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Joint estimation of linear state-space models under non-Gaussian noises

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Abstract - Joint estimation of states and time-varying parameters of linear state space models is of practical importance for fault diagnosis and fault tolerant control. Previous works on this topic haven't considered joint estimation of linear systems in presence of outliers. They can significantly make worse the properties of linearly recursive algorithms which are designed to work in the presence of Gaussian noises. This paper proposes two kinds of strategies of joint parameter-state robust estimation of linear state space models in presence of non-Gaussian noises. Both possible cases are considered, joint robust estimation algorithm in case of parameter-independent matrices as well as in case of parameter-dependent matrices. Because of their good features in robust filtering, the modified and extended Masreliez-Martin filters represent a cornerstone for realization of the robust algorithms for joint state-parameter estimation of linear time-varying stochastic systems in presence of non-Gaussian noises. The good features of the proposed robust algorithms for joint estimation of linear time-varying stochastic systems are illustrated by simulations.

Key words: linear state-space models, joint estimation, robust identification, time-varying parameters, non-Gaussian noises

I. INTRODUCTION

It is well known that is very difficult to determine a large number of physical parameters which are integral part of complex systems. Despite the fact that many system parameters are available with some reasonable accuracy, a large number of parameters are known within a certain range, while some parameters are entirely unknown because manufacturers consider these data as proprietary information [1]. Precise knowledge of system parameters and states is crucial for successful realization of many control techniques.

Joint estimation is of great practical importance for fault diagnosis and fault tolerant control. The main challenge is the detection and isolation of incipient faults in the presence of modeling uncertainty and noise [2]. Inclusion of unknown parameters in the state vector allows easy implementation of the estimation algorithm, because the problem of parameters estimation in this case is solved using the standard filtering theory.

On the other side, there is no such solution available for linear systems in presence of non-Gaussian measurements. The presence of outliers can destroy the good features of linearly recursive algorithms which are designed for estimation in the presence of Gaussian noises. Huber's theory of robust statistics is crucial for the algorithm design [3].

This paper proposes two kinds of strategies to estimate the state and parameter jointly. Firstly, it is considered joint robust estimation of a linear stochastic systems with parameter-independent matrices. The second class of systems considered in this paper is a linear state space system with matrices which are parameter dependent. Conventionally, the robust estimation algorithms in these cases are based on the modified and extended Masreliez-Martin filters.

The designed estimators consider both robustness against noises and sensitivity to all possible faults. The good features of the proposed robust algorithms are illustrated through simulations.

II. JOINT ESTIMATION ALGORITHM IN CASE OF PARAMETER-INDEPENDENT MATRICES

The first class of systems considered in this paper for joint estimation of states and time-varying parameters is in the form of:

$$x(k+1) = A(\mathrm{k})x(k) + B(\mathrm{k})u(k) + C(\mathrm{k})\theta(k) + w(k) \tag{1}$$

$$y(k) = D(\mathrm{k})x(k) + \Gamma(\mathrm{k})\theta(k) + e(k) \tag{2}$$

where $x(k) \in R^n$ and $\theta(k) \in R^p$ are unknown state and parameter vectors, respectively. When the system has all possible sensor and component faults (this is the most common situation), the system model is described as (1)-(2). Component faults and sensor faults are described mathematically as follows, see Fig. 1:

$$f_C(k) = C(\mathrm{k})\theta(k) \tag{3}$$

$$f_S(k) = \Gamma(\mathrm{k})\theta(k) \tag{4}$$

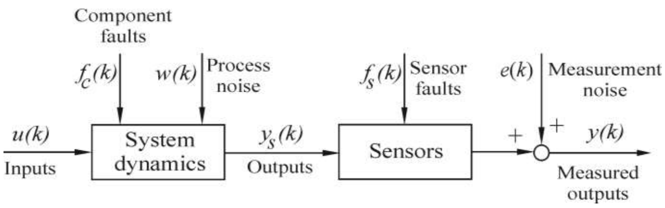

Fig. 1. Open-loop system with component and sensor faults

The component fault represents the case when some condition changes in the system, rendering the dynamic relation invalid, for example a leak in a pneumatic cylinder [4]. Generally speaking, the actual outputs of the system

$y_S(k) = D(\mathrm{k})x(k)$ are not directly accessible, and sensors are then used to measure the system output.

Sensors are the most important components for flight control and aircraft safety due to its roles in flight control and navigation. Any sensor fault must be detected as early as possible to prevent serious accident. To diagnose incipient faults, a fault diagnosis systems have to be made robust against modeling uncertainty and noise [5]. The general form of parameters changing of the stochastic linear system is $\theta(k+1) = G\theta(k) + \eta(k)$ in which G is a priori known nonsingular matrix which is convenient for inclusion of a priori information on the phenomenon which is identified. The stochastic process $\eta(k)$ is zero-mean white noise with covariance matrix $\Phi(k)$. Input and measured output vector of the system are $u(k) \in R^m$ and $y(k) \in R^r$, respectively, while $A(\mathrm{k})$, $B(\mathrm{k})$, $C(\mathrm{k})$, $D(\mathrm{k})$ and $\Gamma(\mathrm{k})$ are known, in general case, time-varying matrices with appropriate dimensions. It is assumed that the process noise is zero-mean Gaussian white noise $w(k): \mathcal{N}(0, Q(k))$, in which $Q(k)$ is the covariance matrix. The measurement noise $e(k)$ has non-Gaussian distribution with approximately normal distribution classes:

$$\mathcal{P}_\varepsilon = \{p(e): p(e) = (1\text{-}\varepsilon)p_1(e) + \varepsilon p_2(e)\} \tag{5}$$

in which the probability density $p(e)$ represents a mixture of primary probability density $p_1(e): \mathcal{N}(0, R_1(k))$ and contaminating probability density $p_2(e): \mathcal{N}(0, R_2(k))$ where contamination degree ε is in range $0 < \varepsilon < 1$, while $R_1(k)$ and $R_2(k)$ are covariance matrices of primary and contaminating term in non-Gaussian distribution (5), respectively.

An obviously easy approach to the joint estimation of $x(k)$ and $\theta(k)$ for system (1)-(2) is to consider the extended system:

$$z(k+1) = \begin{bmatrix} A(\mathrm{k}) & C(\mathrm{k}) \\ 0 & I \end{bmatrix} z(k) + \begin{bmatrix} \mathrm{B}(k) \\ 0 \end{bmatrix} u(\mathrm{k}) + \xi(k) \tag{6}$$

$$y(k) = \begin{bmatrix} D(\mathrm{k}) & \Gamma(\mathrm{k}) \end{bmatrix} z(k) + e(k) \tag{7}$$

or in a more compact form:

$$z(k+1) = F(\mathrm{k})z(k) + \overline{\mathrm{B}}(k)u(\mathrm{k}) + \xi(k) \tag{8}$$

$$y(k) = H(\mathrm{k})z(k) + e(k) \tag{9}$$

in which block matrices are $H(\mathrm{k}) = \begin{bmatrix} D(\mathrm{k}) & \Gamma(\mathrm{k}) \end{bmatrix}$,

$\overline{\mathrm{B}}(k) = \begin{bmatrix} B(\mathrm{k})^T & 0^T \end{bmatrix}^T$,

$F(\mathrm{k}) = \begin{bmatrix} A(\mathrm{k}) & C(\mathrm{k}) \\ 0 & I \end{bmatrix}$, extended state vector is $z(k) = \begin{bmatrix} x(k)^T & \theta(k)^T \end{bmatrix}^T$ and $\xi(k) = [w^T(k) \quad \eta^T(k)]^T$ denotes extended disturbance vector, where $\xi(k): N(0, \Xi(k))$ with $\Xi(k) = diag(Q(k), \Phi(k))$.

Since extended system (8)-(9) is still a linear system, the Masreliez-Martin filter [6] is applicable to the joint estimation of $x(k)$ and $\theta(k)$. Our goal is to derive the robust algorithm for joint state and parameter estimation of stochastic linear systems in the presence of outliers which maintains a low sensitivity in appearance of outliers. For the class of ε-contaminated distributions of probabilities, the nonlinear transformation of prediction error $\psi(\cdot)$ (Huber's function), is obtained as:

$$\psi(\nu(k)) = \min\{|\nu(k), k_\varepsilon|\}\,\mathrm{sgn}(\nu(k)), \tag{10}$$

and its derivative:

$$\psi'(\nu(k)) = \begin{cases} 1 & |\nu(k)| < k_\varepsilon, \\ 0 & otherwise. \end{cases} \tag{11}$$

in which k_ε is appropriately defined parameter of Huber's function, see Fig. 2.

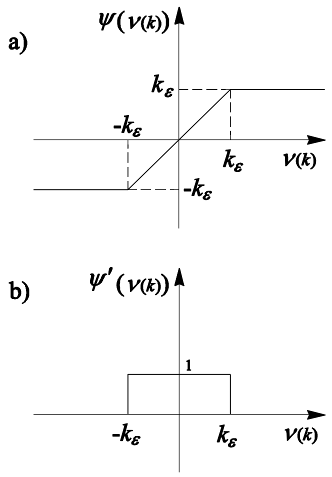

Fig. 2. Huber's function and its derivative

The originally proposed Masreliez-Martin filter include the member in the a posteriori covariance matrix $E_{\mathrm{P}_\varepsilon}\{\psi'[\nu(k)]\}$ which is not easy to determine in practical conditions [6]. In order to improve its applicability, the realization of $\psi'[\nu(k)]$ is introduced instead the member $E_{\mathrm{P}_\varepsilon}\{\psi'[\nu(k)]\}$. Intense simulations justified such interventions. Because of its good features in robust filtering, such modified Masreliez-Martin filter is used as a basis in formulating the joint state and parameter estimator of linear stochastic systems, as follows:

$$
\begin{aligned}
&\hat{z}(k|k-1) = F(k-1)\hat{z}(k-1|k-1) + \overline{B}(k-1)u(k-1) \\
&P(k|k-1) = F(k-1)P(k-1|k-1)F^T(k-1) + \Xi(k-1) \\
&K(k) = \left[N(k) \mid M(k)\right]^T = P^T(k|k-1)H^T(k)T^T(k) \\
&\nu(k) = T(k)\left[y(k) - H(k)\hat{x}(k-1|k-1)\right] \\
&\hat{x}(k|k) = A(k-1)\hat{x}(k-1|k-1) + B(k-1)u(k-1) + \\
&+C(k-1)\hat{\theta}(k-1|k-1) + N(k)\Psi\left(\nu(k)\right) \\
&\hat{\theta}(k|k) = \hat{\theta}(k-1|k-1) + M(k)\Psi\left(\nu(k)\right) \\
&P(k|k) = P(k|k-1) - K(k)\Psi'(\nu(k))K^T(k) \\
&\Psi'\left(\nu(k)\right) = diag\left(\psi'\left(\nu_1(k)\right),\ldots,\psi'\left(\nu_r(k)\right)\right) \\
&T(k) = \left[H(k)P(k|k-1)H^T(k) + R_1(k)\right]^{-\frac{1}{2}}
\end{aligned}
\tag{12}
$$

with initial conditions: $\hat{z}_0 = 0$ and $P_0 = \left[\begin{array}{c|c} P(x_0) & 0 \\ \hline 0 & P(\theta_0) \end{array}\right]$.

In this way, the robust algorithm for the joint states-parameters estimation of linear stochastic systems has been derived.

III. JOINT ESTIMATION ALGORITHM IN CASE OF PARAMETER-DEPENDENT MATRI CES

The second class of systems considered in this paper for joint estimation of states and time-varying parameters is in the form of:

$$x(k+1) = A(\theta(k))x(k) + B(\theta(k))u(k) + w(k) \tag{13}$$

$$y(k) = D(\theta(k))x(k) + e(k) \tag{14}$$

In some cases, the fault $f_p(\mathrm{k})$ could be expressed as a change in the system parameter, for example a change in the i_{th} row and j_{th} column element of the matrix A, the system can then be described as (13)-(14), see Fig. 3. This approach is based on the assumption that the faults are reflected in the physical system parameters such as friction, mass, viscosity, resistance, capacitance, etc. As indicated, the linear state space model is often specified up to the value of some parameters $\theta(k)$. Since matrices A, B and D are dependent of parameters $\theta(k)$ and due to multiplying with state vector $x(k)$, the system (13)-(14) is nonlinear. Hence, to obtain the parameter estimation recursively, we shall consequently face with a general nonlinear filtering problem:

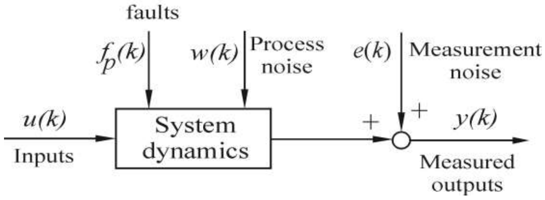

Fig. 3: Open-loop system with parameter faults in the system

$$z(k) = \begin{bmatrix} f_{k-1}(x(k-1), u(k-1), \theta(k-1)) \\ g_{k-1}\left(\theta(k-1)\right) \end{bmatrix} + \begin{bmatrix} w(k-1) \\ \eta(k-1) \end{bmatrix} \tag{15}$$

The extended system is given in a more compact form:

$$z(k) = q_{k-1}(z(k-1), u(k-1), \theta(k-1)) + \xi(k-1) \tag{16}$$

$$y(k) = h_k(z(k), \theta(k)) + e(k) \tag{17}$$

Based on extended robust filter [7], the robust algorithm for joint estimation of linear systems in case of parameter-dependent matrices has the following form:

$$
\begin{aligned}
&\hat{z}(k|k-1) = q_{k-1}(\hat{z}(k-1|k-1), u(k-1), \theta(k-1), 0) \\
&P(k|k-1) = F(k-1)P(k-1|k-1)F^T(k-1) + \\
&+L(k-1)\Xi(k-1)L^T(k-1) \\
&K(k) = P^T(k|k-1)H^T(k)T^T(k) \\
&\nu(k) = T(k)\left[y(k) - h_k(\hat{z}(k|k-1), \theta(k), 0)\right] \\
&\hat{x}(k|k) = f_{k-1}(\hat{x}(k-1|k-1), u(k-1), \hat{\theta}(k-1|k-1)) + \\
&+N(k)\Psi\left(\nu(k)\right) \\
&\hat{\theta}(k|k) = \hat{\theta}(k-1|k-1) + M(k)\Psi\left(\nu(k)\right) \\
&P(k|k) = P(k|k-1) - K(k)\Psi'(\nu(k))K^T(k) \\
&T(k) = \left[H(k)P(k|k-1)H^T(k) + V(k)R_1(k)V^T(k)\right]^{-\frac{1}{2}}
\end{aligned}
\tag{18}
$$

$$F(k) = \left[\begin{array}{c|c} A(k) & F_\theta(k) \\ \hline 0 & I_p \end{array}\right], L(k) = I_{n+p}, H(k) = \left[D(k) \mid H_\theta(k)\right], V(k) = I_r$$

with same initial conditions as in algorithm (12).

IV. SIMULATION RESULTS

The benefits of the proposed robust algorithms for joint estimation of stochastic linear time-varying systems are illustrated through intensive simulations. Firstly, joint robust estimation with parameter-independent matrices is considered. These results demonstrate superiority of the proposed robust algorithm (12) in relation to the joint estimation algorithms based on widely used Kalman filter and Masreliez-Martin filter. Behavior of the algorithms will be considered on:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.1 & 1 & 0 \\ 0.5 & 0.3 & 1 \end{bmatrix}\begin{bmatrix} p_1(\mathrm{k}) \\ p_2(\mathrm{k}) \\ p_3(\mathrm{k}) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix}$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} p_1(\mathrm{k}) \\ p_2(\mathrm{k}) \\ p_3(\mathrm{k}) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix}$$

The features of the proposed robust algorithm are considered on the model whose time-varying parameter vector has expected value $\overline{\theta} = \left[0.96 \quad -1.88 \quad 2.23\right]^T$. The process noise $w(k)$ is zero-mean white noise with covariance matrix $Q(\mathrm{k}) = diag(0.0015, 0.001)$. The covariance matrix of parameters is given by $\Phi(k) = diag(0.003, 0.003, 0.02)$. The non-Gaussian distribution of the measured noise is given by [3]:

$$\mathcal{P}_\varepsilon = \begin{cases} p(\nu_1) = (1-\varepsilon_1)\cdot\mathcal{N}(0;0.005) + \varepsilon_1\cdot\mathcal{N}(0;0.5), \\ p(\nu_2) = (1-\varepsilon_2)\cdot\mathcal{N}(0;0.01) + \varepsilon_2\cdot\mathcal{N}(0;1) \end{cases}. \quad (19)$$

For the purpose of illustrating estimation quality, mean square error (MSE) is used as follows:

$$MSE = \ln\left(\mathrm{E}\left\|\hat{z}(k) - z(k)\right\|^2\right). \quad (20)$$

The system outputs, estimates of states and parameters, as well as mean square errors in the case when contaminations have values $\varepsilon_1 = \varepsilon_2 = 0.1$ are shown on Fig.4.

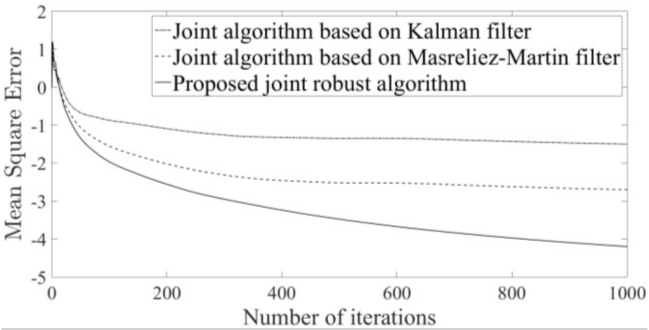

Fig. 4. Mean square errors using robust algorithm (12)

The presented results have shown that the joint estimation algorithm based on widely-used Kalman filter is very sensitive to the presence of non-Gaussian noises, as opposed to the proposed robust joint algorithm.

Following results demonstrate superiority of the proposed robust algorithm (18) in relation to the joint estimation algorithms based on widely-used extended Kalman filter and extended Masreliez-Martin filter. Behavior of the algorithms will be considered on:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} p_1(\mathrm{k}) & 0 \\ 0 & p_2(\mathrm{k}) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} p_3(\mathrm{k}) & 0 \\ 0 & 0.9 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} \quad (22)$$

The features of the proposed robust algorithm are considered on the model whose time-varying parameter vector has expected value $\bar{\theta} = \begin{bmatrix} 1.01 & 0.98 & 0.99 \end{bmatrix}^T$. The process noise $w(k)$ is zero-mean white noise with covariance matrix $Q(\mathrm{k}) = diag(10^{-5}, 10^{-5})$. The covariance matrix of parameters has the form $\Phi(k) = diag(2\cdot10^{-6}, 10^{-6}, 2\cdot10^{-6})$. The non-Gaussian distribution of the measured noise is given by

$$\mathcal{P}_\varepsilon = \begin{cases} p(\nu_1) = (1-\varepsilon_1)\cdot\mathcal{N}(0;0.0001) + \varepsilon_1\cdot\mathcal{N}(0;0.01), \\ p(\nu_2) = (1-\varepsilon_2)\cdot\mathcal{N}(0;0.0001) + \varepsilon_2\cdot\mathcal{N}(0;0.01) \end{cases} \quad (23)$$

The mean square errors in the case when contaminations have values $\varepsilon_1 = \varepsilon_2 = 0.1$ are shown on Fig.5.

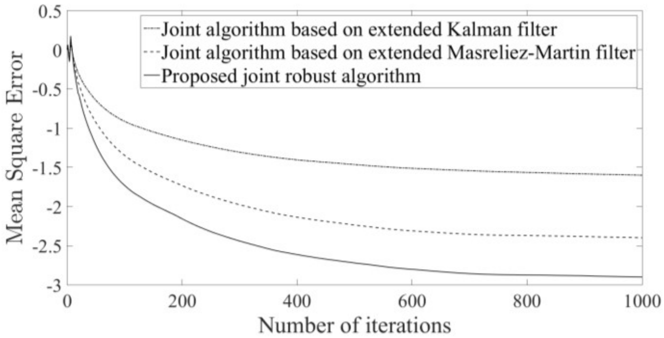

Fig.5. Mean square errors using robust algorithm (18)

The presented results have shown that the widely-used Extended Kalman filter is very sensitive to the presence of non-Gaussian noises, as opposed to the proposed robust joint estimation algorithm.

V. CONCLUSION

The joint state and parameter robust estimation algorithms for stochastic linear time-varying systems, in presence of non-Gaussian noises have been proposed. The proposed algorithms have been used to solve the joint estimation problem of linear stochastic models where the conventional approaches fails. Because of their good features in robust filtering, the modified and extended Masreliez-Martin filters were used as a basis in formulating the joint robust estimator of linear stochastic systems.

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REFERENCES

[1] V. Filipovic, N. Nedic, and V. Stojanovic, Robust identification of pneumatic servo actuators in the real situations, Forschung im Ingenieurwesen - Engineering Research, 75(4) (2011) 183-196.

[2] R. Isermann, Fault-Diagnosis Applications: Model-Based Condition Monitoring: Actuators, Drives, Machinery, Plants, Sensors, and Fault-tolerant Systems, Springer, 2011.

[3] P.J. Huber, Robust statistics, Wiley, New York, 1981.

[4] V. Stojanovic, and N. Nedic, Robust Kalman filtering for nonlinear multivariable stochastic systems in the presence of non-Gaussian noise, International Journal of Robust and Nonlinear Control 26(3) (2016) 445-460.

[5] J. Chen, R.J. Patton, Robust Model-Based Fault Diagnosis for Dynamic Systems, Springer, 1999.

[6] C.J. Masreliez, and R.D. Martin, Robust Bayesian estimation for the linear model and robustifying the Kalman filter, IEEE Transactions on Automatic Control 22(3) (1977) 361–371.

[7] V. Stojanovic, and N. Nedic, Robust Kalman filtering for nonlinear multivariable stochastic systems in the presence of non-Gaussian noise, International Journal of Robust and Nonlinear Control 26(3) (2016) 445-460.