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Separation of constant settling time area with D-composition method for controlled time delay systems

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D-decomposition method in the area of relative stability, developed from Loo [7] in order to separate constant time settling area in parametric space which ensures the system having predefined settling time.[4].This paper develops the methods for synthesis and analysis of controlled-loop system with proportional regulator for circulating reservoir for mixing liquids and presents and investigates the further expansion of last obtained results .

Now it would be possible to separate the region in three-dimensional space(frequency ω (Hz), gain K=1/α and time delay constant (τ), so that adjustable parameters guarantee settling time Ts of controlled system will have a priori defined value .

Useful of this researches is that checking of obtained results can be made with MATLAB software package actually the simulation of dynamic behaviour will be done with this package at the end.

Keywords: loop – controlled time delay system, relative stability, parametric plane, settling time

0. INTRODUCTION

The method for separation the region in the parameter plane, which enables closed- loop system will have pre-settling time was first developed from Loo also particularly developed and explained [4] and this paper will continue extend last mentioned results and their application.

1.1 The class of loop-controlled time delay system -

This method is reserved only for special class of time - delay systems.
The class of closed-loop system with a single delay, when the adjustable parameters α and τ are non-linearly related to polynomial coefficients of quasicharacteristic equation [1],[2],[10] is defined by following open loop transfer function.

$$W_{ok} = \frac{N(s)}{\alpha D(s)} e^{-\tau s} \qquad (1)$$

so that quasicharacteristic equation has the following form:

$$f(s, e^{-\tau s}) = \propto D(s) e^{\tau s} + N(s) = 0 \qquad (2)$$

1.2 Separation the region of pre-settling time

The system will possess an appropriate settling time only if all the roots of quasicharacteristic equation are within this contour C shown on Fig.1

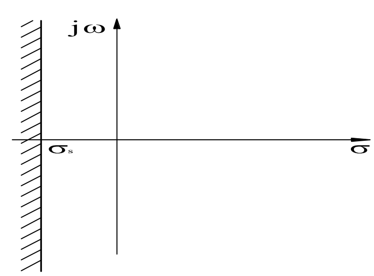

Fig.1.The method is transforming this contour C from complex plane to parametric plane τ-α
It was shown in [3] that setting time is defined by the expression

$$T_s = \frac{1}{\sigma_M} \ln \frac{\delta}{\Delta}, \Delta > 0, \delta < \Delta \qquad (3)$$

where amplitude of step response is minor from δ for time t= Ts of transient.

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1.2.1 Decomposition curve

For ω - Im(s) and σ - Re(s), complex variable s has the form:

$$s = \sigma + j\omega, \tag{4}$$

Their deegres are given with:

$$s^k = x^k + jy^k, k = 0,1,2... \tag{5}$$

where is:

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} \sigma & -\omega \\ \omega & \sigma \end{bmatrix} \cdot \begin{bmatrix} x_k \\ y_k \end{bmatrix}, k = 0,1,2... \tag{6}$$

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{7}$$

From (6) and (7) it is clear that $x_k$and y_k are the real function of σ and ω. So if (5) put into polynomials D(s) and N(s) we got:

$$D(\sigma,\omega)=R_D(\sigma,\omega)+jI_D((\sigma,\omega) \tag{8}$$

$$N(\sigma,\omega)=R_N(\sigma,\omega)+jI_N((\sigma,\omega) \tag{9}$$

So equation (2) gives a form

$$f(\sigma,\omega)=R_F(\sigma,\omega) + jI_F(\sigma,\omega) \tag{10}$$

If (8) and (9) put in (2) we got following equation:

$$R_F(\sigma,\omega)=\alpha\cdot e^{\tau\sigma}\left[R_D(\sigma,\omega)\cos(\tau\cdot\omega)-I_D(\sigma,\omega)\cdot\sin(\omega\cdot\tau)\right]+R_N(\sigma,\omega) \tag{11}$$

$$I_F(\sigma,\omega)=\alpha\cdot e^{\tau\sigma}\left[R_D(\sigma,\omega)\sin(\tau\cdot\omega)+I_D(\sigma,\omega)\cdot\cos(\omega\cdot\tau)\right]+I_N(\sigma,\omega)$$

In polar coordinates (8) and (9) gives a form:

$$D(\sigma,\omega)=r_D(\sigma,\omega)\cdot e^{j\Phi_D(\sigma,\omega)} \tag{12}$$

$$N(\sigma,\omega)=r_N(\sigma,\omega)\cdot e^{j\Phi_N(\sigma,\omega)} \tag{13}$$

and then substituting (12) and (13) in (2) next decomposition curves follows:

$$\alpha=\pm\frac{r_N(\sigma,\omega)}{r_D(\sigma,\omega)}e^{-\tau\sigma} \tag{14}$$

$$\tau=\frac{1}{\omega}\left[\Phi_N(\sigma,\omega)-\Phi_D(\sigma,\omega)+2k\pi+\frac{\pi}{2}\pm\frac{\pi}{2}\right] \tag{15}$$

for $k \in Z, \omega \in [-\infty,+\infty)$

Note:The upper sign of (14) correspondents to the upper sign of (15) and the lower sign of (14) correspondents to the lower sign of (15).

1.2.2 Curve shading

Shading of decomposed curves is determined by the sign of Jacobians (as in systems without delays).
Jacobians of the system as follows:

$$J=\begin{vmatrix} \frac{\partial R_F}{\partial\tau} & \frac{\partial R_F}{\partial\alpha} \\ \frac{\partial I_F}{\partial\tau} & \frac{\partial I_F}{\partial\alpha} \end{vmatrix}=-\alpha\cdot\omega\cdot e^{2\sigma\tau}\cdot r_D^2(\sigma,\omega) \tag{16}$$

1.2.3. Singular lines

Singular lines, in the case of extracting area of pre-settling time is defined for boundary cases ω→-∞ and ω→+∞, in (2), (14) and (15):

$$\lim_{s\to\pm\infty}\frac{N(s)}{D(s)}=-\lim_{\omega\pm\infty}\alpha\cdot e^{\tau\sigma_M}\left[\cos(\tau\omega)+i\cdot\sin(\tau\cdot\omega)\right] \tag{17}$$

Because of nature of expresion (17) it is necessary to be sin(τω)=0 i.e. τ=k∏/ω so (17) becomes:

$$\lim_{s\to\pm\infty}\frac{N(s)}{D(s)}=-\lim_{\omega\pm\infty}\alpha\cdot e^{\frac{k^*\pi}{\omega}\sigma_M}(-1)^{k^*}$$

$$\alpha=(-1)^{k^*+1}\lim_{s\to\pm\infty}\frac{N(s)}{D(s)} \tag{18}$$

τ=0 (19)

Expresion (18) and (19) are singular lines

The procedure of selection periodicity factor k = k * is defined by procedure of selection the area of absolute stability for required automatic control system.[4]. Thus the first of all it is necessary to define area of absolute stability[4],[6],[9] in order to obtain corresponding period k=k*and with that value of k* we are getting (14) and (15) which extract the area of pre-settling time in parametric three-dimensional space.

2.1 Mathematical model of circulating reservoir for mixing liquids

Application of the methods described here will be illustrated with the example of circulating reservoir for mixing two liquids. A mathematical model is developed for control systems with proportional controller gain $K = 1 / \alpha$ and given object (Fig. 2) for some nominal parameter values, with time delay identical for both liquid flows as τ. The open loop transfer of feedback system is:

$$W_{ok} = \frac{(2{,}31\cdot 10^{-4}s+1{,}34\cdot 10^{-7})\cdot e^{-\tau}}{\alpha\cdot(s^{2}+17{,}3\cdot 10^{-4}\cdot s+61{,}5\cdot 10^{-8})} \qquad (20)$$

2.1.1Synthesis of controlled-loop system

According to the methods described in chapter 1,for $\sigma=\sigma_M=$ 1sec equations (14), (15) and (16) becomes:

$$\alpha = \pm\sqrt{\frac{5{,}34\cdot 10^{-8}\omega^2+5{,}33\cdot 10^{-8}}{\omega^4+2\cdot\omega^2+1}}e^{\tau}$$

$$\tau = \frac{1}{\omega}\begin{bmatrix} arctg\,\dfrac{5{,}33\cdot 10^{-8}+5{,}34\cdot 10^{-8}\cdot\omega^2}{\omega^4+2\cdot\omega^2+1}+ \\ +2k^{*}\pi+\dfrac{\pi}{2}\pm\dfrac{\pi}{2}\end{bmatrix} \qquad (21)$$

$$J = -\alpha\cdot\omega\cdot e^{-2\tau}\cdot\left[\omega^4+2\omega^2+1\right] \qquad (22)$$

Singular lines are: α=0 and τ=0 and $\tau\Rightarrow\infty$
k^*represents k for absolute stability in previous paper k*=0,-1

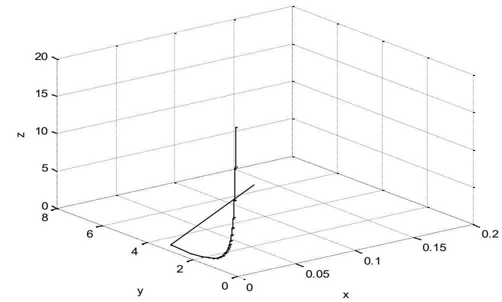

Figure 2. Area for σ_M=1s for α>0

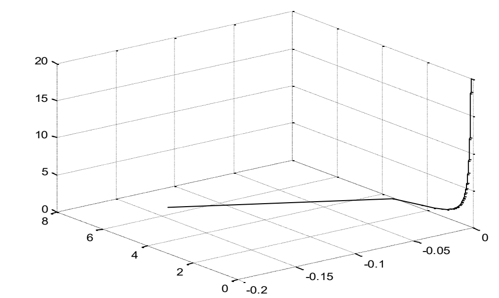

Figure 3.Area for settling time σ_M=1s for α<0

It could be possible to define region for 3D space consisting of parametric plane α-τ (x-y axis) and the axis ω=z for σ_M=1s from (3) and [3] obtains Ts = 58s, which for σ =0 represents the boundary of region of absolute stability. In that case results obtained with this method are the same like the well- known ones. [9], [10].

B.Dynamic analysis of synthesized system

We highlight the point which determines the controller parameters α = 1/180 and τ = 3s from the region of σ_M=1s which is defined with extracting lines (21) and singular lines α=0 and τ=0, and on the basis of (20) receives the open loop transfer function of the system. Simulation of the system behavior is done with MATLAB software with step function. Simulation result of step response is shown on Fig 4.

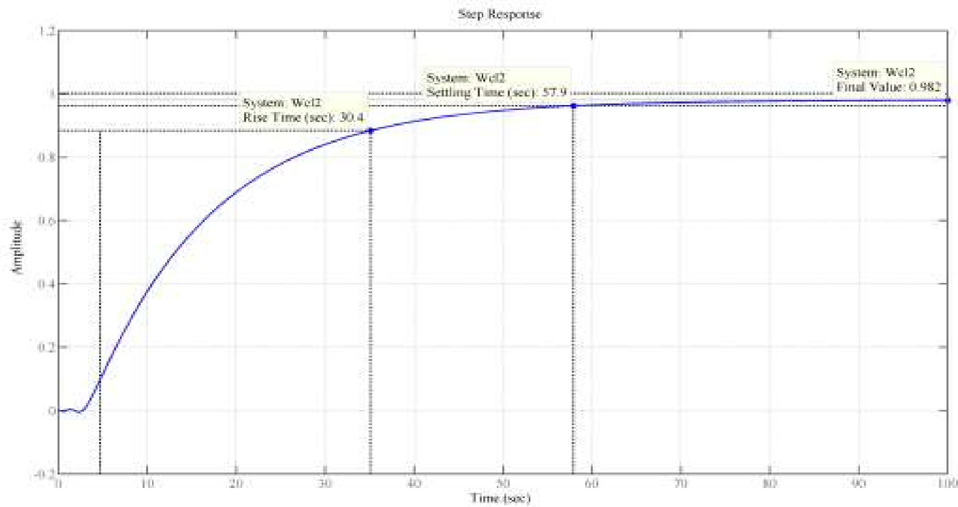

Figure 4. Step response

IV.CONCLUSION

The results obtaining with MATLAB have proven the value of pre-settling time Ts=57,9s of synthetiezed system. This software package enables to us more precisious results and proven of this theory than obtained in [4].

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