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Some Considerations on Application of Relay Feedback Test to the Multivariable Systems

S. Lj. Prodanović +, N. N. Nedić ++ and Lj. M. Dubonjić ++

+ University of East Sarajevo, Faculty of Mechanical Engineering, Vuka Karadžića 30, 71123 East Sarajevo, Bosnia and Herzegovina
Phone: (+387 57) 340-847, Fax: (+387 57) 340-847, E-Mail: sasa.prodanovic77@gmail.com , WWW:http://www.maf.ues.rs.ba/

++ University of Kragujevac, Faculty of Mechanical and Civil Engineering in Kraljevo, Dositejeva 19, 36000 Kraljevo, Serbia
Phone: (+381 36) 383-377, Fax: (+381 36) 383-378, E-Mail: nedic.n@mfkv.kg.ac.rs , dubonjic.lj@mfkv.kg.ac.rs , WWW:http://www.mfkv.kg.ac.rs/

Abstract* - Wide usage of relay feedback test for controller tuning naturally leads to numerous investigations for its improvement and extension of its application to the various objects and control strategies. Present paper contains testing possibilities for application relay feedback test to the already decoupled two input two output system. The aim is to protect system from influence of the interaction during procedure of tuning of PID controller or its shorter variants. Survey has been supported by simulations in order to prove efficiency of presented approach on the two examples of systems. The results confirm that relay feedback test can be applied simultaneously to the all loops of the previously decoupled multivariable system.

***Key words:* Relay feedback test, PID control, decoupler, multivariable systems**

I. INTRODUCTION

It is well known fact that relay feedback test is very suitable method for process identification and tuning of PID controllers. Its advantages are reflected in its applicability in real time without interrupting the system's operation, then in obtaining necessary information about the system based on only two parameters (ultimate gain K_u and ultimate period T_u), as well as the benefits of using auto-tuning. Computer simulations are widely accepted approach in the testing of system. Therefore, this research has been carried out on the mathematical model of the object. Usage of relay feedback test in the SISO (single-input single-output) systems in recent decades enabled the experiences that naturally should be applied to the multivariable systems. In that case interaction among control loops increase problems already present due to disturbance. The main trouble is determining wrong values for ultimate gain and ultimate period. There are several approaches in introducing this test into multivariable systems [1]. First, *independent single relay test*. The relay test is applied at one moment only to one loop, while all others are open. Second, *serial relay test*. Relay test is applied to one loop and it is continue to operate with a simple controller and remains closed. This procedure is conducted for each loop in a row. Third, *decentralized relay test*. The relay test is simultaneously realized in all loops.

Many researchers [2-13] were dealt to extend relay feedback test to the multivariable systems and they have given significant contribution to this problematic. There are important breakthroughs in the recent investigations, too. Application of the ultimate-point method for multivariable systems has been successfully carried out in [14], taking care about stability margins. In [15] a specific approach has been made to synthesize the relay feedback model of the undesirable response as functions of its ultimate properties, and all with the goal to determine as exact as possible model of system in the process of identification. Investigation in [16] contains suitable model of the oscillatory process that has been proposed and studied in the frequency-domain. This approach leads to the simple and accurate procedure for the computation of the oscillation parameters in each control loop. Comprehensive overview of relay feedback test application for stable and unstable MIMO (multi-input multi-output) systems has been given in the [17].

In the mentioned studies, relay feedback test has nowhere been applied to the already decoupled system. Present paper deal with possibilities for simultaneous introduction of relay feedback test into all control loops, where interaction between them has already been compensated using previously designed decoupler. Of course, P, PI or PID controller can be designed based on obtained ultimate points. The paper has been structured as follows: section II describes developed methodology. Results of simulations performed on two examples are given in section III. Section IV contains conclusions.

II. EXPLANATION OF APPROACH

Better dynamical behavior of multivariable systems that can be achieved using decoupler had a large influence to generation of main idea in the present paper. Procedure for decoupler design will not be discussed here. It can be determined without mathematical model of object, for example using iterative method, before application of relay feedback test. Afterward, as in many other cases, parameters of controller should be determined and improved based on relay feedback test. Structure that makes this strategy more clear is shown in the Fig. 1 [18]. Where r_i – reference variable, e_i – error, v_i – controller (relay) output, u_i – manipulated variable, y_i – responses. Since 2x2 systems are discussed in the paper (i = 2), transfer matrix of the object is:

$$G(s)=\begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \quad (1)$$

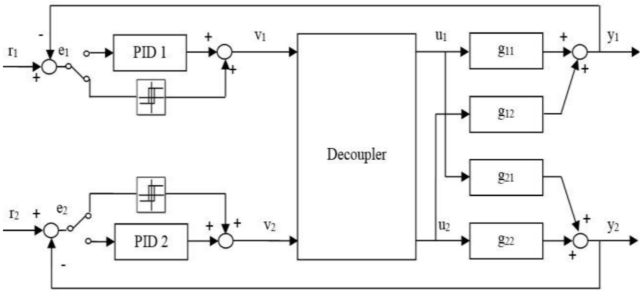

Fig. 1. Configuration for applying a relay feedback test for a multivariable system [18]

Display of relay output (u) and process response (y) on the stability margin are shown in Fig. 3. where h – ideal relay amplitude, L – process delay, a – amplitude of process oscillation.

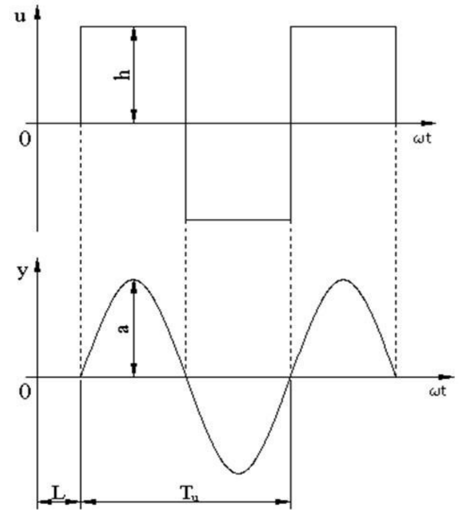

Fig. 2. General display of relay output (u) and process response (y) [19]

Ultimate period T_u and amplitude of process oscillation a, is determined from Fig. 2., while the ultimate gain is calculated from expression $K_u=4h/\pi a$. According these two values parameters of the controller can be easy calculated based on Ziegler and Nichols method given in Table I.

TABLE I

Parameters of PID controllers according Ziegler-Nichols method [20]

Controller	K_p	T_i	T_d
P	$0{,}5K_u$	-	-
PI	$0{,}4K_u$	$0{,}8T_u$	-
PID (parallel)	$0{,}4K_u$	$0{,}5T_u$	$0{,}125T_u$
PID (serial)	$0{,}3K_u$	$0{,}157T_u$	$0{,}25T_u$

III. EXAMPLES

Presented procedure is tested on two examples, i.e. one mechanical system without time delay and the other process that contain time delay.

Example 1.

This example is represent of the mechanical systems. Electrohydraulic servosystem for structural testing is considered here. Reference variables are forces F_{r1} and F_{r2}. Object outputs are variables F_1 and F_2 and their changes are measured by transducers. Intensity and character of the forces are controlled by flow rates through the servovalves that represent manipulated variables. Its mathematical model has been derived in [21]. It follows:

$$G(s)=\frac{1}{\Delta(s)}\begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

$$\begin{aligned}
g_{11}(s) &= 2.926\cdot10^{2}s^{4}+1.9152\cdot10^{4}s^{3}+ \\
&\quad +1.2667\cdot10^{7}s^{2}+5.5825\cdot10^{7}s+4.7959\cdot10^{9} \\
g_{12}(s) &= -3.8382\cdot10^{4}s^{3}-1.7068\cdot10^{7}s^{2}- \\
&\quad -8.3584\cdot10^{7}s-6.4967\cdot10^{9} \\
g_{21}(s) &= -4.4533\cdot10^{3}s^{3}-3.2461\cdot10^{6}s^{2}- \\
&\quad -1.4362\cdot10^{7}s-1.2403\cdot10^{9} \\
g_{22}(s) &= 2.506\cdot10^{2}s^{4}+1.6229\cdot10^{4}s^{3}+ \\
&\quad +6.6134\cdot10^{6}s^{2}+3.0476\cdot10^{6}s+2.4813\cdot10^{9} \\
\Delta(s) &= s^{5}+1.2308\cdot10^{2}s^{4}+6.993\cdot10^{4}s^{3}+ \\
&\quad +1.5098\cdot10^{6}s^{2}+3.5504\cdot10^{8}s+8.2333\cdot10^{-6}
\end{aligned} \quad (2)$$

Responses obtained after simulation of relay feedback test, when h=0,1 are shown in Fig. 3. and 4.

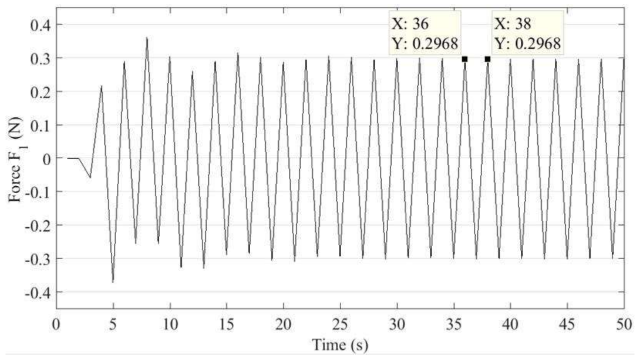

Fig. 3. First output of the system (force F_1)

Parameters for design of PID 1 are obtained from Fig. 3. $a_1=0{,}2968$ (N) , $T_{u1}=2$ (s) , $K_{u1}=0{,}429$. Hence:

- P controller: $K_p=0{,}2145$
- PI controller: $K_p=0{,}1716$; $K_i=0{,}107$
- PID controller: $K_p=0{,}1716$; $K_i=0{,}1716$; $K_d=0{,}0429$

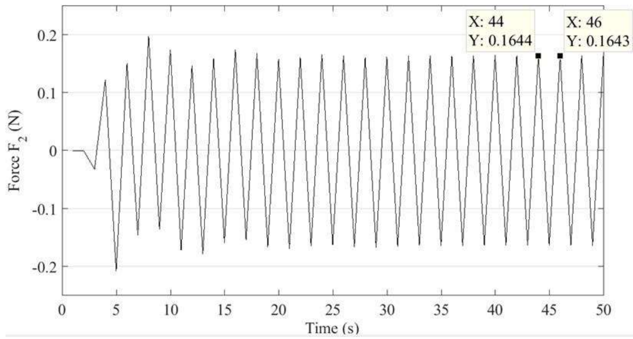

Fig. 4. Second output of the system (force F_2)

Parameters for design of PID 2 are obtained from Fig. 4. a_2=0,1644 (N) , T_{u2}=2 (s) , K_{u2}=0,775 . Hence:

- P controller: K_p=0,3875
- PI controller: K_p=0,31 ; K_i=0,194
- PID controller: K_p=0,31 ; K_i=0,31 ; K_d=0,0775

Calculated parameters of controllers are introduced into entire multivariable system in order to simulate its functioning. Previously system has been decoupled using inverted static decoupler (3) and (4) [18].

$$D(s)=\begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix} \quad (3)$$

$$d_{12}(s)\Big|_{s=0}=\frac{-g_{12}(0)}{g_{11}(0)}=1.35$$
$$d_{21}(s)\Big|_{s=0}=\frac{-g_{21}(0)}{g_{22}(0)}=0.5 \quad (4)$$

Control under P, PI, PID controller and case without decoupler [21] has been compared. Responses are shown in Fig. 5.

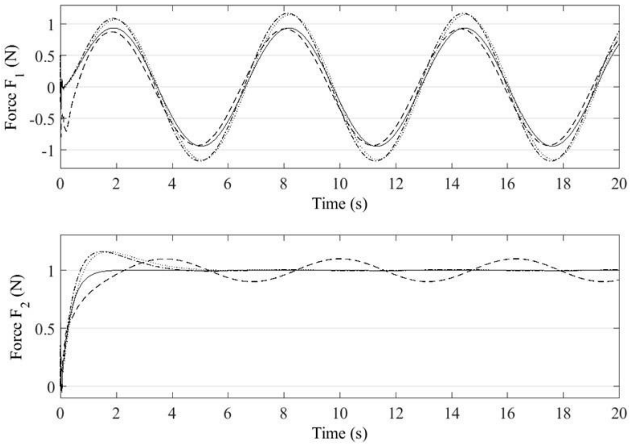

Fig. 5. Forces on cylinders: _ _ _ Singer and Meashio [21] (___ P, ... PI, _._._ PID controllers designed using relay feedback test for decoupled system), first input sine and second input step function

These responses confirm that it is possible to design multivariable P, PI and PID controllers using presented method. It is also noted that the best dynamic behavior of the system is achieved by applying the P controller. PI and PID are worse due to presence of overshoot and larger settling time. System without decoupler [21] has bad second response.

Example 2.

Binary distillation column (water-methanol) is taken as a represent of processes with time delay. Its mathematical model has been derived by Wood and Berry [22]. It is given by (5) and (6). Outputs (controlled variables) are: $X_D(s)$ – percentage of methanol in the distillate, $X_B(s)$ – percentage of methanol in the bottom products. Manipulated variables are: R(s) – reflux flow rate and S(s) – steam flow rate in the reboiler.

$$\begin{bmatrix} X_D(s) \\ X_B(s) \end{bmatrix}=G(s)\cdot\begin{bmatrix} R(s) \\ S(s) \end{bmatrix} \quad (5)$$

$$G(s)=\begin{bmatrix} \frac{12,8}{16,7s+1}e^{-s} & \frac{-18,9}{21s+1}e^{-3s} \\ \frac{6,6}{10,9s+1}e^{-7s} & \frac{-19,4}{14,4s+1}e^{-3s} \end{bmatrix} \quad (6)$$

Simulation of relay feedback test has been carried out and responses are shown in Fig. 6. and 7. In this case, for the second output, the value of the height of the ideal relay characteristic h was adjusted to bring the process to the stability margin. Therefore, for first loop h=0,1, and for the second one mentioned adjustment has been done h=0,01.

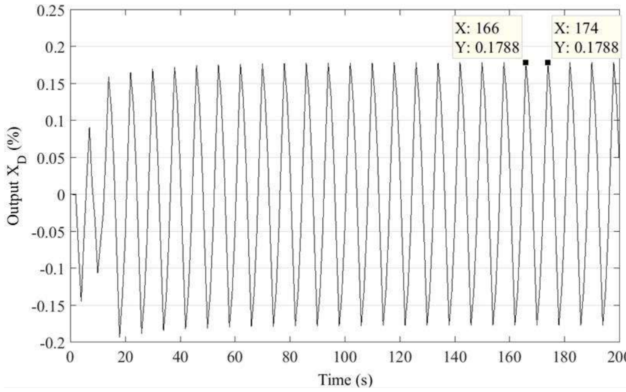

Fig. 6. First output of the system X_D

Parameters for design of PID 1 are obtained from Fig. 6. a_1=0,178 (N) , T_{u1}=8 (s) , and properly calculated K_{u1}=0,712 . Hence:

- P controller: K_p=0,356
- PI controller: K_p=0,2848 ; K_i=0,0445
- PID controller: K_p=0,2848 ; K_i=0,0712 ; K_d=0,2848

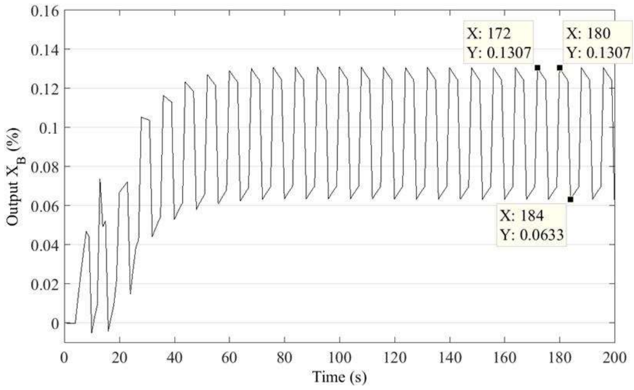

Fig. 7. Second output of the system X_B

Parameters for design of PID 2 are obtained from Fig. 7. a_2= 0,0337 (N) , T_{u2}=8 (s) , K_{u2}=0,378 . Hence:

- P controller: K_p=-0,189
- PI controller: K_p=-0,1512 ; K_i=-0,0236

• PID controller: K_p=-0,1512 ; K_i=-0,0378 ; K_d=-0,1512

It is obvious that negative PID parameters couldn't be calculated from expression $K_u=4h/\pi a$. So they should be take as absolute values in order to get stable system. Therefore both variants positive and negative have to be taken into account.

Interaction in this process has been canceled using direct decoupler given by (7) and (8) [18].

$$D(s)=\begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix}=\begin{bmatrix} 1 & \dfrac{-g_{12}(s)}{g_{11}(s)} \\ \dfrac{-g_{21}(s)}{g_{22}(s)} & 1 \end{bmatrix} \tag{7}$$

$$\begin{aligned} d_{12}(s)&=1{,}47\frac{16{,}7s+1}{21s+1}e^{-2s} \\ d_{21}(s)&=0{,}34\frac{14{,}4s+1}{10{,}9s+1}e^{-4s} \end{aligned} \tag{8}$$

Simulations of the system with above controllers has been performed. Responses are shown in Fig. 8. Obtained responses also confirm possibility for design multivariable P, PI and PID controllers using presented method. It is evident that PI controller enables the best responses of this distillation column. PID controller cause bigger overshoot and settling time, while responses under P type are bad because of large steady-state error. In general, it is important to obtain at least one type of controller that can give good system behavior.

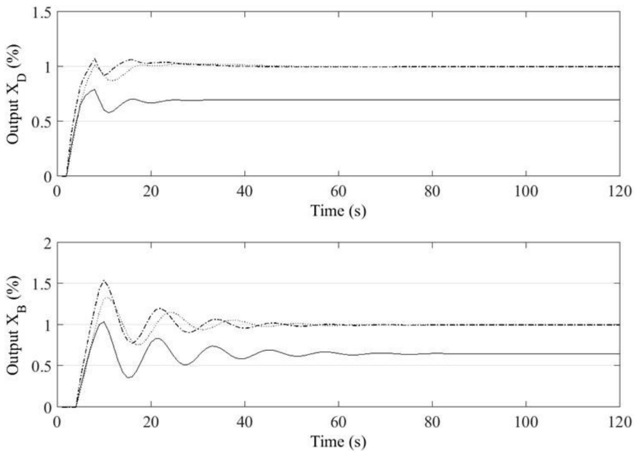

Fig. 8. Outputs of the distillation column (___ P, ... PI, _._._ PID controllers designed using relay feedback test for decoupled system), both inputs are step function

IV. CONCLUSIONS

This research has proved as possible simultaneous application of relay feedback test to the already decoupled system. Presented method prevents influence of interaction between control loops during carrying out of the test and in that way enables correct obtaining of ultimate point. The aim was to make the system come to a stability margin, in order to ultimate gain and ultimate period can be determined from both responses. It has been shown that in some cases it is necessary to correct the height of the ideal relay amplitude in order to achieve that. Feasibility of suggested method was in focus. Therefore, attention is not devoted to fine controller tuning and responses surely can be a little bit better. That can be topic of the future research as well as which performances can be obtained using saturation relay.

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