy. In this paper is shown that by using index of performance, which is uniformly bounded, it is possible to design MPC switching controllers which guarantee stability of feedback system. Further investigation is oriented toward the unstable plants with constraints.

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