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Adaptive Input Design for Identification of Output Error Model with Constrained Output

V. Stojanovic and V. Filipovic

Department of Energetics and Automatic Control, University of Kragujevac, Faculty of Mechanical Engineering Kraljevo, Dositejeva 19, 36000 Kraljevo, Serbia, E-Mail: vladostojanovic@open.telekom.rs

Abstract ***- This paper considers the identification of output error (OE) model, for the case of constrained output variance. The constraint plays a very important role in the process industry, in the reduction of degradation of product quality. In this paper, it is shown, in the form of theorem, that the optimal input signal, with constrained output, is achieved by a minimum variance controller together with stochastic reference. The key problem is that the optimal input depends on the system parameters to be identified. In order to overcome this problem, it is the proposed two-stage adaptive procedure***

a) obtaining an initial model using PRBS as input signal

b) application of adaptive minimum variance controller together with the stochastic variable reference, in order to generate input signals for system identification

Theoretical results are illustrated by simulations.

Key words: ***System identification, adaptive input design, output error model, constrained output variance, minimum variance controller***

I. INTRODUCTION

The design of controllers is largely based on the use of mathematical models that are obtained in the process of system identification [1-3]. The main task of the theory of identification is the extraction of maximum information from the measurements that are available. This requirement is realized by optimal experiment design [4-5]. The basic approach consists in minimizing the scalar function of Fisher information matrix [4].

The key problem in the optimal input design is that the optimal input depends on the unknown system properties to be identified. Namely, the Fisher information matrix typically depends on system parameters. There are two basic approaches to overcome this problem. The first approach is based on robust optimal experiment design. In this case the procedure is slightly sensitive to the uncertainty of a priori information about the system [6-7]. The second approach is based on adaptation. One such, two-stage procedure is proposed in [8]. In the first stage, in a short time interval, the data are collected using PRBS input. Based on these data system model is identified, and that is initial model for optimal input design. In the second stage, the obtained input signal, by using minimum variance controller and stochastic reference, is used to generate a new data set. Adaptive input design for the ARX models is discussed in [9].

In many practical cases, constraints on the fluctuation of input and/or output signals are very important [10]. For example, in the industrial production, product quality must be within certain limits (constraints on the fluctuation of the output signal).

If the constraint is related to the variance of the output signal, it is shown that the experiment design is D-optimal and that the input signal is generated using a minimum variance controller together with an external stochastic signal [11-12].

In reference [13], it is discussed the robust identification of a pneumatic cylinder, which is modeled as a stochastic system with non-Gaussian noise. Input design is based on the ideas from model predictive control and a bandlimited "$1/f$" noise.

This paper considers the optimal experiment design for output error (OE) models. There is a constraint on the output power. It has been shown that the optimal input signal can be obtained by minimum variance controller whose reference is white noise sequence with known variance. In order to be able to implement the algorithm, adaptive approach was applied. It was used direct adaptive minimum variance controller. The algorithm has two stages. In the first stage, the process parameters are estimated. In the second stage, based on thus obtained parameters, it has been formed the minimum variance controller that generates the input signal of the process by which the identification is made. Because the reference signal is in the form of white noise, parameter estimation is consistent (the true values of parameters are obtained with probability 1). The paper's results are supported by simulations.

II. OPTIMAL ALGORITHM DESIGN

In this paper we consider the model

$$y(k) = \frac{b_1}{F(q^{-1})} u(k-1) + e(k) . \tag{1}$$

In this case the parameters b_1 and $f_i\,(i=1,\ldots,n)$ are estimated, where $F(q^{-1}) = 1 + f_1 q^{-1} + \ldots + f_n q^{-n}$.

Let us consider the system (1). Based on N measurements of output, the following vector can be formed

$$Y = \left[y(1) \quad \dots \quad y(N) \right]^T . \tag{2}$$

Using the vector Y, the Fisher information matrix can be defined as

$$M = E_{Y|\beta} \left\{ \left(\frac{\partial \log p(Y|\beta)}{\partial \beta} \right)^T \left(\frac{\partial \log p(Y|\beta)}{\partial \beta} \right) \right\}, \tag{3}$$

where $E_{\cdot|\cdot}$ denotes conditional mathematical expectation and $M \in R^{N \times N}$. Parameter vector β has the form

$$\beta = \left[\theta^T \quad \sigma^2 \right]^T . \tag{4}$$

where

$$\theta^T = \left[f_1 \quad \dots \quad f_n \quad b_1 \right]^T , \tag{5}$$

and σ^2 is the variance of the noise $e(k)$.

In this paper the average value of the Fisher information matrix (3) will be used

$$\overline{M} = \frac{1}{N} M . \tag{6}$$

The following criterion form will be used

$$J = -\log \det \overline{M} . \tag{7}$$

We will now formulate the main result of this paper in the form of the Theorem.

Theorem 1. Suppose that for the OE model (1) the following conditions are fulfilled

1° Stochastic noise $e(k)$ has a Gaussian distribution with variance σ^2 and zero mean.

2° Constraint on the output is $Ey^2(k) \le W$, $W \in (0, \infty)$.

3° $Ey^2(k) > \sigma^2$.

Then the criterion $-\log \det \overline{M}$ achieves its minimum value if the system input $u(k)$ is generated by a minimum variance controller which reference is stochastic process $\{\eta(k)\}$ with probability density function

$$p(\eta) = \frac{b_1}{\sqrt{2\pi W}} e^{-\frac{(\eta b_1)^2}{2W}} . \qquad \blacksquare \tag{8}$$

Proof: Let us define $\omega(k)$ as

$$\omega(k) = y(k) + f_1 y_M(k-1) + \dots + f_n y_M(k-n) - b_1 u(k-1) . \tag{9}$$

According to the condition 1° of the **Theorem 1**, it follows that

$$p(Y|\beta) = \frac{1}{\sigma^N (2\pi)^{N/2}} e^{-\frac{1}{2\sigma^2} \sum_{k=1}^{N} \omega^2(k)} . \tag{10}$$

From the relation (10) one can obtain

$$\begin{aligned} \frac{\partial \log p(Y|\beta)}{\partial \beta} &= -\frac{1}{\sigma^2} \sum_{k=1}^{N} \frac{\partial \omega(k)}{\partial \beta} \omega(k) - \\ &- \frac{1}{2\sigma^2} \frac{\partial \sigma^2}{\partial \beta} \left(N - \frac{1}{\sigma^2} \sum_{k=1}^{N} \omega^2(k) \right) \end{aligned} . \tag{11}$$

After some calculations, the relation (3) can be expressed as

$$M = E_{Y|\beta} \left\{ \frac{1}{\sigma^2} \sum_{t=1}^{N} \left(\frac{\partial \omega(k)}{\partial \beta} \right)^T \frac{\partial \omega(k)}{\partial \beta} \right\} + \frac{N}{2\sigma^4} \left(\frac{\partial \sigma^2}{\partial \beta} \right)^T \frac{\partial \sigma^2}{\partial \beta} . \tag{12}$$

The mean value of the Fisher information matrix has the form

$$\overline{M} = E_{Y|\beta} \left\{ \frac{1}{N\sigma^2} \sum_{t=1}^{N} \left(\frac{\partial \omega(k)}{\partial \beta} \right)^T \frac{\partial \omega(k)}{\partial \beta} \right\} + \frac{1}{2\sigma^4} \left(\frac{\partial \sigma^2}{\partial \beta} \right)^T \frac{\partial \sigma^2}{\partial \beta} \tag{13}$$

where

$$\begin{aligned} \frac{\partial \omega(k)}{\partial \beta} &= \left[\frac{\partial \omega(k)}{\partial f} \;\vdots\; \frac{\partial \omega(k)}{\partial b_1} \;\vdots\; \frac{\partial \omega(k)}{\partial \sigma^2} \right] \\ &= \left[y_M(k-1) \quad \dots \quad y_M(k-n) \quad -u(k-1) \quad 0 \right]. \end{aligned} \tag{14}$$

$$\frac{\partial \sigma^2}{\partial \beta} = \left[\frac{\partial \sigma^2}{\partial f} \;\vdots\; \frac{\partial \sigma^2}{\partial b_1} \;\vdots\; \frac{\partial \sigma^2}{\partial \sigma^2} \right] = \left[0 \quad \dots \quad 0 \quad 0 \quad 1 \right]. \tag{15}$$

From relations (13), (14) and (15), it follows that

$$\overline{M} = \frac{1}{\sigma^2} E \left[\begin{array}{ccc|c|c} y_M(k-1)^2 & \dots & y_M(k-1) \cdot y_M(k-n) & -u(k-1) y_M(k-1) & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ y_M(k-n) \cdot y_M(k-1) & \dots & y_M(k-n)^2 & -u(k-1) y_M(k-n) & 0 \\ \hline -u(k-1) y_M(k-1) & \dots & -u(k-1) y_M(k-n) & u(k-1)^2 & 0 \\ \hline 0 & \dots & 0 & 0 & 1/2\sigma^2 \end{array} \right] . \tag{16}$$

The relation (16) can be written in more compact form

$$\overline{M} = \frac{1}{\sigma^2} \left[\begin{array}{c|c|c} A & B & 0 \\ \hline B^T & C & 0 \\ \hline 0 & 0 & 1/2\sigma^2 \end{array} \right] . \tag{17}$$

The following task is to determine elements of the matrix $\overline{M}$ in the relation (17).

<u>Step 1</u>(Determining the matrix A)

Let us define

$$E\{ y_M(k-i) y_M(k-j) \} \triangleq \rho_{|i-j|} . \tag{18}$$

Based on the relation (18) the matrix A can be presented in the following form

$$A = \begin{bmatrix} \rho_0 & \rho_1 & \dots & \rho_{n-1} \\ \rho_1 & \rho_0 & \dots & \rho_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n-1} & \rho_{n-2} & \dots & \rho_0 \end{bmatrix} . \tag{19}$$

<u>Step 2</u> (Determining the matrix B)

Using an auxiliary model

$$y_M(k) = \frac{B(q^{-1})}{F(q^{-1})}u(k), \quad (20)$$

relation (1) can be expressed as

$$y(k) = -f_1 y_M(k-1) - \ldots - f_n y_M(k-n) + b_1 u(k-1) + e(k) \quad (21)$$

After multiplying the relation (21) with

$$y_M(k-i),\ i = 1,\ldots,n \quad (22)$$

and applying the mathematical expectation operator to the relations (22), one can obtain

$$V = -Af + Bb_1, \quad (23)$$

where $f^T = [f_1 \quad \ldots \quad f_n]$, $V = [\rho_1 \quad \rho_2 \quad \ldots \quad \rho_n]^T$.
Finally, it follows from the relation (23) that

$$B = \frac{1}{b_1}\left(Af + V\right). \quad (24)$$

Step 3 (Determining the scalar C)
From the relation (20) one can get the input signal $u(\cdot)$

$$u(k-1) = \frac{1}{b_1}\left[y_M(k) + f_1 y_M(k-1) + \ldots + f_n y_M(k-n)\right]. \quad (25)$$

From the relation (25), after some calculations one can obtain expression for the scalar C

$$C = E\left\{u(k-1)^2\right\} = \frac{1}{b_1^2}\left(\rho_0 + 2fV^T + f^T Af\right). \quad (26)$$

Since, all elements of the matrix $\overline{M}$ are now known (relation (17)), one can obtain

$$\det \overline{M} = \frac{1}{2\sigma^4}\det\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} = \frac{1}{2\sigma^4}\det A \cdot \det\left(C - B^T A^{-1} B\right). \quad (27)$$

It follows, from the last relation, that

$$-\log\det \overline{M} = \log 2\sigma^4 - \log\left(\det A\right) - \log\det\left(C - B^T A^{-1} B\right). \quad (28)$$

From relations (26) - (28), it follows that

$$\begin{aligned} -\log\det \overline{M} &= \log 2\sigma^4 - \log\left(\det A\right) - \\ &- \log\left(\rho_0 - V^T A^{-1} V\right) + \log b_1^2. \end{aligned} \quad (29)$$

If $\rho_i = 0,\ i > 0$, we finally get

$$\min\left\{-\log\det \overline{M}\right\} = \log\frac{2\sigma^4 b_1^2}{W^{n+1}} \quad (30)$$

To achieve that $\rho_i = 0,\ i > 0$, it is necessary that the following condition be fulfilled

i) $\{y(k)\}$ is a uncorrelated sequence.

The condition is fulfilled if the input signal is chosen in the following form

$$u(k) = \frac{1}{b_1}\left[f_1 y_M(k) + \ldots + f_n y_M(k-n+1)\right] + \eta(k), \quad (31)$$

where $\eta(k)$ is a reference signal that represents white noise with variance W/b_1^2.

The relation (31) represents the minimum variance controller for the model (1). This theorem is proved. ■

The main result of the **Theorem 1** is the fact that the synthesis of input signals, which induce the predetermined output power, is performed by using the feedback. In this case the feedback consists of a minimum variance controller and stochastic reference. This result also shows that the optimal input design requires knowledge of the true system parameters. In practical conditions, however, such a requirement is contradictory because the optimal input design is performed in order to speed up the identification process (determination of model parameters). However, the result of the theorem has the following important implications

i. what, at the best case, can be achieved by using the optimal input signal,

ii. application of the adaptation philosophy for creating a practical algorithm for the optimal input design.

III. ADAPTIVE INPUT DESIGN

In practical conditions, the true parameters of the system (1) are unknown. Primarily, let us observe that from relations (1) and (31), it follows that

$$y_M(k+1) = b_1\eta(k). \quad (32)$$

In this paper, the following two-stage procedure will be used

A. By using PRBS signal as input, through N_{init} iterations, the initial model of the process is determined,

B. After that, adaptation is applied for the controller defined in **Theorem 1**.

IV. SIMULATION RESULTS

The proposed two-stage identification algorithm has been tested on the following OE model

$$y(k) = \frac{0.5q^{-1}}{1 - 1.5q^{-1} + 0.7q^{-2}}u(k) + e(k). \quad (33)$$

The system identification example, is based on measured 1000 input-output data points obtained during the simulations. The measurement noise $e(k)$ has Gaussian distribution, $p_N(e) \sim N(0, 0.2)$.

To demonstrate the superiority of the proposed two-stage identification algorithm, a comparison with open loop identification algorithm, when input signal is PRBS signal, is made.

The simulation results are compared in terms of mean square error (MSE), defined by

$$MSE = \ln\left(\left\|\hat{\theta}(k) - \theta(k)\right\|^2\right) \quad (34)$$

Figs. 1 to 3 shows parameter estimates, and mean square errors in the case where the output variance cannot be greater then $W = 0.5$.

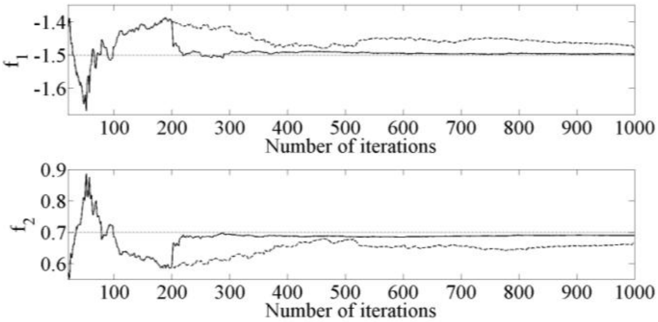

Fig.1. Estimates of parameters f_1 and f_2 (solid line: Parameter estimates using adaptive excitation signal, dash-dot: Parameter estimates using PRBS, dotted line: True parameter values)

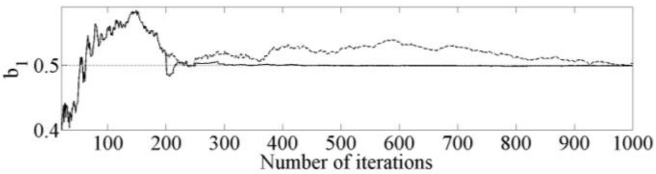

Fig.2. Estimate of parameter b_1 (solid line: Parameter estimate using adaptive excitation signal, dash-dot: Parameter estimate using PRBS, dotted line: True parameter value)

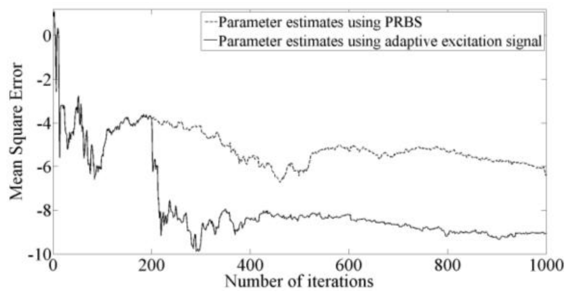

Fig.3. Mean square error

Based on these figures, it can be concluded that experiment design increases the convergence speed of parameters to true values, keeping the given output variance W.

Also, through intensive simulations, it is shown that relaxation of constraints on output reduces the error of estimated parameters. This conclusion is clearly demonstrated in the following figure.

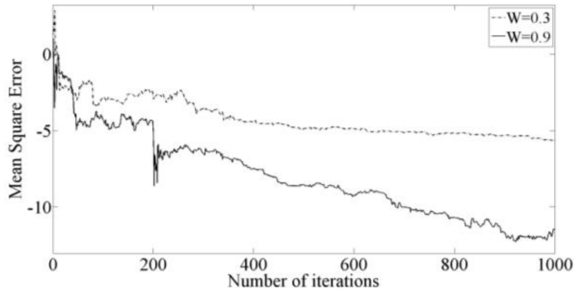

Fig.4. Mean square error for different values of output variance constraint W

V. CONCLUSION

In this paper the optimal input design for the identification of OE model, in the case of constrained output variance, is considered. It is shown, in this case, that the optimal experiment is obtained by using minimum variance controller and stochastic reference signal. The adaptive two-stage procedure for generating the input signal has been proposed. The initial model of the process is firstly obtained using PRBS input signal, after which the minimum variance controller is applied to generate the input signal. Simulation results show the superiority of identification using adaptive methodology for generating the input signal in the relation to the identification of system parameters in the open loop using PRBS input. It is also shown that with relaxation of output constraints, smaller error of estimated parameters is obtained.

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