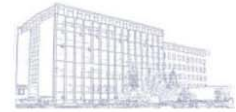




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Analysis and Optimization Design of Welded I-girder of the Single-beam Bridge Crane

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Abstract— The paper presents the analysis and optimization of the geometric parameters of I-profile of the main girder of the single-beam bridge crane. The reduction of the cross-sectional area of the welded I-girder is set as the objective function. The criteria of permissible stresses in the characteristic points of I-section, stability of the upper plate of I-profile, permissible static deflection of the girder, minimum plate thicknesses and some geometric limits, were applied as the constraint functions for this optimization problem. The obtained results of optimization were verified on examples of implemented solutions of the single-beam bridge cranes, showing how much the mass can be saved in application of the welded I-profiles in comparison to standard rolled I-profiles, which are most commonly applied.

Keywords— I-section, optimum design, single-beam bridge crane, welded girder

I. INTRODUCTION

Single-beam bridge cranes are widely used in industrial plants, primarily for small, medium and large loads and spans, and as main girder, in addition to different types of standard I-profiles, can be used I-shaped welded girder. I-girders are widely used in industry due to their small size, their weight and the ensuing price.

Optimization is a procedure through which the best possible values of decision variables are obtained under the given set of constraint functions and in accordance to objective function. The most common optimization procedure applies to a design that will minimize the total cost or the total mass or any other specific objective.

There are a large number of papers and publications who dealing with the problems of optimization and analysis of stresses and deformations of the main girder of the single-beam bridge cranes and monorail structures with I-profile.

Most papers treat the problems of optimization and analysis using by FEM, in different software packages ([1]-[3], [5], [7], [9], [11]). In the paper [2], existing solution of single-beam bridge crane was analyzed, comparing the results of stresses and strains obtained on analytically way and numerically (using the modern FEM software package). In [11], the mathematical model and numerical simulations of the mechanical phenomena in I-

profile of the single-beam gantry crane, using by FEM analysis was performed.

In the paper [3], comparative analysis of local bending stresses calculated using by different analytically methods was performed, and these results were compared with the results obtained by FEM analysis. The analysis of local bending stresses was carried out in [7], where analytical results according to Eurocodes were compared to the results obtained by FEM analysis in ABAQUS software package. Similar analysis was performed in [5], whereby the analytical results and the results from FEM analysis were compared with the experimental ones. Local bending stresses are very import in analysis for these types of structures. The paper [10] shown loading capacities curves for I-profiles of the single-beam bridge crane subjected to local bending stresses. Similarly to previous, the paper [6], shown loading capacities curves for I-profiles subjected to stress, deflection and lateral buckling criteria. The criterion of lateral buckling was presented in [12] and [14], too. In addition to ABAQUS software package, ANSYS software package very much represented in the optimization processes. The mentioned software package was used in the process of optimization in [9].

In addition to optimization and analysis using by FEM, most numerical methods are present for different types of optimization problems ([1], [4], [13], [14]). In the paper [1], optimization procedure for I-profile was performed, subjected to stress, deflection, fatigue, yield and buckling criteria, using by GA in MathCad software package.

GRG algorithm has a wide application in processes of optimization. The usage of GRG algorithm can be seen in [4] and [13] where mass of I-girder was reduced by 7,44% and 8,74%, respectively, relative to start value. This method was performed in the paper [14], too on examples for the single-beam bridge cranes with of the rectangular box section.

In addition to weight optimization, cost optimization is important, too, and this procedure was carried out in the paper [8], for high strength steel in I-welded beams, using by the particle swarm optimization (PSO) method.

Having in mind these results, the aim of this paper is to define the optimum values of geometric parameters of I-profile of the single-beam bridge crane.

II. MATHEMATICAL FORMULATION OF THE OPTIMIZATION PROBLEM

The optimization problem is defined in following way:

$$\min f(X) \quad (1)$$

subject to:

$$g_i(X) \leq 0, \quad i = 1, \dots, m \quad (2)$$

and

$$l_j \leq X_j \leq u_j, \quad u_j > l_j, \quad j = 1, \dots, n \quad (3)$$

where:

$f(X)$ - the objective function (target function)

X - the design vector vector made of 6 design variables

$g_i(X)$ - the constraint function

l_j, u_j - lower, i.e. upper boundary

i - number of constraint functions

j - number of design variables

Design variables are the values that should be defined during the optimization procedure. Each design variable is defined by its upper and lower boundaries.

The objective and constraint functions are presented in the next chapters.

A. Objective (target) function

The objective function is represented by the area of the cross-section of I-girder (Fig. 1).

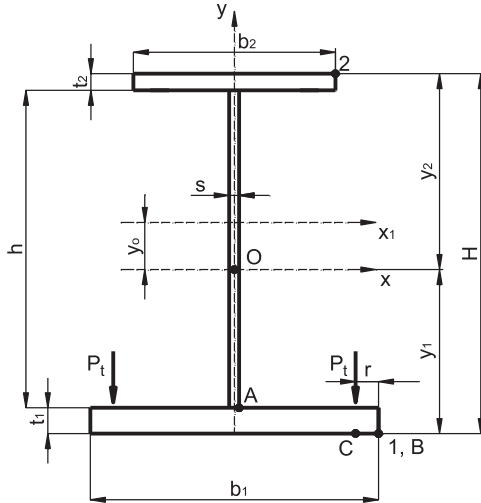


Fig. 1 I-profile

The vector of the given parameters is:

$$\vec{x} = (Q, L, m_t, b_t, e_1, n_t, R_e, k_a, K \dots) \quad (4)$$

where:

Q - the carrying capacity of the crane

L - the span of the crane

m_t - the mass of the trolley

b_t - the distance between the wheels of the trolley

e_1 - the distance between the wheel 1 and resulting force in the vertical plane

n_t - number of the trolley wheels

R_e - the minimum yield stress of the plate material

k_a - the dynamic coefficient of crane load in the horizontal plane, [15]

K - the coefficient who depends on the purpose of the crane and control condition of the crane, [15]

The paper treats six geometric variables:

$$X = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = [b_1 \ b_2 \ h \ t_1 \ t_2 \ s]^T \quad (5)$$

The area of the cross-section, i.e. the objective function, is:

$$A = b_1 \cdot t_1 + b_2 \cdot t_2 + h \cdot s \quad (6)$$

where:

b_1 - bottom flange width

b_2 - top flange width

h - web height

t_1 - bottom flange thickness

t_2 - top flange thickness

s - web thickness

The values of the corresponding forces and bending moments in the corresponding planes and the value of the transverse force are calculated in the following way (Fig. 2):

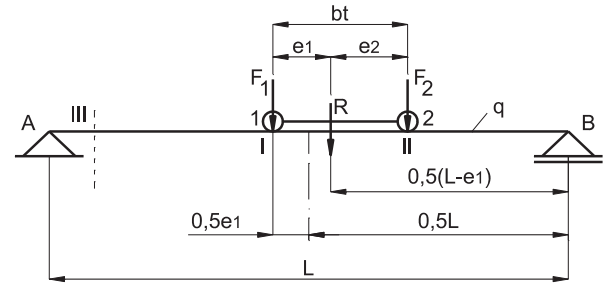


Fig. 2 Static model of the single-beam bridge crane

$$e_2 = \frac{b_t}{2} - e_1 \quad (7)$$

$$R = F_1 + F_2 = \gamma \cdot (\psi \cdot Q + m_t) \cdot g \quad (8)$$

$$F_1 = \frac{e_2}{b_t} \cdot R \quad (9)$$

$$F_2 = \frac{e_1}{b_t} \cdot R \quad (9)$$

$$M_{Vl} = \frac{R}{4 \cdot L} \cdot (L - e_1)^2 + \gamma \cdot \frac{q \cdot L^2}{8} \quad (10)$$

$$q = \rho \cdot g \cdot A \quad (11)$$

$$F_{st} = F_{1,st} + F_{2,st} = (Q + m_t) \cdot g \quad (12)$$

$$F_{1,st} = \frac{e_2}{b_t} \cdot F_{st} \quad (13)$$

$$F_{2,st} = \frac{e_1}{b_t} \cdot F_{st} \quad (14)$$

$$R_h = F_{1,h} + F_{2,h} = k_a \cdot (Q + m_t) \cdot g \quad (15)$$

$$F_{1,h} = k_a \cdot F_{1,st} \quad (16)$$

$$F_{2,h} = k_a \cdot F_{2,st} \quad (17)$$

$$M_{HI} = \gamma \cdot \left[\frac{R_h}{4 \cdot L} \cdot (L - e_1)^2 + k_a \cdot \frac{q \cdot L^2}{8} \right] \quad (18)$$

$$P_t = \frac{2 \cdot F_{\max}}{n_t} \quad (19)$$

$$F_{\max} = \max(F_1, F_2) \quad (20)$$

where:

e_2 - the distance between the wheel 2 and resulting force in the vertical plane

γ - the coefficient which depends on the classification class, [15]

ψ - the dynamic coefficient of the influence of load oscillation in the vertical plane, [15]

F_1, F_2 - forces acting upon girder beneath the trolley wheel 1 and trolley wheel 2, respectively

R - resulting force in the vertical plane

ρ - density of material of the girder

F_{st} - resulting static force in the vertical plane

R_h - resulting force in the horizontal plane

q - specific weight per unit of length of the girder

M_{VI}, M_{HI} - the bending moments in the vertical and horizontal planes, respectively

P_t - the vertical load of the trolley wheel (the maximum pressure of the wheel)

n_t - number of the trolley wheels

The geometrical properties in the specific points of I-profile shall be determined by well-known expressions ($I_x, I_y, W_{x1}, W_{y1}, W_{x2}, W_{y2}, W_{xA}, W_{xB}, W_{yB}, W_{xC}, W_{yC}$).

B. Constraint functions

1) *The criterion of permissible stresses:* Strength check is conducted in specific points of the girder. Actual stress has to be lower than critical stress, which depends on point position.

Maximum normal stress at point 1, B:

$$\sigma_{1,B,u} = \sigma_{zV1} + \sigma_{zH1} = \frac{M_{VI}}{W_{x1}} + \frac{M_{HI}}{W_{y1}} \leq \sigma_{k1} \quad (21)$$

$$\sigma_{k1} = \frac{R_e}{\nu_1} \quad (22)$$

where:

$\sigma_{zV1}, \sigma_{zH1}$ - normal stresses in the vertical and horizontal planes, respectively

σ_{k1} - critical stress for load case 1

$\nu_1 = 1,5$ - the factored load coefficient for load case 1

Maximum normal stress at point 2:

$$\sigma_{2,u} = \sigma_{zV2} + \sigma_{zH2} = \frac{M_{VI}}{W_{x2}} + \frac{M_{HI}}{W_{y2}} \leq \sigma_{k1} \quad (23)$$

Maximum equivalent stress at point A:

$$\sigma_{A,u} = \sqrt{\sigma_{zA}^2 + \sigma_{x,A}^2 - \sigma_{zA} \cdot \sigma_{x,A}} \leq \sigma_{k1} \quad (24)$$

$$\sigma_{zA} = \sigma_{z,A}^P + \sigma_{zVA} \quad (25)$$

$$\sigma_{zVA} = \frac{M_{VI}}{W_{xA}} \quad (26)$$

$$\sigma_{x,A}^P = K_{Ax} \cdot \frac{P_t}{t_1^2} \leq \sigma_{k1} \quad (27)$$

$$\sigma_{z,A}^P = K_{Az} \cdot \frac{P_t}{t_1^2} \leq \sigma_{k1} \quad (28)$$

$$K_{Ax} = 1,85414 - 0,06529 \cdot \xi + 1,20813 \cdot \xi^2 \quad (29)$$

$$\xi = \frac{c}{a} \quad (30)$$

$$a = \frac{b_1 - s}{2} \quad (31)$$

$$c = a - r \quad (32)$$

$$K_{Az} = 0,40677 + 0,47403 \cdot \xi - 0,00991 \cdot \xi^2 \quad (33)$$

where:

σ_{zA} - maximum normal stress at point A

σ_{zVA} - normal stress in the vertical plane

$\sigma_{x,A}^P, \sigma_{z,A}^P$ - the normal stresses due to the local pressure of the trolley wheel at point A, [15]

K_{Ax}, K_{Az} - the corresponding coefficients for calculating stresses at point A, [10]

Maximum equivalent stress at point C:

$$\sigma_{C,u} = \sqrt{\sigma_{zC}^2 + \sigma_{x,C}^2 - \sigma_{zC} \cdot \sigma_{x,C}} \leq \sigma_{k2} \quad (34)$$

$$\sigma_{k2} = \frac{R_e}{\nu_2} \quad (35)$$

$$\sigma_{zC} = \sigma_{z,C}^P + \sigma_{zVC} + \sigma_{zHC} = \sigma_{z,C}^P + \frac{M_{VI}}{W_{xC}} + \frac{M_{HI}}{W_{yC}} \quad (36)$$

$$\sigma_{x,C}^P = K_{Cx} \cdot \frac{P_t}{t_1^2} \leq \sigma_{k1} \quad (37)$$

$$\sigma_{z,C}^P = K_{Cz} \cdot \frac{P_t}{t_1^2} \leq \sigma_{k1} \quad (38)$$

$$K_{Cx} = -0,53296 + 12,90617 \cdot \xi - 35,27425 \cdot \xi^2 + 45,32993 \cdot \xi^3 - 22,25349 \cdot \xi^4 \quad (39)$$

$$K_{Cz} = -2,88333 + 43,90676 \cdot \xi - 179,1317 \cdot \xi^2 + 358,79953 \cdot \xi^3 - 344,40559 \cdot \xi^4 + 128,20513 \cdot \xi^5 \quad (40)$$

where:

σ_{zC} - maximum normal stress at point C

σ_{zVC} , σ_{zHC} - normal stress in the vertical and horizontal planes, respectively

$\sigma_{x,C}^P$, $\sigma_{z,C}^P$ - the normal stresses due to the local pressure of the trolley wheel at point C, [15]

K_{Cx} , K_{Cz} - the corresponding coefficients for calculating stresses at point C, [10]

σ_{k2} - critical stress for load case 2

The effect of shearing is neglected in this analysis

2) *The criterion of local stability of the top flange:* Safety check for local stability of the top flange of the girder is done in compliance with [16], [17] and [18], for I-girders. So, it has to be fulfilled:

$$\sigma_{2,u} \leq \sigma_{p,k} \quad (41)$$

$$\sigma_{p,k} = 1,14 \cdot \sigma_{i,dop} \quad (42)$$

$$\sigma_{i,dop} = \chi_p \cdot \sigma_{k1} \quad (43)$$

$$\chi_p = \frac{2}{\beta_v + \sqrt{\beta_v^2 - 4 \cdot \bar{\lambda}^2}} \text{ for } \bar{\lambda} > 0,2 \quad (44)$$

$$\chi_p = 1 \text{ for } \bar{\lambda} \leq 0,2$$

$$\beta_v = 1 + 0,489 \cdot (\bar{\lambda} - 0,2) + \bar{\lambda}^2 \quad (45)$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_v} \quad (46)$$

$$\lambda_v = \pi \cdot \sqrt{\frac{E}{R_e}} \quad (47)$$

$$\lambda = \frac{l_i}{i_p} \quad (48)$$

$$i_p = \sqrt{\frac{I_p}{A_p}} \quad (49)$$

$$I_p = \frac{b_2^3 \cdot t_2}{12} \quad (50)$$

$$A_p = b_2 \cdot t_2 \quad (51)$$

$$l_i = \beta \cdot L \quad (52)$$

$$\beta = \sqrt{\frac{1}{3,18}} \quad (53)$$

where:

$\sigma_{p,k}$ - critical stress, according to [16]

β_v , χ_p - non-dimensional coefficients, according to [17]

$\bar{\lambda}$ - non-dimensional girder slenderness, according to [17]

β - non-dimensional girder slenderness, according to [18]

3) *The criterion of permissible static deflection:* In order to satisfy this criterion, it is necessary that the static deflection in the vertical plane have the value smaller than the permissible value:

$$f_{\max} = \frac{F_{1,st} \cdot L^3}{48 \cdot B} \cdot [1 + \alpha \cdot (1 - 6 \cdot \beta_t^2)] + \frac{5 \cdot q \cdot L^4}{384 \cdot B} \leq f_{dop} \quad (54)$$

$$f_{dop} = K \cdot L \quad (55)$$

$$\alpha = \frac{F_{2,st}}{F_{1,st}} \quad (56)$$

$$\beta_t = \frac{b_t}{L} \quad (57)$$

$$B = E \cdot I_x \quad (58)$$

where:

f_{\max} - the deflection in the vertical plane

B - flexural rigidity of a beam

f_{dop} - the permissible deflection in the vertical plane

III. NUMERICAL REPRESENTATION OF THE OBTAINED RESULTS

The optimization is done by GRG2 algorithm, using Solver Tool in Analysis module from Ms Excel software package.

Variable parameters are cross-section height and width and plates thicknesses. All constraints shown in previous chapters are taken into analysis.

Minimum thickness of the web plate is adopted to be 5 mm and minimum thickness of the bottom and top flange is adopted to be 6 mm, which are also the constraint functions. Another additional criterion was taken that the maximum values of the width of the bottom and top flange is less than 300 mm, which corresponds to the maximum values for standard I-profiles. Minimum value of the width of the bottom flange depends of trolley dimensions. Number of trolley wheels for all examples is four.

Other input parameters are taken according to basic characteristics for existing solutions of the the single-beam bridge cranes (Table I) and according to [15].

Table II shows the results of the optimization (optimal values and savings) for eight solutions of the single-beam bridge cranes with standard I-profile.

where:

A_p - value of the area of standard I-profile (Table I)

A_{opt} - optimal value of the area of welded I-girder (Table II)

TABLE I BASIC CHARACTERISTICS OF EXISTING SOLUTIONS OF SINGLE-GIRDER BRIDGE CRANES

	Location	Q (t)	L (m)	m_t (kg)	b_t (mm)	e_t (mm)	Cl. class	$b_{1,min}$ (mm)	k_a	Profile	Material	A_p (cm ²)
1	Radijator - Kraljevo	5	16,78	350	405	202,5	II	100	0,1	HEA-700	S275	260
2	Track Expert - Svilajnac	5	14,31	330	405	202,5	II	100	0,05	HEA-550	S235	212
3	Jedinstvo - Grocka	2	4,81	180	116	58	I	55	0,05	IPE-330	S235	62,6
4	Statik - Kovin	6,3	15,915	370	420	225	II	100	0,1	HEB-600	S235	270
5	Gasteh - Indija	5	14,78	340	196	98	II	100	0,05	HEA-550	S235	212
6	Farmakom - Sombor	3,2	10	340	196	98	II	82	0,1	HEA-360	S235	143
7	RAPP Zastava - Kragujevac	10	7,75	610	708	354	II	100	0,05	HEB-500	S235	239
8	Ferro Corpo - Belgrade	6,3	5,92	380	420	225	II	100	0,1	HEB-360	S235	181

TABLE II THE VALUES OF OPTIMUM PARAMETERS AND SAVINGS, USING BY GRG2 METHOD

	h (mm)	s (mm)	b_1 (mm)	t_1 (mm)	b_2 (mm)	t_2 (mm)	A_{opt} (cm ²)	Saving (%)
1	705,9	5	100	15	298,5	29	136,85	47,36
2	700,4	5	100	19	299,2	16	101,90	51,94
3	229,7	5	55	11	173,7	6	27,96	55,34
4	723,9	5	100	20	298,9	33	154,83	42,65
5	716,7	5	100	19	300	17	105,83	50,08
6	505,4	5	82	15	300	9	64,57	54,85
7	704,3	5	100	27	299,5	10	92,16	61,44
8	460,2	5	100	22	297,5	8	68,83	61,98

IV. CONCLUSIONS

The paper presented optimum dimensions of I-section of the mean girder of the single-beam bridge crane, according to national standards, using by GRG2 optimization method. The criteria of permissible stresses in the characteristic points of I-section, stability of the upper plate of I-profile, permissible static deflection of the girder, minimum plate thicknesses and other geometric limits were applied as the constraint functions. The objective function was minimum cross-sectional area, whereby given constraint conditions were satisfied.

Based on the optimization theory and the procedure for calculation of box girder of the single-beam bridge crane, this paper combines the optimum design philosophy and the design of the box girder. The cross-sectional area is optimized making full use of the optimization functions in Ms Excel software package. The result shows that Solver Tool is reliable, convenient and rapid.

The optimization task – minimization of the cross-sectional area was successfully realized, which is seen in the comparison of the results obtained with solutions made in practice.

Justification of application of this method resulted in significant savings in material, within the range of 42,65 ÷ 61,44 %.

It can be noted that in all cases the minimum width for installation is obtained for the width of the bottom flange.

Taking into account the possibilities offered by presented procedure, the imposing conclusion is that further research should include additional constraint functions, such as: influence of manufacturing technology,

types of material, conditions of crane control and operation, oscillation of the girder, material fatigue and price for welding of the girder.

The obtained results can be of great importance both for crane designers and for the researchers who dealing with similar optimization problems.

This is the basis for further investigation and optimization related to material savings and reduction of production costs, which would demand more complex analysis. Previously mentioned is the basis for further research in order to save material, and also to minimize the cost of the girder manufacturing.

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