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The Method for Extracting Region of Absolute Stability-Loop - Controlled Time Delay Systems

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This paper presents D-decomposition method in the area of absolute stability, developed from Neimark [10] in order to extract region of absolute stability in two-parameters plane for special class of time – delay system.. It means that the open loop transfer function has specific expression with linearly connected open loop gain K=1/α,where α is one variable parameter and the second one is time delay constanτ which is connect with is in non-linear function in open loop transfer function.[4].Now we develop the method for synthesis and analysis of controlled-loop system with proportional regulator for circulating reservoir for mixing liquids and presents and investigates the further expansion of last obtained results .With this method it is possible to separate the region in three-dimensional space(frequency ω (Hz), gain K=1/α and time delay constant (τ), sothat adjustable parameters guarantee absolute stability of synthetized automatic control system.

Keywords: loop – controlled time delay system, absolute stability, parametric plane, circulating reservoir for mixing liquids

0. INTRODUCTION

This method which Russian scientist Neimark [9],[10] was first started to develop is the method for testing stability of time-delay systems so-called D-composition method. All works about this method was given in [1]. The method for separation the region in the parameter plane, which enables closed- loop system will have pre-settling timewas also particularly developed and explained in [4] and this paper will continue extend last mentioned results and their application.

1.1Definition the class of loop-controlled time delaysystem-mathematical model of circulating reservoir for mixing liquids

This part has already been presented in[1], [2].The class of closed-loop system with a single delay, when the adjustable parameters are non-linearly related to polynomial coefficients ofquasicharacteristic equation [10] is defined by following open loop transfer function.

$$W_{ok} = \frac{N(s)}{\alpha D(s)} e^{-\tau s} \qquad (1)$$

so that quasicharacteristic equation has the following form:

$$f(s, e^{-\tau s}) = \propto D(s) e^{\tau s} + N(s) = 0 \qquad (2)$$

where K = 1/α is proportional regulator gain, so α is a regulator parameter linearly related to polynomial coefficients of quasicharacteristic polynomial. Pure time delay is τ, which in the case of circulating reservoir for mixing liquids [4] introduced into the control parts of the object, through valves and pipes to reservoir, as described in the definition of a mathematical model of this system [4].

1.2 Separation the region of absolute stability

The system will possess an absolute stabil.ity only if all the roots of quasicharacteristic equation are in left part of complex plane.
The method is transforming this part of complex plane to parametric plane τ-α:

1.2.1 Decomposition curve

For ω- Im(s) complex variable s has the form:

$$s = j\omega, \qquad (4)$$

Then N(s) and D(s) become:

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$$N(j\omega) = N_1(\omega) + jN_2(\omega) \quad (5)$$

$$D(j\omega) = D_1(\omega) + jD_2(\omega)$$

when (5) puts in (2) it gives:

$$N_1(\omega) + \alpha[D_1(\omega)\cos\omega t - D_2(\omega)\sin\omega t] = 0 \quad (6)$$

$$N_2(\omega) + \alpha[D_1(\omega)\sin\omega t + D_2(\omega)\cos\omega t] = 0 \quad (7)$$

When α from (6) puts in (7) it gives:

$$tg(\omega t) = \frac{N_2(\omega)D_1(\omega) - N_1(\omega)D_2(\omega)}{N_1(\omega)D_1(\omega) - N_2(\omega)D_2(\omega)} \quad (8)$$

Conditions for (8) are:

$$N_1(\omega) \neq 0, D_1^2(\omega) + D_2^2(\omega) \neq 0, \quad (9)$$

From (6) we got

$$\alpha = \frac{-N_1(\omega)}{\cos(\omega t)\left[D_1(\omega) - D_2(\omega)tg(\omega t)\right]} \quad (10)$$

So we know that:

$$\cos(\omega t) = \frac{1}{\pm\sqrt{1 + tg(\omega t)}} \quad (11)$$

and then substituting (8) in (11) next decomposition curves follows:

$$\alpha = \pm\sqrt{\frac{N_1^2(\omega) + N_2^2(\omega)}{D_1^2(\omega) + D_2^2(\omega)}} \quad (14)$$

$$\tau = \frac{1}{\omega}\left[arctg\frac{N_2(\omega)D_1(\omega) - N_1(\omega)D_2(\omega)}{N_1(\omega)D_1(\omega) + N_2(\omega)D_2(\omega)} + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right] \quad (15)$$

for $k \in Z, \omega \in [-\infty, +\infty)$

Note:The upper sign of (14) correspondents to the upper sign of (15) and the lower sign of (14) correspondents to the lower sign of (15).

1.2.2 Curve shading

Shading of decomposed curves is determined by the sign of Jacobians (as in systems without delays).

$$F(j\omega) = R_F(\omega) + jI_F(\omega) \quad (15)$$

$$R_F = N_1(\omega) + \alpha[D_1(\omega)\cos(\omega\tau) - D_2(\omega)\sin(\omega\tau)]$$

$$I_F = N_2(\omega) + \alpha[D_1(\omega)\sin(\omega\tau) + D_2(\omega)\cos(\omega\tau)]$$

Jacobians of the system as follows:

$$J = \begin{vmatrix} \frac{\partial R_F}{\partial \tau} & \frac{\partial R_F}{\partial \alpha} \\ \frac{\partial I_F}{\partial \tau} & \frac{\partial I_F}{\partial \alpha} \end{vmatrix} = -\alpha \cdot \omega \cdot [D_1^2(\omega) + D_2^2(\omega)] \quad (16)$$

1.2.3. Singular lines

Singular lines, in the case of extracting area of pre-settling time is defined for boundary cases ω→-∞ and ω→+∞, in (2), (14) and (15):

$$\lim_{s\to\pm\infty}\frac{N(s)}{D(s)} = -\lim_{\omega\pm\infty}\alpha\cdot\left[\cos(\tau\omega) + j\cdot\sin(\tau\cdot\omega)\right] \quad (17)$$

Because of nature of expresion (17) it is necessary to be sin(τω)=0 i.e. τ=k∏/ω so (17) becomes:

$$\lim_{s\to\pm\infty}\frac{N(s)}{D(s)} = -\lim_{\omega\pm\infty}\alpha\cdot(-1)^k$$

$$\alpha = (-1)^{k+1}\lim_{s\to\pm\infty}\frac{N(s)}{D(s)} \quad (18)$$

τ=0 and τ ⇨∞ (19)

Expresion (18) and (19) are singular lines

The decomposition curves shows that we are getting for every single values k hatched area area between those curves and singular lines (18) and (19). So lets mark hatched area for some vakue marked k with s_k [4] it is clear that the hatched area adequate to left part of complex plane is defined with following expression:

$$s = \bigcup_k \{s_k\} \quad (20)$$

Now we can choose in parametric three-dimensional space parameter τ and α from that hatched area s (better view in τ - α projecting plane)

2.1 **Aplication the method – circulating reservoir for mixing liquids**

Application of the methods described here will be illustrated with the example of circulating reservoir for mixing two liquids. A mathematical model is developed for control systems with proportional controller which gain is K = 1 / α and given object [4] for some nominal parameter values, with time delay identical for both liquid flows as τ. The open loop transfer of feedback system is:

$$W_{ok} = \frac{(2{,}31\cdot 10^{-4}s + 1{,}34\cdot 10^{-7})\cdot e^{-\tau s}}{\alpha\cdot(s^{2}+17{,}3\cdot 10^{-4}\cdot s + 61{,}5\cdot 10^{-8})} \qquad (21)$$

2.1.1Synthesis of controlled-loop system

According to the methods described in chapter 1 equations (14), (15) and (16) become:

$$\alpha = \pm\sqrt{\frac{5{,}34\cdot 10^{-8}\omega^{2} + 1{,}8\cdot 10^{-14}}{3{,}78\cdot 10^{-13} + 1{,}76\cdot 10^{-6}\cdot\omega^{2} + \omega^{4}}}$$

$$\tau = \frac{1}{\omega}\left[\begin{array}{l} -arctg\dfrac{9{,}02\cdot 10^{-11}\omega + 2{,}31\cdot 10^{-4}\cdot\omega^{3}}{3{,}78\cdot 10^{-13} + 2{,}65\cdot 10^{-7}\cdot\omega^{2}} + \\ +2k\pi + \dfrac{\pi}{2} \pm \dfrac{\pi}{2} \end{array}\right] \quad (21)$$

$$J = -\alpha\cdot\omega\cdot\left[\omega^{4} + 1{,}76\cdot 10^{-6}\omega^{2} + 3{,}78\cdot 10^{-13}\right]$$

Singular lines are: α=0 τ=0 and and τ ⇨+∞

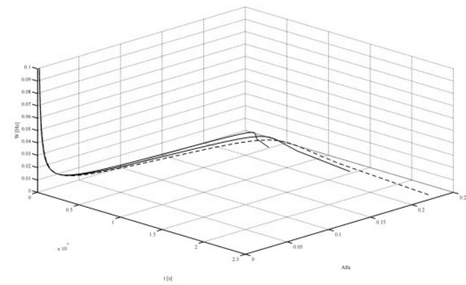

Figure 2. Separation the area of absolute stability

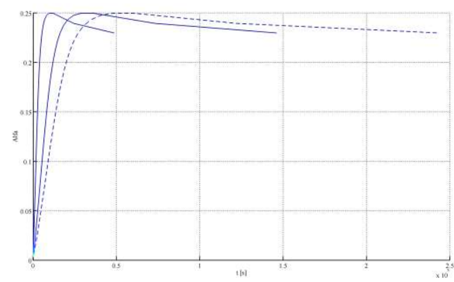

Figure 3.Area of absolute stability in parametric plane

Line - - - - - - for k=0,-1

Line ______ for k=1,-2

Line ______ for k=2,-3

From the union of this areas we can highlight the point α=0,3 and τ=10 000s (22)

Dashed line represents the boundary of region of absolute stability.

B.Dynamic analysis of synthesized system

With chosen parameters from absolute stability region and on the basis of (21) we receive the open loop transfer function of the system. Simulation of the system behavior is done with MATLAB software with step function.Simulation result of step response is shown on Fig 4.

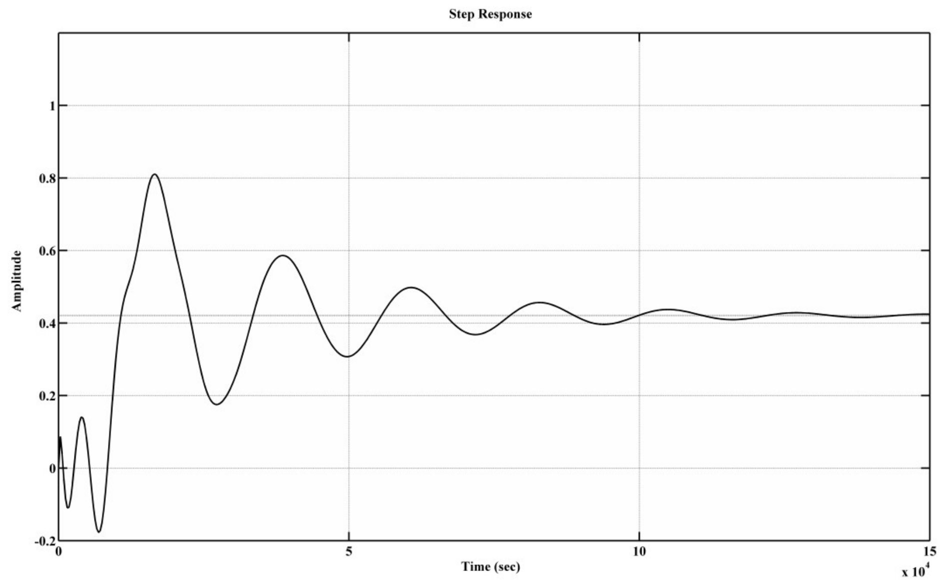

Figure 4. Step response

IV.CONCLUSION

This part could be the same like ones in [1].New software package MATLAB enables to obtain more precocious method for separation the field of absolute stability in the parametric plane of α-τ [4]. The results presenting here overview variation and dependences of parameters in three-dimensional form(α, τ, ω_n) where could be possible to choose from the given space curve the values for parameters which guarantee much better accuracy in methodology than the last obtained results [4].

REFERENCES:

[1]V.S.Brašić, Lj.Dubomjić,N.Nedić „Parametric Methods in Analysis and Synthesis of Controlled Time Delay System– Circulating Reservoir for Mixing Liquids“, SAUM 2010

[2]D.Lj.Debeljković, V.S. Brašić, S.A.Milinković and M.B.Jovanovć ”On relative stability of linear stationary feedback control systems with time delay”, *IMA Journal of Mathematical Control&Information, Oxford University Press,* pp.13-

[3]Milojković B,Grujić Lj. „Automatsko upravljanje“, Mašinski fakult. Beograd, 1977.17, 1996.

[4] V. S. Brašić, ”Analysis and synthesis of feedback control systems with transport lag”, Report, Department of Control Engineering, Faculty of Mech. Eng., Belgrade, June, 1994.

[5] Y. Chu, *Trans.Am.Inst. Elec. Eng*.**71** (2)

[6] L. Eisenberg, “Stability of linear systems with transport lag”*IEEE Trans*AC **11,**247, 1966.

[7]S. Loo, “Stability of linear stationary systems with time delay”*Int.J.Control***9**,103,1969.

[8] O. Mikić, “Some extensions of Mitrovicev’s method in analysis and synthesis of feedback control systems with time delay”, *Proc*.USAUM, Belgrade.Pp.52-67,1982.

[9] Yu. Neimark, “O Opredelenii Značenii Parametrov, prikatorih SAR ustoičiva. Avt.Telem, **3**,190,1949.

[10] Yu. Neimark,“D-razbienie prostransvov kvazipolinomov” *Prikl. Mat.i Meh.13*, 349, 1949.

[11] D. Šiljak, “Generalization of Mitrovicev’s method”,*IEEE Trans.Ind. Appl*.314, September 1964.

[12] D. Šiljak, “Analysis and synthesis of feedback control systems in the parametric plane –Parts I, II, III. *IEEE Trans.Ind.Appl.449, November1964*

[13] D. Šiljak, “Generalization of the parametric plane method”, *IEEE Trans*AC**11**,