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Stochastic Model of a Pneumatic Actuator

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Abstract: *Intensive research in the field of mathematical modelling of the pneumatic cylinder has shown that its mathematical model is nonlinear and that a lot of important details cannot be included in the model. Selection of the model and the identification method have been conditioned by the following facts:*

a) The nonlinear model of the system can be approximated by a linear model with time-variant parameters

b) There is the influence of the combination of heat coefficient, unknown discharge coefficient and change of temperature on the pneumatic cylinder model. Therefore it is assumed that the parameters of the pneumatic cylinder are random (stochastic parameters)

c) In practical conditions, observations have a non-Gaussian distribution.

Due to the abovementioned reasons, it is assumed that the pneumatic cylinder model is a linear stochastic model with variable parameters. The Masreliez-Martin filter (robust Kalman filter) was used for identification of parameters of the model. For the purpose of increasing the practical value of the filter, the some heuristic modifications were performed. The behaviour of the new approach to identification of the pneumatic cylinder is illustrated by simulations.

Keywords: *pneumatic actuator, stochastic model, time-variant parameters, non-Gaussian distribution, robust filter*

1 INTRODUCTION

Since pneumatically driven systems have a lot of distinct characteristics of energy-saving, cleanliness, simple structure and operation, and high efficiency and are suitable for working in a harsh environment, they have been extensively used for many years in robot driven systems and industrial automation [1].

However, the problem with complex nonlinear models, such as the pneumatic servo cylinder, is that it is difficult to choose the large number of physical parameters involved in the model. Although a lot of parameter values are known a priori with reasonable accuracy, a large number of parameters are only known within a certain range, and some are even completely unknown. This may be due to manufacturing tolerances, or due to the fact that manufacturers do not provide parameter values because they consider them as proprietary information.

Furthermore, it is extremely difficult to accurately acquire some system parameters (such as component dimensions, internal leakage coefficients, static and dynamic friction forces, etc.) because the mentioned parameters cannot be directly measured or calculated. This causes a great difficulty in system modelling and control.

The consequence of these problems is that the theoretical model is often not useful for quantitative analysis of the pneumatic servo-system behaviour.

The purpose of this paper is to use the theory and findings of system identification to obtain a mathematical model, so that the controller can be designed on the basis of the model.

Östring et al. [2] identified the behaviour of an industrial robot in order to model its mechanical flexibilities, while Johansson et al. [3] used a state-space model to identify the robot manipulator dynamics. Assuming most parameters in pneumatic servo system do not change during operation, Shih and Tseng [4] performed the identification offline and adjusted servo-control before the operation accordingly. Furthermore, they investigated the impact of different parameters (sampling time, order model, different supply pressures, etc.) in the identification process.

The mentioned references consider the linear models of the pneumatic cylinder which are ad hoc adopted, without considering justification

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of such an approach. It is necessary to notice the following details:

i. The pneumatic cylinder is a nonlinear system (presence of friction force)

ii. There is a significant influence of the combination of the heat coefficient, unknown discharge coefficient and change of temperature on the behaviour of the pneumatic cylinder [5]. The mentioned influences cannot be easily included in the cylinder model and have random character.

On the other hand, recent research has shown that the nonlinear model of the system can be approximated by a linear system with time-variant parameters [6]. In this paper it is assumed that the parameters of the pneumatic cylinder model change randomly. The change of parameters is described by the random walk method, where the corresponding noise is modelled as the Gaussian stochastic process. The output error (OE) method is used as the identification algorithm. It is assumed that the measurement noise is non-Gaussian. Justification of this approach was confirmed in practice [7]. Namely, in measurements there are rare, inconsistent observations with the largest part of population of observations (outliers). Therefore, synthesis of robust algorithms is of primary interest.

The Masreliez-Martin filter is the natural frame for realization of the described algorithm. The model in the state space in which the process noise has a Gaussian distribution, and the measurement noise has a non-Gaussian distribution corresponds to the adopted model for the pneumatic cylinder.

2 MODELLING OF A PNEUMATIC SERVO SYSTEM

The system under consideration consists of an electro-pneumatic position control servo drive and a pneumatic actuator with a load as shown in Fig. 1.

Applying Newton's second law to the forces on the piston, the resulting force equation is

$$A_aP_a - A_bP_b = m\ddot{y} + \beta_e\dot{y} + F_f(\dot{y}) + k_e y + F_{ext} \tag{1}$$

where P_a and P_b denote the pressure of the chamber a and b, respectively, m denotes the total mass of the piston and the load referred to the piston, y is the piston displacement, β_e is the nonlinear viscous friction coefficient, k_e denotes the load spring gradient; and F_{ext} denotes the load force disturbance on the piston. The term F_f in equation (1) describes the summing nonlinear effects of static and Coulomb friction forces of the system.

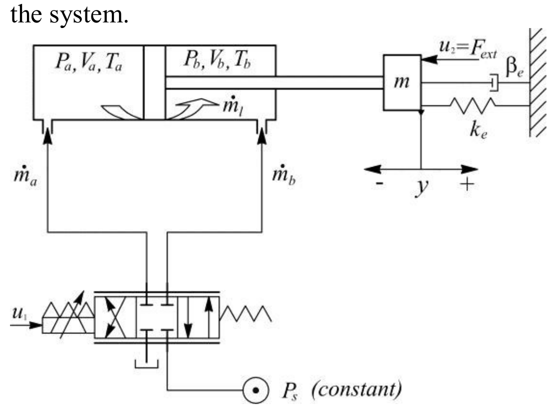

Fig. 1 *Schematic representation of the valve controlled asymmetric piston*

Pressure dynamics in the chambers, for $i = a,b$, is given by [5]:

$$\frac{dP_i}{dt} = -\alpha(t)g_i(P_i, y, \dot{y}) + \beta(t)h_i(t, P_i, y)u_1 \tag{2}$$

in which:

$$g_i(P_i, y, \dot{y}) = \frac{P_i\dot{V}_i(\dot{y})}{V_i(y)} \tag{3}$$

and

$$h_i(t, P_i, y) = \frac{\sqrt{RT_S}}{V_i(y)}Wf(P_i)\,\mathrm{sgn}(u_1) \tag{4}$$

where

R is the universal gas constant, W is a spool constant, T_S is ambient absolute temperature.

If the state variables and the input variables are defined as $x_1 = y$, $x_2 = \dot{y}$, $x_3 = P_a$, $x_4 = P_b$, u_1 (valve input) $u_2 = F_{ext}$ (external disturbance) then a completely nonlinear model of the pneumatic servo-system, can be written as:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m}\big(A_a x_3 - A_b x_4 - \beta_e x_2 - F_f(x_2) - \\
&-k_e x_1 - u_2 \big) \\
\dot{x}_3 &= -\alpha(t) g(x_1, x_2, x_3) + \beta(t) h(t, x_1, x_3) \\
\dot{x}_4 &= -\alpha(t) g(x_1, x_2, x_4) + \beta(t) h(t, x_1, x_4)
\end{aligned} \tag{5}$$

Uncertain heat coefficient $\alpha(t)$ depends on the actual heat transfer occurring during the process. As it can be seen from [5], $\alpha(t)$ takes values between 1 and 1.3997.

Uncertain bound parameter $\beta(t)$, which takes values between 0.075 and 1.3297 (see [5]), is used to characterize the combination of the heat coefficient $\alpha(t)$, the unknown valve discharge coefficient $C_d(t)$ and the variation of the temperature $\tau(t)$. Thus, $\beta(t)$ is generally expressed by:

$$\beta(t) = \alpha(t) C_d(t) \sqrt{\tau(t)} \tag{6}$$

Since uncertain heat coefficient $\alpha(t)$ and uncertain bound parameter $\beta(t)$, are only known in the certain range, it can be considered that their changes have random character. Since mentioned uncertain coefficients are involved (directly or indirectly) in the state variables, previous analysis has justified the assumption that the system is considered as stochastic.

3 STOCHASTIC MODEL OF THE PNEUMATIC ACTUATOR

The previous section shows that the mathematical model of the pneumatic cylinder in nonlinear and that it is not possible to include a large number of important details in the model. The natural way of solving this problem is to apply the identification theory. In that case the following problems arise:

- Type of the model (linear, nonlinear, deterministic, stochastic)
- Nature of disturbance (uniformly constrained, stochastic)

The following three facts have conditioned the choice of the model:

a) Recent research has shown that the nonlinear model of the system can be correctly approximated by a system with time variant parameters [6]

b) A more detailed analysis of the pneumatic cylinder model described in the previous section shows that the combination of heat coefficient, unknown discharge coefficient and change of temperature influences the model of cylinder [5]. Those influences are random and therefore it is assumed that the parameters of the pneumatic cylinder are random.

c) Practical and theoretical research has shown that in a stochastic model of the system there are some observations that are inconsistent with the largest part of the population (outliers) [7], and that is why the disturbance in the model (measurement noise) is non-Gaussian.

The mentioned reasons lead to the assumption that the model of the pneumatic cylinder is a stochastic linear model with time variant parameters.

Taking into account the physics of the problem, it will be assumed that the change of the parameters has the form of random walk

$$\theta(k+1) = \theta(k) + \omega(k) \tag{7}$$

where the stochastic process $\omega(k)$ is Gaussian with the mean value zero and the covariance matrix $W(k)$.

The output error method based on systems with a reference model will be used as a model which describes the dynamics of the pneumatic cylinder.

The output of the model without disturbance will be denoted as $y_n(k)$. The dynamics of the model in that case is described as

$$\begin{aligned}
y_n(k) &= -a_1(k) y_n(k-1) - \ldots - a_n(k) y_n(k-n) \\
&+ b_1(k) u(k-1) + \ldots + b_m(k) u(k-m)
\end{aligned} \tag{8}$$

Let us introduce the following vectors

$$\theta(k) = [a_1(k),\ldots,a_n(k),b_1(k),\ldots,b_m(k)]^T \quad (9)$$

$$\varphi_0(k) = [-y_n(k-1),\ldots,-y_n(k-n), \\ u(k-1),\ldots,u(k-m)]^T \quad (10)$$

In that case the dynamics of the system with disturbance is given by the following relation

$$y(k) = \theta^T(k)\varphi_0(k) + \nu(k) \quad (11)$$

The disturbance $\nu(k)$ is non-Gaussian and includes the presence of outliers.

The problem with the relation (8) is that the values $y_n(k-i)$, $(i = 1,2,\ldots,n)$ cannot be measured. Therefore, these values are calculated by using the current estimates of the parameters θ. It results in

$$\hat{y}_n(k) = -\hat{a}_1(k)\hat{y}_n(k-1) - \ldots - \hat{a}_n(k)\hat{y}_n(k-n) \\ +\hat{b}_1(k)u(k-1) + \ldots + \hat{b}_m(k)u(k-m) \quad (12)$$

the following vectors are introduced

$$\hat{\theta}(k) = [\hat{a}_1(k),\ldots,\hat{a}_n(k),\hat{b}_1(k),\ldots,\hat{b}_m(k)]^T \quad (13)$$

$$\varphi(k) = [-\hat{y}_n(k-1),\ldots,-\hat{y}_n(k-n), \\ u(k-1),\ldots,u(k-m)]^T \quad (14)$$

the relation

$$\hat{y}_n(k) = \hat{\theta}^T(k)\varphi(k) \quad (15)$$

is obtained. At the moment k, before the estimate $\hat{\theta}(k)$ is known, the prediction of the model is [8]

$$\hat{y}(k) = \hat{\theta}^T(k-1)\varphi(k) \quad (16)$$

The natural definition of the prediction error is

$$\nu(k) = y(k) - \hat{y}(k) \quad (17)$$

Let us assume that the system in the state space can be described as

$$x(k+1) = F(k)x(k) + w(k) \quad (18)$$

$$y(k) = H(k)x(k) + \nu(k) \quad (19)$$

where

$$x(\cdot) \in R^n,\ F(\cdot) \in R^{n\times n},\ w(\cdot) \in R^n$$

$$y(\cdot) \in R^1,\ H(\cdot) \in R^{1\times n},\ \nu(\cdot) \in R^1$$

The value $x(\cdot)$ is the state vector, $y(\cdot)$ is the system output, and $w(\cdot)$ and $e(\cdot)$ are the process noise and the measurement noise, respectively. It is assumed that the process noise is Gaussian $\mathrm{N}(0,\mathrm{W}(k))$, where $\mathrm{W}(k)$ is the covariance matrix, and $\nu(\cdot)$ is the measurement noise which has non-Gaussian distribution.

In reference [9] Masreliez and Martin proposed the robust Kalman filter for the mentioned situation. This filter has small sensitivity to the presence of outliers in comparison with the standard Kalman filter deduced for the case when the values $w(\cdot)$ and $\nu(\cdot)$ have Gaussian distribution.

The originally proposed robust Kalman filter [9] includes two values which are not easy to determine in practical conditions. They are the scalar transformation $T(k)$ as well as the member in the a posteriori covariance matrix $E_{f_0}\{\psi'(\nu(k))\}$. The mentioned member represents Fisher information for the least favourable probability density [10]

$$I(p) = \int_{-\infty}^{\infty} \frac{p'^2(\zeta)}{p(\zeta)} d\zeta \quad (20)$$

In order to increase the practical values of the robust Kalman filter [9] the following heuristics were performed:

a) For the scalar transformation $T(k)$ it has been adopted that $T(k) = 1$

b) The member $E_{P_\varepsilon}\{\psi'(\nu(k))\}$ was approximated by the realization of $\psi'(\nu(k))$

Intense simulations justified such interventions. Now the proposed robust Kalman filter [9] obtains the following modified form:

$$\hat{x}(k|k) = F(k-1)\hat{x}(k-1|k-1) + P(k|k-1)\cdot \\ H^T(k)\cdot\psi[y(k) - H(k)F(k-1)\hat{x}(k-1|k-1)] \quad (21)$$

$$P(k|k-1) = F(k-1)P(k-1|k-1)F^T(k-1) + \\ +W(k-1) \quad (22)$$

$$P(k|k) = P(k|k-1) - P(k|k-1)H^T(k)H(k)\cdot P(k|k-1)\cdot \cdot\psi'\left(y(k) - H(k)F(k-1)\hat{x}(k-1|k-1)\right) \quad (23)$$

It is important to notice that the second heuristic modification increases the rate of convergence (21)-(23) in the initial iterations. Namely, the relations (21)-(23) for the robust Kalman filter gain result in:

$$K(k) = F(k-1)\big[P(k-1|k-2) - -P(k-1|k-2)H^T(k)H(k)P(k-1|k-2)\cdot \cdot\psi'\left(\nu(k)\right)\big]F^T(k-1)H^T(k) + W(k-1)H^T(k) \quad (24)$$

If $|\nu(k)| > k_\varepsilon$, the relation (24) becomes

$$K(k) = F(k-1)P(k-1|k-2)F^T(k-1)\cdot H^T(k-1) + W(k-1)H^T(k) \quad (25)$$

It means that the bigger the estimation errors, the higher the filter gain and thus the higher rate of estimation convergence.

By comparing the relations (7) and (11) with the relations (18) and (19) and taking care that the vector $\varphi_0(k)$ should be replaced with $\varphi(k)$ and by substituting for the values

$$F(k) = I\,,\ H(k) = \varphi^T(k)\,,\ \hat{x}(k|k) = \hat{\theta}(k) \quad (26)$$

a recursive algorithm for estimation of time-variant parameters is obtained in the relation (21)-(23).

4 ILUSTRATIVE EXAMPLE

The model of a pneumatic cylinder whose time varying parameter vector has the expected value:

$$\bar{\theta} = [-0.9131 \quad -0.3523 \quad 0.1118 \quad 0.2318 \quad -0.0413 \quad 0.0766 \quad 0.0115 \quad 0.0647]^T \quad (27)$$

is considered for the purpose of demonstrating the performance of the proposed robust procedure for parameters estimation. The process noise $\omega(k)$ is Gaussian with the zero mean value and the covariance matrix

$W(k) = E\{\omega(k)\omega(k)^T\}$, where

$$\omega(k) = [w_1(k) \quad w_2(k) \quad w_3(k) \quad w_4(k) \quad w_5(k) \quad w_6(k) \quad w_7(k) \quad w_8(k)]^T \quad (28)$$

If the probability density is denoted as $p_N(\cdot) \sim N(m,\sigma_N^2)$ where m is the mean value, and σ_N^2 is the dispersion, then:

$$\begin{array}{ll} p_N(w_1) \sim \mathrm{N}(0;2\cdot10^{-6}), & p_N(w_5) \sim \mathrm{N}(0;2\cdot10^{-8}), \\ p_N(w_2) \sim \mathrm{N}(0;3\cdot10^{-6}), & p_N(w_6) \sim \mathrm{N}(0;2.2\cdot10^{-8}), \\ p_N(w_3) \sim \mathrm{N}(0;2.5\cdot10^{-6}), & p_N(w_7) \sim \mathrm{N}(0;2.5\cdot10^{-8}), \\ p_N(w_4) \sim \mathrm{N}(0;2.2\cdot10^{-6}), & p_N(w_8) \sim \mathrm{N}(0;3\cdot10^{-8}). \end{array}$$

Figures 2 to 5 show the system output, parameter estimates, and mean square error in the case when the contamination $\varepsilon = 0.05$.

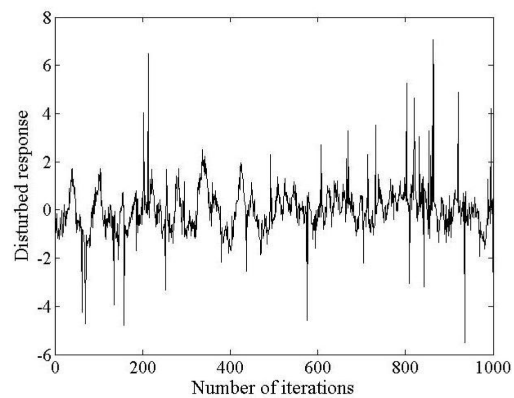

Fig. 2 *Simulation of the measured output signal of the system with the contamination* $\varepsilon = 0.05$

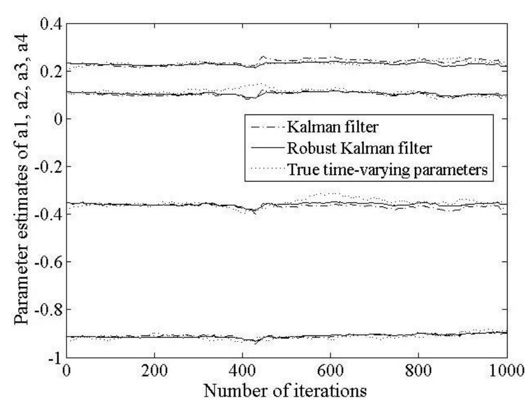

Fig. 3 *Estimates of the parameter* a_i *obtained in a non-Gaussian noise environment with the contamination* $\varepsilon = 0.05$

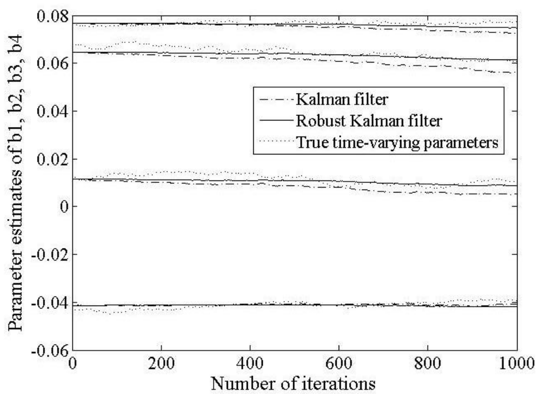

Fig. 4 *Estimates of the parameter* b_i *obtained in a non-Gaussian noise environment with the contamination* $\varepsilon = 0.05$

The simulation results are compared in terms of mean square error (MSE), defined by

$$MSE = \log\left(\left\|\hat{\theta}(k) - \theta(k)\right\|^2\right) \tag{29}$$

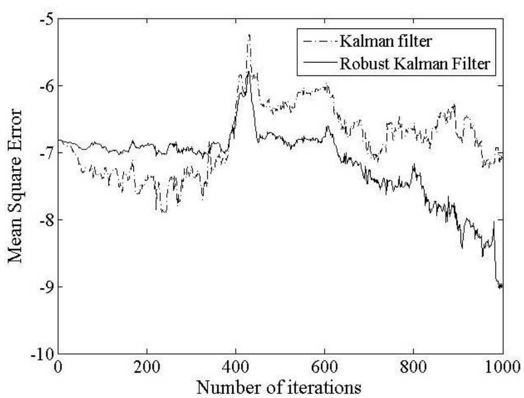

Fig. 5 *Mean square error, obtained in a non-Gaussian noise environment with the contamination* $\varepsilon = 0.05$

Remark 1**:** The presented results have shown that the classical Kalman filter is very sensitive to the non-Gaussian measurement noise presence, as opposed to the proposed robust Kalman filter.

5 CONCLUSION

The paper considers a new mathematical model of the pneumatic cylinder. Change of parameters of the model is described by random walk. It is assumed that the cylinder is described by means of the output error model, where the measurement noise is non-Gaussian. Since the system is described with a stochastic model with variable parameters, the natural frame for identification is the Masreliez-Martin filter (the robust Kalman filter). Heuristic modifications of the mentioned filter which considerably increase its practical values were performed. The results of this paper can be the starting point for design of an adaptive regulator.

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