

# Modeling and simulation of Hydraulic Long Transmission Line by Bond Graph

Dragan Pršić<sup>1,\*</sup>, Novak Nedić<sup>1</sup>, Ljubiša Dubonjić<sup>1</sup>

<sup>1</sup>University of Kragujevac, Faculty of Mechanical Engineering Kraljevo, Kraljevo 36 000, Serbia

*This paper address the issue of modeling of the hydraulic long transmission line. In its base, such model is nonlinear with distributed parameters. Since general solution in closed-form for such model is not available, certain simplifications have to be introduced. The pipeline in the paper has been divided to a cascaded network of  $\pi$  segments so that a model with lumped parameters could be reached. For segment modeling, a standard library of bond graphs element has been used. On the basis of models with lumped parameters, the effect of the number of segments, pipeline length and effective bulk modulus on the dynamics of long transmission line has been analyzed.*

**Keywords:** Long transmission line, lumped parameters,  $\pi$  segments, bond graph

## 1. INTRODUCTION

Existence of a long transmission line (LTL) in hydraulic systems makes their dynamics significantly complex. This is especially emphasized with building and mining machines, agricultural machines, transportation machines, machine tools and other devices where connection between the actuators energy source achieved by a long hydraulic line. Physical variables, pressure and volumetric flow featuring the energy transfer along the hydraulic line, besides the time coordinate, depend on spatial coordinate as well. These physical variables' dependency on spatial coordinate conditions spatial distribution cannot be neglected in long hydraulic line modeling. Therefore they are described with models with distributed parameters. Models with distributed parameters are described with partial differential equations and they are of infinitesimally high order. Use of such a model in analysis of dynamic behavior in time domain is not practical because it requires work with transcendent transfer functions and their approximations using Bessel functions.

In this paper we have used the method of description of a long hydraulic pipe with lumped parameters in a way that it was divided to  $n$  equal segments of  $L_s$  length.  $\pi$  and  $T$  model with lumped parameters have been used whose electrical analogies are given in the paper. On the basis of equivalent electric circuits, adequate bond graphs of these circuits were made and connected into a cascade of  $n$  segments, which

defines the mathematical model of the hydraulic pipe. On the basis of the mathematical model and simulation, we have analyzed the impact of certain parameters on the character of the transfer process and its results are given in the paper.

## 2. MODELING OF HYDRAULIC LONG TRANSMISSION LINE

One-dimensional flow of compressible, viscous fluid through the LTL is represented by a set of nonlinear partial differential equations [1]. Applying physical principles of mass conservation, Newton's second law and energy conservation leads to:

$$\frac{A}{a^2} \frac{\partial p}{\partial t} = -\rho \frac{\partial Q}{\partial x} \quad (1a)$$

$$\frac{1}{A} \frac{\partial Q}{\partial t} + g \sin \alpha + \frac{\lambda}{2dA^2} Q^2 = -\rho \frac{\partial p}{\partial x} \quad (1b)$$

That is a pair of quasilinear hyperbolic partial differential equations describing pressure change  $p$  and volumetric flow rate  $Q$  depending on time  $t$  and distance  $x$  along the pipeline. Generally, there is no closed-form solution for these equations. The problem is particularly expressed in case of turbulent flow which introduces stochastic parameters. Models with distributed parameters are described by differential equations and the model thus obtained is of infinitesimally high order [2-5].

\*Corr. Author's Address: Faculty of Mechanical Engineering Kraljevo, Kraljevo 36000, Serbia, prsic.d@mfvk.rs

In order to find analytical or numerical solution, certain simplifications have to be introduced. One approach is based on division of non-homogenous fields ( $p(x,t)$ ,  $Q(x,t)$ ) on segments with homogenous fields ( $p(t)$ ,  $Q(t)$ ) of all physical variables. In other words, instead of models with distributed parameters we move to the model with lumped parameters. Exactly this is the approach used in this study. LTL is observed as a cascaded network of small lumped elements (Fig.1) where dynamics of each of them is described by common differential equations.



Fig.1. A cascaded network representing a transmission line

Each of lumped elements presents spatial abstractions of distributed physical properties. Physical dimensions of each segment are much smaller than the shortest wavelength of interest. Connections between lumped elements represent physical constraints on the physical quantities associated to the elements [6]. Practically, we need to do a model of one segment and then serially connect those models into the system model (LTL).

What we usually find in literature are two single-lump approximations for a short fluid line [7,8]. Those two approximations are known as  $\pi$  and T circuits. Their electrical analogies are given in Fig. 2.

Same elements for modeling of the lumped elements were used in both models. Using capacitor (C element), we model fluid compressibility, using coil (L element) we model fluid inertia, and with resistor (R element) we model losses due to friction. Difference is in the way those elements are connected. In the first case, there are two distinctive pressures:  $P_1$  on one end and  $P_2$  on the other end of the segment. Same flow rate  $Q_i$  is adopted along the whole segment length. In the second case, there is one distinctive pressure of Pi segment and two flows  $Q_1$  and  $Q_2$  on the segment's ends.

Which model is to be used depends on the remaining part of the circuit where the given pipeline is located. Namely, each segment is

connected with the surrounding with two energy flows.

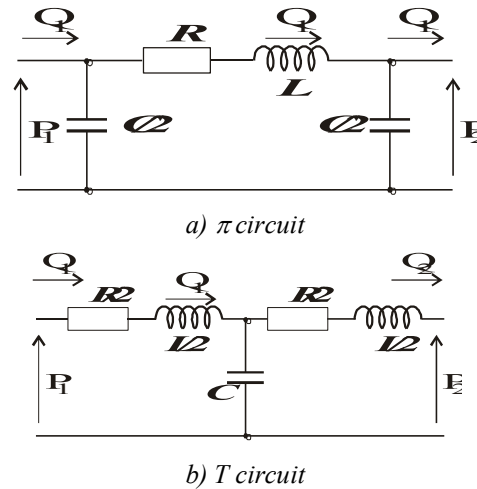


Fig.2. Equivalent electrical analogy of short fluid line

On one end, the energy flow is  $P_1Q_1$ , and on the other  $P_2Q_2$ . This means that interaction of segments with the environment can be described with four variables. In regard to the segment, two values must be independent, and the other two dependent, provided that the independent values must be on different segment ends. For example, it is not allowed for the values  $P_1$  and  $Q_1$  to be independent. Therefore, it is possible to make four combinations of independent-dependent variables:  $Q_1Q_2 \rightarrow P_1P_2$ ,  $Q_1P_2 \rightarrow Q_2P_1$ ,  $P_1Q_2 \rightarrow Q_1P_2$  and  $P_1P_2 \rightarrow Q_1Q_2$ . Model of the segment, i.e. the whole pipeline depends on the combination appearing in the circuit. It should be noted that when the segment cascade is made, regardless if we use  $\pi$  or  $T$  model, there are blocks inside the model shown in Fig. 3 that periodically repeat.

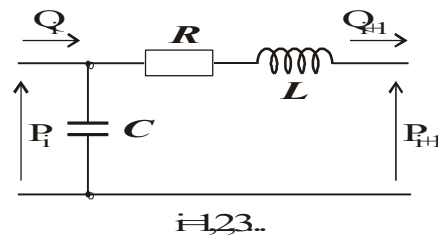


Fig.3. Block of LTL model repeating periodically

Difference in models comprising of  $\pi$  and  $T$  segments is in the end blocks – interfaces according to the remaining of systems symmetric in regard to internal blocks. Equivalent electrical circuits of end blocks of the cascade with  $\pi$  segments ( $\pi$  interfaces) are shown in Fig. 4 and cascades with  $T$  segments ( $T$  interfaces) in Fig. 5.

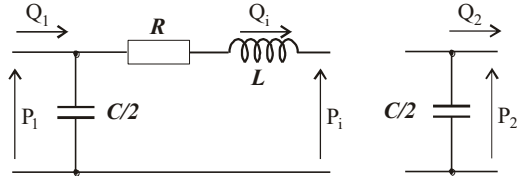


Fig.4. LTL model interfaces with  $\pi$  segments ( $\pi$  interfaces)

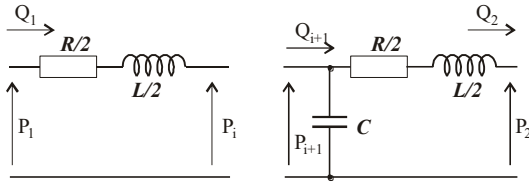


Fig.5. LTL model interfaces with  $T$  segments ( $T$  interfaces)

Suitable bond graphs of the electric circuits in Fig. 3, 4 and 5 are shown in Fig. 6, 7 and 8 in respectively.

On the basis of drawn causality it can be seen that all internal blocks are second rate models. The end blocks for the  $\pi$  cascade have internal causality for one-junction and arbitrary causality for zero-junctions. It is similar with the  $T$  cascade where zero-junction has integral causality and one-junctions the arbitrary one. This means that the LTL model can have minimum order of  $(n-1)*2+1$  and maximum  $(n-1)*2+3$ . For example, cascade of  $n=5$  segments can be minimally 9 order and maximum 11. Order of the model depends on the hydraulic circuit where the given LTL is found. In order to avoid the differential causality and reduce the numerical problem, recommendation is to use the  $\pi$  cascade if the flow is independent value. If pressure is

independent value, differential causality is avoided with use of the  $T$  cascade.

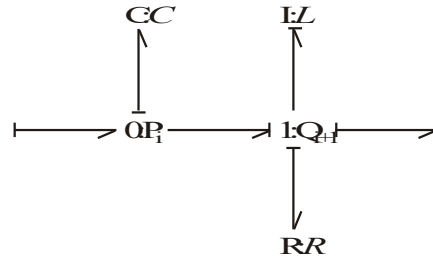


Fig.6. LTL model bond graph repeating periodically

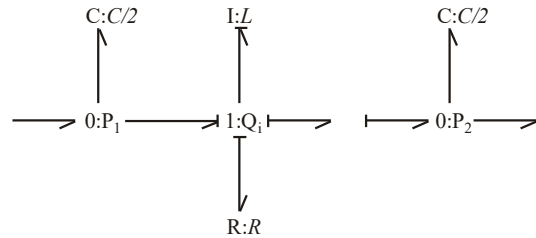


Fig.7. LTL model interface bond graph with  $\pi$  segments ( $\pi$  interfaces)

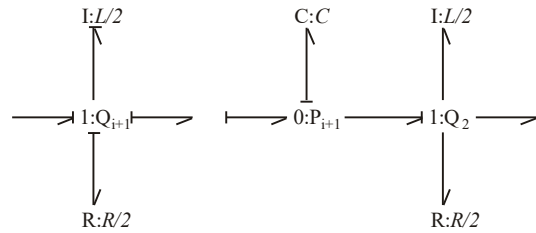


Fig.8. LTL model interface bond graph with  $T$  segments ( $T$  interfaces)

In this study, we observe the response of the system on the independent flow change. Diagram of hydraulic installation is shown in Fig. 9.

As a source of trigger, variable flow rate source is used. LTL is represented as a cascade of  $n$  segments. At the end of the circle there is a orifice of constant resistance. Model order is determined with  $2n+1$ .

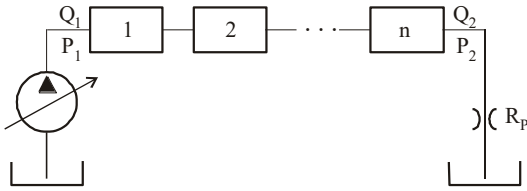


Fig.9. Schematic of a circuit for LTL simulation

### 3. SIMULATION ANALYSIS

On the  $\pi$  model with interface shown in Fig. 4, behavior of LTL in circuit shown in Fig. 9 is simulated. Aim is to analyze the impact of several parameters on the transfer process.

The parameters at which the simulation was performed:  $Be = 1.4 \cdot 10^9 Pa$ ,  
 $Q_{ref} = 3.5 \cdot 10^{-4} m^3 / s$ ,  $d = 1 \cdot 10^{-2} m$ ,  
 $\rho = 860 kg / m^3$ ,  $\nu = 9.7 \cdot 10^{-5} m^2 / s$

Before the beginning of simulation, we need to define initial conditions for each segment's flow and pressure. It is assumed that streaming is in the steady-state regime before the beginning of the transition process.

Results on figures 10, 11 and 12 show the impact of the number of segments in the LTL model. For pipe length  $L=16 m$  relevant  $Q_2$  flow changes and  $P_1$  and  $P_2$  pressures are shown to a unit step input of flow rate  $Q_1$ .

Three cases were observed: a)  $n=1$ ,  $L_s=16 m$ ; b)  $n=4$ ,  $L_s=4 m$  and c)  $n=16$ ,  $L_s=1m$ . Figure 10 shows relevant change of  $Q_2$  flow to step change of  $Q_1$  flow. Figure 11 shows relative pressure change for the same change of  $Q_1$  flow.

With increase in the number of segments, non-linearity in response comes to expression, overshoot is bigger, frequency of higher frequencies appears and delay in response becomes greater.

The following three figures (Fig. 13, 14,15) show impact of the pipeline length on transfer processes in LTL. In all three cases  $\pi$  segments of same length  $L_s=2m$  are used. Difference is only in the number of segments.

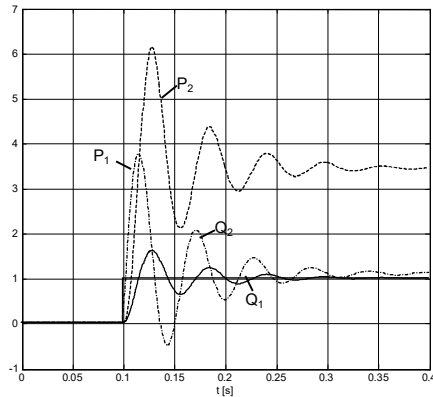


Fig.10. Transient processes for  $n=1$  and  $L_s=16 m$

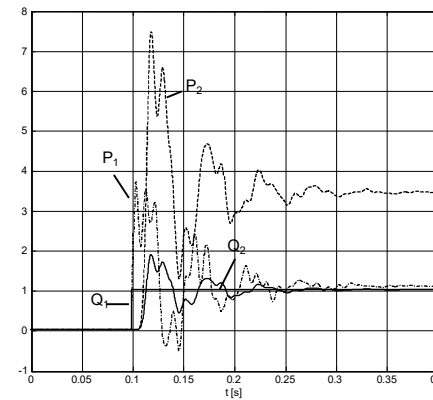


Fig.11. Transient processes for  $n=4$  and  $L_s=4 m$

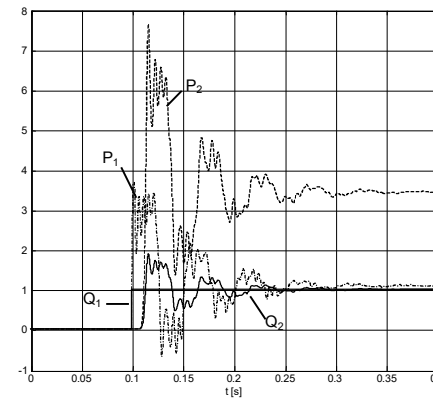


Fig.12. Transient processes for  $n=8$  and  $L_s=2 m$

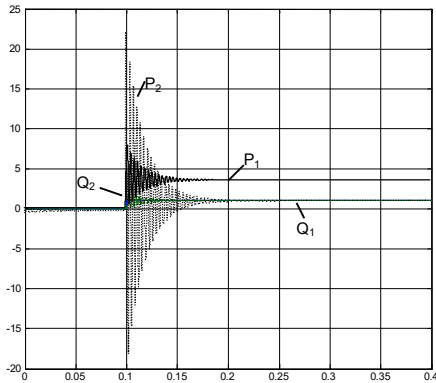


Fig.13. Transient processes for  $n=1$  and  $L_s=2$  m

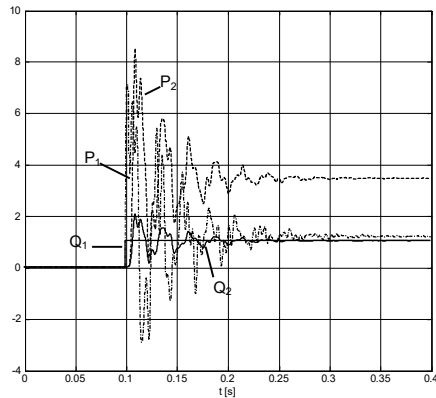


Fig.14. Transient processes for  $n=4$  and  $L_s=2$  m

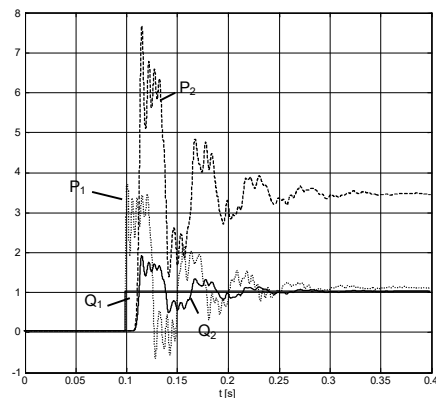


Fig.15. Transient processes for  $n=8$  and  $L_s=2$  m

With the increase of pipeline length response velocity reduces, frequency of damped

oscillations reduces and response delay increases. Particularly noted is reduction of intensification on pressure  $P_2$ .

The last parameter which influence on the transfer process we are analyzing is effective bulk modulus. In previous examples we assumed that the elasticity of hydraulic pipe wall can be neglected. We also assumed that there is no entrapped vapor or gas in the fluid. Effective bulk modulus can be approximately determined using the expression [9]:

$$\frac{1}{B_e} = \frac{1}{B_c} + \frac{1}{B_l} + \frac{V_g}{V_t} \frac{1}{B_g} \quad (2)$$

where:

$B_c$  - bulk modulus of hydraulic pipe,  $B_l$  - bulk modulus of liquid,  $V_g/V_t$  - portion of vapor or gas in total volume,  $B_g$  - bulk modulus of entrapped gas.

If we assume that  $B_c = 3.45 \cdot 10^{10} Pa$ ,  $B_l = 1.4 \cdot 10^9 Pa$ ,  $B_g = 2 \cdot 10^5 Pa$  then transient processes for different gas percentage in the fluid look like in figures 16, 17 and 18 ( $n=8, L_s=2m$ ).

Simulation results show that presence of air in LTL can have great influence on dynamics. Even with small volume share of air, character of the transfer process changes completely. Instead of oscillations, we have an aperiodic transfer process and significantly slower system reaction.

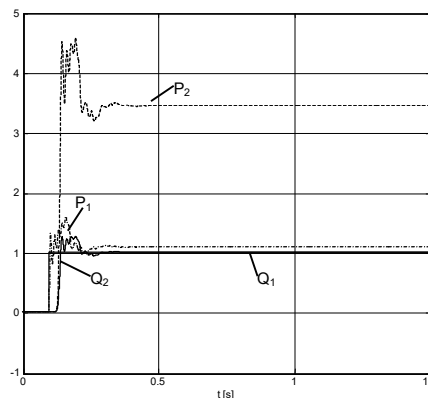
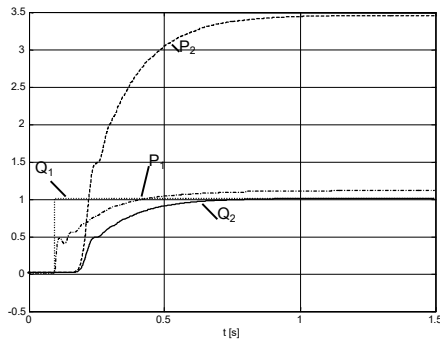
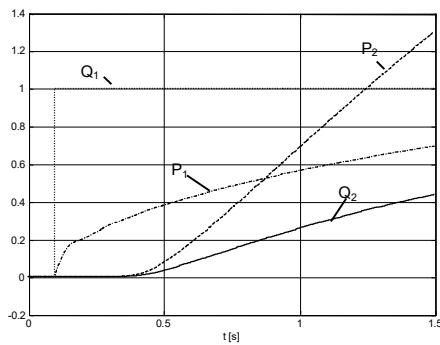


Fig.16. Transient processes for  $V_g/V_t=0.001$

Fig.17. Transient processes for  $Vg/Vt=0.01$ Fig.18. Transient processes for  $Vg/Vt=0.1$ 

#### 4. CONCLUSIONS

Model of long transmission line is derived by using the method of pipeline division on final length segments. Inside each of the segments, spatial change is abstracted and value time change is observed. Friction, inertia and compressibility are included in the model with use of bond graph elements. Simulation shows that accuracy of results depends on the number of segments the pipeline is approximated. That number depends on the hydraulic circuit where the pipeline is located. Practically, the number of segments should be chosen so that length of one segment is no less than 2 m. This is how the mathematical model of the final order is derived with, which can further be reduced to a lower order and there with correctly describe the transient process. Besides the pipeline length, effective compressibility of fluid also greatly affects the dynamics. Presence of entrapped vapor

or gas slows the transfer process and reduces oscillations in the system.

#### 5. REFERENCES

- [1] Matko, D., Geiger, G., Gregoritz, W., Pipeline simulation techniques, *Mathematics and Computers in simulation*, 52 (2000) 211-230.
- [2] Stecki, J.S. and Davis, D.C.: Fluid transmission lines – distributed parameter models. Part 1: A review of the state of the art, *Proceedings of the Institution of Mechanical Engineers. Part A: Journal of Power and Energy*, Vol. 200, No. 4, pp. 215-228, 1986.
- [3] Stecki, J.S. and Davis, D.C.: Fluid transmission lines – distributed parameter models. Part 2: Comparison of models, *Proceedings of the Institution of Mechanical Engineers. Part A: Journal of Power and Energy*, Vol. 200, No. 4, pp. 229-236, 1986.
- [4] Hullender, D.A. and Healey, A.J.: Rational polynomial approximations for fluid transmission line models, in: *Proceedings of the ASME Winter Annual Meeting – Fluid Transmission Line Dynamics*, 15-20.11.1981, Washington, USA, pp. 33-55.
- [5] Hullender, D.A., Woods, R.L. and Hsu, C.-H.: Time domain simulation of fluid transmission lines using minimum order state variable models, in: *Proceedings of the ASME Winter Annual Meeting – Fluid Transmission Line Dynamics*, 13-18.11.1983, Boston, USA, pp. 78-97.
- [6] Söderman U. *Conceptual Modelling of Physical Systems - a frame of reference* Ph.D. Thesis. Linköping, Sweden: Linköping University; 1995.
- [7] Watton, J., *Fluid power systems: modeling, simulation, analog and microcomputer control*, Prentice Hall International, 1989.
- [8] Nedić, N., Filipović, V., Dubonjić, Lj., *Design of Controllers With Fixed Order for Hydraulic Control System With a Long Transmission Line*, *FME Transactions* (2010) 38, 79-86.
- [9] Herbert Merritt, *Hydraulic Control Systems*, John Wiley & Sons, 1967.