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TARX Model for Pneumatic Cylinder and Identification

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Abstract ***- Pneumatic cylinder is nonlinear system with some uncertain parameters. Owing that fact we introduce next assumptions***

a) the nonlinear model of the pneumatic cylinder can be approximated with time-varying ARX model (TARX model)

b) owing the influence of the combination of heat coefficient, unknown discharge coefficient and change of temperature, we suppose that parameters of the pneumatic cylinder are random (stochastic parameters)

c) observations have a Gaussian distribution

Due to the abovementioned reasons, it is assumed that the pneumatic cylinder model is a linear stochastic model with variable parameters. We will use a recursive algorithm with forgetting factor. Such algorithms overcome shortcoming which is inherent to Kalman filter approach. That is a priori knowledge of stochastic disturbance variance and covariance matrix of the parameters noise.

Key words: ***System identification, pneumatic cylinder, stochastic model, time-variant parameters, Gaussian distribution***

I. INTRODUCTION

Since pneumatically driven systems have a lot of distinct characteristics of energy-saving, cleanliness, simple structure and operation, high efficiency and they are suitable for working in a harsh environment, they have been extensively used for many years in robot driven systems and industrial automation [1].

However, the problem with complex nonlinear models, such as the pneumatic servo cylinder, is that it is difficult to choose the large number of physical parameters involved in the model. Although a lot of parameter values are known a priori with reasonable accuracy, a large number of parameters are only known within a certain range, and some are even completely unknown. This may be due to manufacturing tolerances, or due to the fact that manufacturers do not provide parameter values because they consider them as proprietary information.

Furthermore, it is extremely difficult to accurately acquire some system parameters (such as component dimensions, internal leakage coefficients, static and dynamic friction forces, etc.) because the mentioned parameters cannot be directly measured or calculated. This causes a great difficulty in system modelling and control.

The consequence of these problems is that the theoretical model is often not useful for quantitative analysis of the pneumatic servo-system behaviour.

The purpose of this paper is to use the theory and findings of system identification to obtain a mathematical model, so that the controller can be designed on the basis of the model.

Östring et al. [2] identified the behaviour of an industrial robot in order to model its mechanical flexibilities, while Johansson et al. [3] used a state-space model to identify the robot manipulator dynamics. Assuming most parameters in pneumatic servo system do not change during operation, Shih and Tseng [4] performed the identification offline and adjusted servo-control before the operation accordingly. Furthermore, they investigated the impact of different parameters (sampling time, order model, different supply pressures, etc.) in the identification process.

The mentioned references consider the linear models of the pneumatic cylinder which are ad hoc adopted, without considering justification of such an approach. It is necessary to notice the following details:

- The pneumatic cylinder is a nonlinear system (presence of friction force)
- There is a significant influence of the combination of the heat coefficient, unknown discharge coefficient and change of temperature on the behaviour of the pneumatic cylinder [5]. The mentioned influences cannot be easily included in the cylinder model and have random character.

On the other hand, recent research has shown that the nonlinear model of the system can be approximated by a linear system with time-variant parameters [6]. In this paper it is assumed that the nonlinear model of the pneumatic cylinder can be approximated with time-varying ARX model (TARX model). We will use a recursive algorithm with forgetting factor. Such algorithms overcome shortcoming which is inherent to Kalman filter approach. That is a priori knowledge of stochastic disturbance variance and covariance matrix of the parameters noise.

II. MODELLING OF A PNEUMATIC CYLINDER

The system under consideration consists of an electro-pneumatic position control servo drive and a pneumatic cylinder as shown in Fig. 1.

Applying Newton's second law to the forces on the piston, the resulting force equation is

$$A_a P_a - A_b P_b = m\ddot{y} + \beta_e \dot{y} + F_f(\dot{y}) + k_e y + F_{ext} \quad (1)$$

where P_a and P_b denote the pressure of the chamber a and b, respectively, m denotes the total mass of the piston and the load referred to the piston, y is the piston displacement, β_e is the nonlinear viscous friction coefficient, k_e denotes the load spring gradient; and F_{ext} denotes the load force disturbance on the piston. The term F_f in equation (1) describes the summing nonlinear effects of static and Coulomb friction forces of the system.

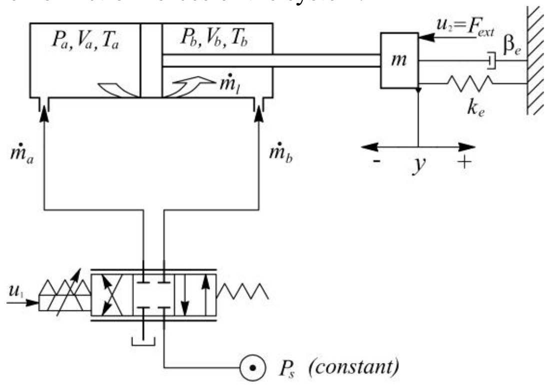

Fig. 1 Schematic representation of the pneumatic servo cylinder

Pressure dynamics in the chambers, for $i = a, b$, is given by [5]:

$$\frac{dP_i}{dt} = -\alpha(t) g_i(P_i, y, \dot{y}) + \beta(t) h_i(t, P_i, y) u_1 \quad (2)$$

in which:

$$g_i(P_i, y, \dot{y}) = \frac{P_i \dot{V}_i(\dot{y})}{V_i(y)} \quad (3)$$

and

$$h_i(t, P_i, y) = \frac{\sqrt{RT_S}}{V_i(y)} W f(P_i) \operatorname{sgn}(u_1) \quad (4)$$

where

R is the universal gas constant, W is a spool constant, T_S is ambient absolute temperature.

Uncertain heat coefficient $\alpha(t)$ depends on the actual heat transfer occurring during the process. As it can be seen from [5], $\alpha(t)$ takes values between 1 and 1.3997.

Uncertain bound parameter $\beta(t)$, which takes values between 0.075 and 1.3297 (see [5]), is used to characterize the combination of the heat coefficient $\alpha(t)$, the unknown valve discharge coefficient $C_d(t)$ and the variation of the temperature $\tau(t)$. Thus, $\beta(t)$ is generally expressed by:

$$\beta(t) = \alpha(t) C_d(t) \sqrt{\tau(t)} \quad (5)$$

Since uncertain heat coefficient $\alpha(t)$ and uncertain bound parameter $\beta(t)$, are only known in the certain range, it can be considered that their changes have random character. Since mentioned uncertain coefficients are involved (directly or indirectly) in the system parameters, previous analysis has justified the assumption that the system is considered as stochastic.

III. STOCHASTIC MODEL OF THE PNEUMATIC CYLINDER AND PARAMETER ESTIMATION

The previous section shows that the mathematical model of the pneumatic cylinder in nonlinear and that it is not possible to include a large number of important details in the model. The natural way of solving this problem is to apply the identification theory. In that case the following problems arise:

- Type of the model (linear, nonlinear, deterministic, stochastic)
- Nature of disturbance (uniformly constrained, stochastic)

The following facts have conditioned the choice of the model:

1) Recent research has shown that the nonlinear model of the system can be correctly approximated by a system with time variant parameters [6]
2) A more detailed analysis of the pneumatic cylinder model described in the previous section shows that the combination of heat coefficient, unknown discharge coefficient and change of temperature influences the model of cylinder [5]. Those influences are random and therefore it is assumed that the parameters of the pneumatic cylinder are random.
3) observations have a Gaussian distribution

The mentioned reasons lead to the assumption that the model of the pneumatic cylinder is a stochastic linear model with time variant parameters.

Time-varying ARX model (TARX model) will be used as a model which describes the dynamics of the pneumatic cylinder.

$$A\left(q^{-1}, k\right) y(k) = B\left(q^{-1}, h\right) u(k) + e(k) \quad (6)$$

where

$$A\left(q^{-1}, k\right) = 1 + a_1(k) q^{-1} + \ldots + a_n(k) q^{-n}$$

$$B\left(q^{-1}, k\right) = b_1(k) q^{-1} + \ldots + b_m(k) q^{-m}$$

It is assumed that measurement noise $e(k)$ has Gaussian distribution.

Taking into account the physics of the problem, it will be assumed that the change of the parameters has the form of random walk

$$\theta(k+1) = \theta(k) + \omega(k) \quad (7)$$

where the stochastic process $\omega(k)$ is Gaussian noise.

Let us introduce the following parameter vector and measurement vector

$$\theta(k) = [a_1(k), \ldots, a_n(k), b_1(k), \ldots, b_m(k)]^T \quad (8)$$

$$\varphi(k) = [-y(k-1), \ldots, -y(k-n), u(k-1), \ldots, u(k-m)]^T \quad (9)$$

In that case the dynamics of the system with disturbance is given by the following relation

$$y(k) = \theta^T(k)\varphi_0(k) + e(k) \quad (10)$$

Let's introduce the following loss function to be minimized, [7-8].

$$V_t(\theta) = \frac{1}{2}\sum_{s=1}^{t} \lambda^{t-s}\varepsilon^2(s) \quad (11)$$

where $\varepsilon(s)$ is prediction error.

The prediction error estimate $\hat{\theta}(t)$ is then defined by minimization of $V_t(\theta)$:

$$\hat{\theta}(t) = \arg\min_{\theta} V_t(\theta) \quad (12)$$

It can be shown that recursive prediction error identification algorithm with forgetting factor has the form

$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\phi(k)\left[y(k) - \hat{\theta}^T(k-1)\phi(k)\right]$$
$$P(k) = \frac{1}{\lambda(k)}\left[P(k-1) - \frac{P(k-1)\phi(k)\phi^T(k)P(n-1)}{\lambda(k) + \phi^T(k)P(n-1)\phi(n)}\right]$$
$$\hat{\theta}(k) = 0, \quad P(0) = 10^4 I \quad (13)$$

where $\lambda(k)$ represents forgetting factor.

When the properties of the system may change (slowly) with time, the recursive identification algorithm should be able to track the time varying parameters describing such a system.

The loss function contains forgetting factor λ, where

$$0 < \lambda \le 1 \quad (14)$$

Usually, forgetting factor is a number somewhat less than 1 (for example $\lambda = 0.99$ or $\lambda = 0.95$).

This means that with increasing t the measurements obtained previously are discounted. The forgetting factor gives a larger weight to more recent data in order to cope with the system dynamics. Generally, the faster a system parameter varies with time, the smaller the forgetting factor to guarantee a good tracking performance.

IV. EXPERIMENTAL RESULTS

To demonstrate the performance of proposed identification algorithm for a pneumatic cylinder, we used the real pneumatic cylinder, which is located in faculty laboratory.

Pneumatic cylinder is approximated with next TARX model

$$A(q^{-1},k)y(k) = B(q^{-1},h)u(k) + e(k) \quad (15)$$

where

$$A(q^{-1},k) = 1 + a_1(k)q^{-1} + a_2(k)q^{-2}$$
$$B(q^{-1},k) = b_1(k)q^{-1} + b_2(k)q^{-2}$$

The system identification example, is based on measured 1000 input-output data points obtained during the experiments. PRBS voltage signal is used for input signal, while position of the pneumatic cylinder is observed for output signal.

Fig.2. and Fig.3. shows system input and corresponding system output.

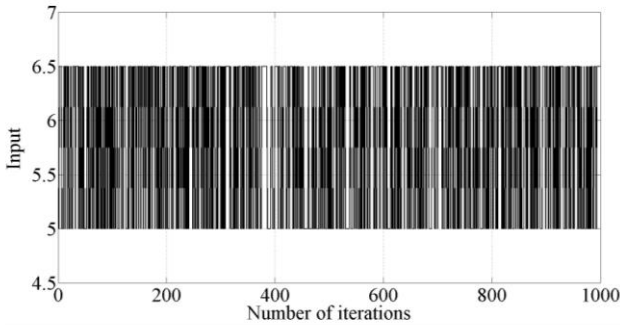

Fig.2. PRBS excitation signal

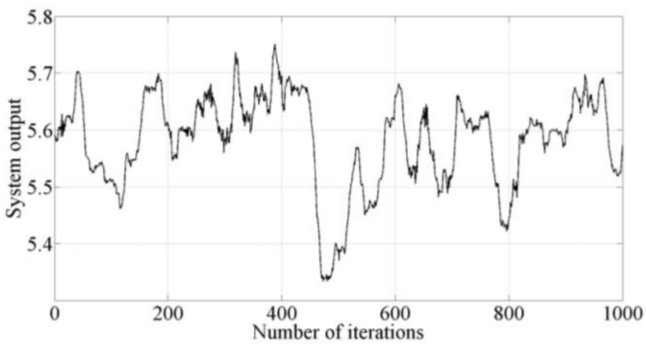

Fig.3. Measured system output using PRBS excitation signal

Parameter estimation was conducted for different values of forgetting factor. It has been shown that for a given pneumatic cylinder, under given conditions, the most accurate parameter estimates are obtained for $\lambda = 0.99$. Figs. 4 and 5 shows parameter estimates of the pneumatic cylinder, for the case when $\lambda = 0.99$.

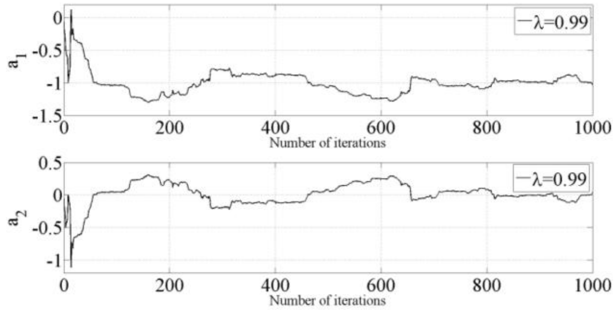

Fig.4. Estimates of parameters a_1 and a_2

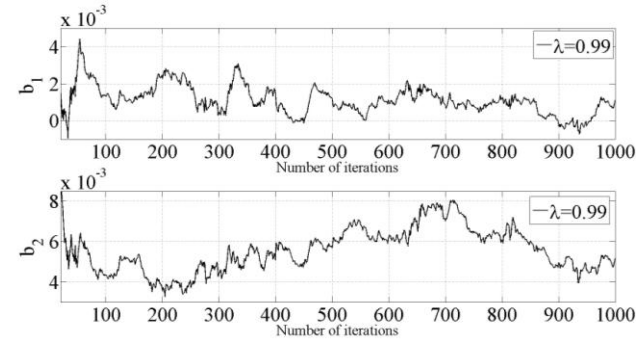

Fig.5. Estimates of parameters b_1 and b_2

The model validation is shown in the following figure, in which is presented a comparative view of the system and model outputs.

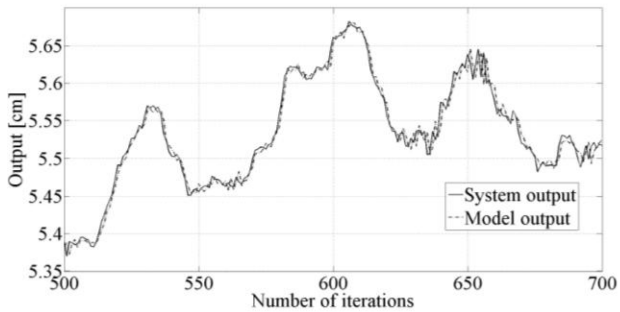

Fig.6. Model validation, with the forgetting factor $\lambda = 0.99$

Figs. 7-9 shows parameter estimates of the pneumatic cylinder and model validation for the case when $\lambda = 0.9$.

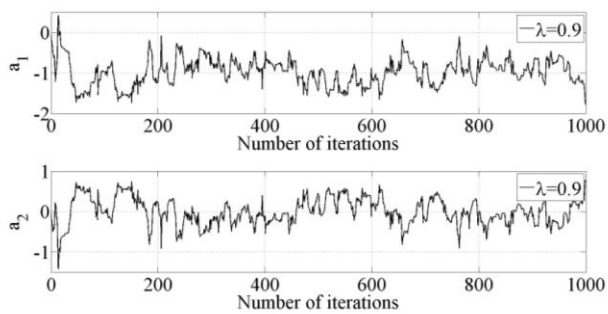

Fig.7. Estimates of parameters a_1 and a_2

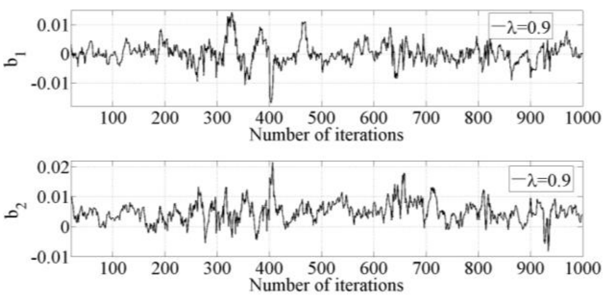

Fig.8. Estimates of parameters b_1 and b_2

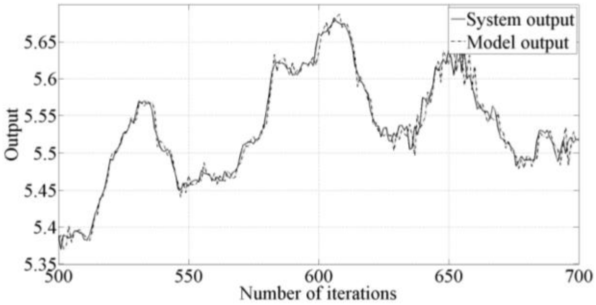

Fig.9. Model validation, with the forgetting factor $\lambda = 0.9$

V. CONCLUSION

The paper considers a stochastic mathematical model of the pneumatic cylinder and parameter estimation. Owing the influence of the combination of heat coefficient, unknown discharge coefficient and change of temperature, we suppose that parameters of the pneumatic cylinder are random. The nonlinear model of the pneumatic cylinder was approximated with time-varying ARX model.

For parameter estimation of this time-varying model, a recursive algorithm with forgetting factor is used. The good behavior of proposed TARX model and identification procedure for the pneumatic cylinder is illustrated on the example of the real system.

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