Optimization of the Box Section of the Main Girders of the Bridge Crane for the Case of Placing the Rail in the Middle of the Top Flange

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This paper considers the problem of optimization of the box section of the main girder of the bridge crane for the case of placing the rail in the middle of the top flange. Reduction of the girder mass is set as the objective function. The method of Lagrange multiplier was used as the methodology for approximate determination of optimum dependences of geometrical parameters of the box section. The criterion of strength were applied as the constraint function. The analysis of the optimization results and the solutions was the basis for recommendations which are significant for designers during construction of cranes.

Keywords: Box section, Bridge crane, Lagrange multiplier, Optimization, Strength

1. INTRODUCTION

The main task in the process of designing the carrying structure of the bridge crane is determination of optimum dimensions of the main girder box section. The mass of the main girder has the largest share in the total mass of the bridge crane, so it is very important to perform its optimization in order to reduce the total costs of manufacturing the whole carrying structure. That is the reason why the selection of the optimum shape and geometrical parameters which influence the reduction of mass and costs of manufacturing is the subject of research of a lot of authors ([2], [3], [5], [7], [8], [9], [10], [11], [12], [14], [15], [16], [17] and [18]).

Most authors set permissible stress or two constraint functions: permissible stress and permissible deflection as the constraint function.

The analysis of cost structure for manufacturing metal structures made in [2], showed that the participation of material costs in the total costs is the largest (30-73) %, and that the other costs are lower**.**

Having in mind all the above mentioned results and conclusions, the aim of this paper is to define optimum values of geometrical parameters of the box girder cross-section that will lead to the reduction of its mass.

2. MATEMATHICAL FORMULATION OF THE OPTIMIZATION PROBLEM

The task of optimization is to define geometrical parameters of the cross section of the girder as well as their mutual relations, which result in its minimum area.

Minimization of the mass corresponds to minimization of the volume, i.e. the area of the cross section of the girder, where the given boundary conditions must be satisfied. The area of the cross section primarily depends on: height and width of the girder, thickness of plates and their mutual relations.

The optimization problem defined in this way can be given the following general mathematical formulation:

minimize $f(\boldsymbol{X})$ subject to $g(\boldsymbol{X}) \le 0$.

where:

$f(\boldsymbol{X})$ the objective function,

$g(\boldsymbol{X}) \le 0$ the constraint function,

$\boldsymbol{X} = \{x_1, ..., x_D\}^T$ represents the design vector made of D design variables. Design variables are the values that should be defined during the optimization procedure.

In this paper optimization for the criterion of strenght:

$$g = \sigma_{\max} - \sigma_k \le 0 \qquad (1)$$

where:

$\sigma_{\max}$ - the calculation stress,

σ_k - the permissible stress.

The Lagrange function is defined in the following way:

$$\Phi = A + \lambda \cdot g \qquad (2)$$

$$\frac{\partial \Phi}{\partial b} = 0; \frac{\partial A}{\partial b} + \lambda \cdot \frac{\partial g}{\partial b} = 0 \qquad (3)$$

$$\frac{\partial \Phi}{\partial h} = 0; \frac{\partial A}{\partial h} + \lambda \cdot \frac{\partial g}{\partial h} = 0 \qquad (4)$$

$$\frac{\partial \Phi}{\partial \lambda} = 0; \Rightarrow g = 0 \qquad (5)$$

3. OBJECTIVE AND CONSTRAINT FUNCTIONS

3.1. Objective function

The objective function is represented by the area of the cross section of the box girder (Fig. 1). The paper treats two optimization parameters *(h, b).* The wall thicknesses t_1 and t_2 are not treated as optimization parameters for the purpose of simplification of the procedure. Their values were adopted in accordance with the recommendations of crane manufacturers [6].

$$A(h,b) = f(h,b) = \frac{2}{s} \cdot (e \cdot b \cdot h + h^2) \qquad (6)$$

where:

$e = \dfrac{t_1}{t_2}$ - the ratio between thicknesses of plates at the flange and at the web,

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$s=\dfrac{h}{t_2}$ - the ratio between the height and thickness of the plate at the web,

$k=\dfrac{h}{b}$ - the ratio between the height and width of the girder.

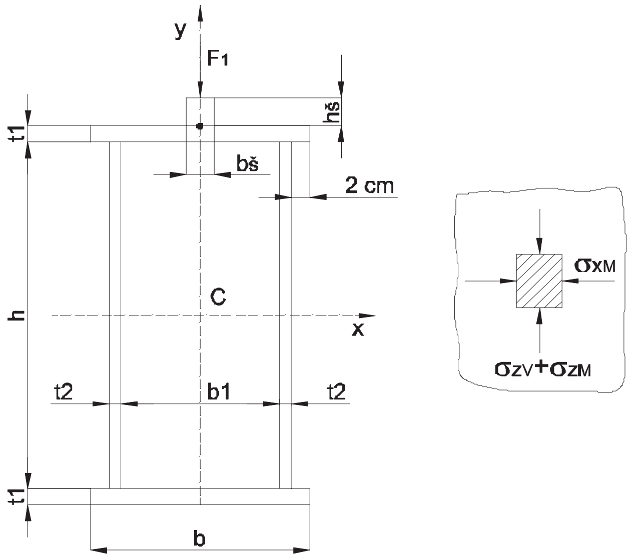

Figure 1: The box section of the main girder of the bridge crane

To know the optimal value of the ratio between the height and width of the girder *k* is of particular significance for the designer, especially in the initial design phase.

The expressions for the moments of inertia around the *x* and *y* axes are:

$$I_x=\frac{1}{6}\cdot\frac{h^4}{s}+\frac{1}{2}\cdot e\cdot b\cdot\frac{(s+e)^2}{s^3}\cdot h^3 \qquad (7)$$

$$I_y=\frac{1}{6}\cdot e\cdot\frac{h}{s}\cdot b^3+\frac{1}{2}\cdot\frac{h^2}{s}\cdot\frac{(f\cdot b\cdot s+h)^2}{s^2} \qquad (8)$$

where:

$f=\dfrac{b_1}{b}<1$ - the ratio between the distance of web plates and the width of flange plates of the box girder.

Since the expressions for the moments of inertia *(I_x, I_y)* and the section moduli *(W_x, W_y)* are complex, it is common to take approximate values of expressions by neglecting the members of the lower order ([8], [16] and [18]):

$$W_x=\alpha_x\cdot h\cdot A \qquad (9)$$

$$W_y=\alpha_y\cdot b\cdot A \qquad (10)$$

where:

α_x, α_y - the dimensionless coefficient of the resistance moment of inertia for the *x* and *y* – axes.

The coefficient α_x are obtained from the conditions of equality of the equation (7) and the expression (9) and relation between moment of inertia and section moduli:

$$\alpha_x=\frac{k+3\cdot e}{6\cdot(e+k)} \qquad (11)$$

By repeating the procedure for the section moduli for the *y* – axis, the following values of coefficient are obtained:

$$\alpha_y=\frac{3\cdot k\cdot f^2+e}{6\cdot(e+k)} \qquad (12)$$

3.2. Constraint function

The maximum equivalent stress which occurs in the main girder of the bridge crane for the case of placing the rail in the middle of the top flange is under the rail (Fig. 1). The constraint function according to this criterion is:

$$\sigma_{\max}=\sqrt{(\sigma_{zV}+\sigma_{zM})^2+\sigma_{xM}{}^2-(\sigma_{zV}+\sigma_{zM})\cdot\sigma_{xM}}\le\sigma_k \qquad (13)$$

Partial conditions must also be fulfilled:

$$\sigma_z=\sigma_{zV}+\sigma_{zM}\le\sigma_k \qquad (14)$$

$$\sigma_{xM}\le\sigma_k \qquad (15)$$

where:

$$\sigma_k=\frac{f_y}{\nu_1} \qquad (16)$$

where:

f_y - the minimum yield stress of the plate material,

ν_1 - the factored load coefficient for load case 1,

σ_{zM} - the normal stress due to local bending in the longitudinal direction of the girder,

σ_{xM} - the normal stress due to transverse bending of the web plate.

$$\sigma_{zV}=\frac{M_{cv}+c\cdot A}{\alpha_x\cdot h\cdot A} \qquad (17)$$

where:

M_{cv} - the bending moment in the vertical plane,

c – the coefficient of influence of the dead weight of the girder on the bending moment.

Local bending of the plate and occurrence of a biaxial state of normal stresses arise due to the contact between the rail and the web plate during passage of the trolley.

The normal stress due to local bending in the longitudinal direction of the girder, which is obtained on the basis of equality between rail deformations and the web plate is:

$$\sigma_{zM}=\frac{6\cdot K_3\cdot N}{t_1^2} \qquad (18)$$

The normal stress due to transverse bending of the web plate is:

$$\sigma_{xM}=\frac{6\cdot K_2\cdot N}{t_1^2} \qquad (19)$$

where:

N - the part of the maximum force of wheel pressure which, due to rail rigidity, goes for the plate and depends on the ratio a_1/b_1 (Fig. 2),

K_2, K_3 - the dimensionless coefficients,

a_1 - the distance between short vertical stiffeners.

At the very beginning it is necessary to analyze certain ratios of geometrical parameters.

$$a_1=\frac{a}{3}=\frac{2\cdot h}{3} \qquad (20)$$

The following ratio is observed:

$$\frac{a_1}{b_1}=\frac{2\cdot h}{3\cdot b_1}=\frac{2\cdot k}{3\cdot f} \qquad (21)$$

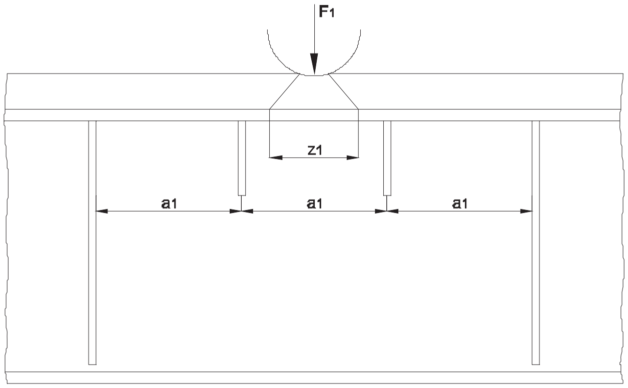

Figure 2: Action of the wheel on the rail of the main girder of the bridge crane

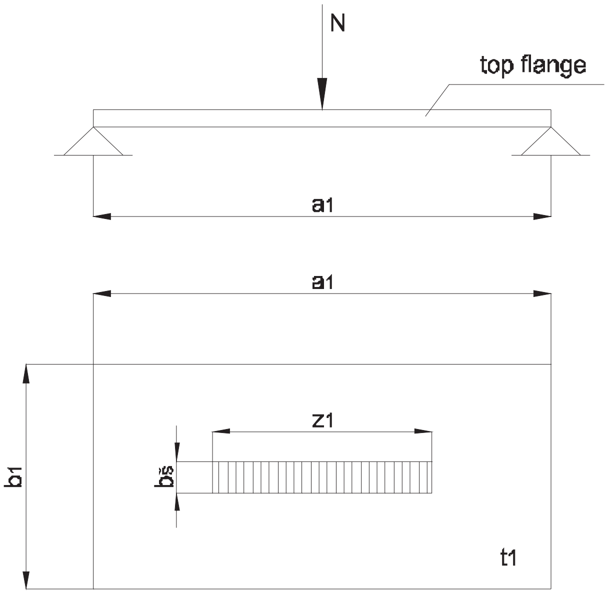

Figure 3: The zone of distribution of a part of the maximum force of wheel pressure

As in this case K=2÷3, f<1, it follows that this ratio is higher than 1, i.e. it is obtained that $a_1 > b_1$, i.e. the force N is taken according to the formula (22).

$$N = \frac{\gamma \cdot F_1}{1 + \frac{96 \cdot b_1^2 \cdot I_{\check{s}1} \cdot K_1}{a_1^3 \cdot t_1^3} \cdot \frac{1}{c_o}} \tag{22}$$

where:

γ - the coefficient of the classification class of the bridge crane [1],

F_1 - the maximum force of pressure of the wheel on the main girder of the bridge crane ,

$c_o \approx 1$ - the coefficient which depends on the manner of connecting the rail to the flange,

$I_{\check{s}1}$ - the moment of inertia of the rail for its own axis,

K_1 - the coefficient which depends on the ratio a_1 / b_1.

The members of the formula (22) will now be analyzed.

It is seen that this ratio depends both on k and on f. As the limit for the expected values of k is known, it is necessary to consider the values taken for the parameter f.

$$f = 1 - \frac{2 \cdot k \cdot b + 4 \cdot s}{s \cdot b} \tag{23}$$

As f is treated as constant, it is necessary to adopt a mean value of it.

As f depends on the slenderness s, mean values will be adopted, so that s = 210 is taken for S235 , s = 170 is taken for S355 , and a mean value will be taken for k=2.5.

The following value of the parameter f is adopted for the expected range of values of the width b.

$f_{sr} = 0.87$ - for S355 , $f_{sr} = 0.88$ - for S235 . It is seen that these values are approximate.

Now the ratio a_1 / b_1 should be analyzed.

$$\frac{a_1}{b_1} = \frac{2 \cdot k}{3 \cdot f} = 1.53 \div 2.3 \tag{24}$$

For this interval of ratio values, the approximate value of the coefficient K_1 can be adopted, and its value is $K_1 \approx 0.176$, where deviations of this value with the upper and lower limits are smaller than 5%, [13].

The same will now be done for the coefficients K_2 and K_3. Their dependence is little more complex in relation to the previous coefficient. These coefficients depend both on the ratio a_1 / b_1, and the ratios $b_{\check{s}} / b_1$ and z_1 / b_1, where:

$$z_1 = 2 \cdot h_{\check{s}} + 5\,cm \tag{25}$$

where:

z_1 - the width of the zone of action of the wheel on the rail (Fig. 3),

$h_{\check{s}}$ - the height of the rail,

$b_{\check{s}}$ - the width of the rail.

In order to treat the coefficients K_2 and K_3 as constant and not variable values (which would considerably complicate the model), it is adopted that the rail is of a square cross section, where $h_{\check{s}} = b_{\check{s}}$ and it is adopted that $b_{\check{s}} \approx b/8$, as the carrying capacities higher than Q=16t are not observed.

The ratio $b_{\check{s}} / b_1$ is now observed.

$$\frac{b_{\check{s}}}{b_1} = \frac{b}{8 \cdot f \cdot b} = \frac{1}{8 \cdot f} < 0{,}2 \tag{26}$$

$$\frac{z_1}{b_1} = \frac{2 \cdot h_{\check{s}} + 5}{b_1} = \frac{b_1 + 20 \cdot f}{4 \cdot f \cdot b_1} \tag{27}$$

Taking into account the spans, carrying capacities and classification classes that are analyzed in this case, the expected values for b_1 will be found in the following range $b_1 = 30 \div 45\,cm$. In that case, the ratio z_1 / b_1 is within the following limits: $z_1 / b_1 = 0.456 \div 0.400$, [13].

For this interval of ratio values, the approximate value of the coefficient K_2 can be adopted, and its value is $K_2 \approx 0.213$, where deviations of this value with the upper and lower limits are smaller than 5%. The situation is similar for the coefficient K_3 and its value is $K_3 \approx 0.149$, [13].

These deviations can be tolerated because exceeding of stresses up to 10% is tolerated, according to [4].

The members of the formula (22) are further observed.

It is now necessary to consider the expressions for stresses (18) and (19), which should be written as functions of h and b, i.e. the ratio $N/t_1^{\,2}$.

The following ratio is observed first:

$$Kn = \frac{27 \cdot K_1 \cdot f^2 \cdot s_1^{\,3}}{64^2 \cdot c_o} \tag{28}$$

where:

$$s_1 = \frac{s}{e} \tag{29}$$

By replacing in the expression (22), it is obtained that:

$$N_1 = \frac{N}{t_1^{\,2}} = \frac{F \cdot h^4}{h^6 + Kn \cdot b^6} \tag{30}$$

where:

$$F = \gamma \cdot F_1 \cdot s_1^{\,2} \tag{31}$$

The expressions (18) and (19) now become:

$$\sigma_{zM} = 6 \cdot K_3 \cdot N_1 = \frac{6 \cdot K_3 \cdot F \cdot h^4}{h^6 + Kn \cdot b^6} \tag{32}$$

$$\sigma_{xM} = 6 \cdot K_2 \cdot N_1 = \frac{6 \cdot K_2 \cdot F \cdot h^4}{h^6 + Kn \cdot b^6} \tag{33}$$

The constraint functions in this case have the following forms:

$$g_{11} = \sqrt{(\sigma_{zV} + \sigma_{zM})^2 + \sigma_{xM}^{\;2} - (\sigma_{zV} + \sigma_{zM})\sigma_{xM}} - \sigma_k \le 0 \tag{34}$$

$$g_{12} = \sigma_{zV} + \sigma_{zM} - \sigma_k \le 0 \tag{35}$$

$$g_{13} = \sigma_{xM} - \sigma_k \le 0 \tag{36}$$

By applying the well-known method of Lagrange multipliers to the expression (34), it is obtained that:

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g_{11}}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_{11}}{\partial b} \tag{37}$$

After rearrangement, it is obtained that:

$$\left[2(\sigma_{zV} + \sigma_{zM}) - \sigma_{xM}\right]\left[\left(\frac{\partial \sigma_{zV}}{\partial h} + \frac{\partial \sigma_{zM}}{\partial h}\right)\frac{\partial A}{\partial b} - \left(\frac{\partial \sigma_{zV}}{\partial b} + \frac{\partial \sigma_{zM}}{\partial b}\right)\frac{\partial A}{\partial h}\right] = \left[(\sigma_{zV} + \sigma_{zM}) - 2\sigma_{xM}\right]\left(\frac{\partial \sigma_{xM}}{\partial h}\frac{\partial A}{\partial b} - \frac{\partial \sigma_{xM}}{\partial b}\frac{\partial A}{\partial h}\right) \tag{38}$$

By applying the well-known method of Lagrange multipliers to the expression (35), it is obtained that:

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g_{12}}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_{12}}{\partial b} \tag{39}$$

i.e.:

$$\left(\frac{\partial \sigma_{zV}}{\partial h} + \frac{\partial \sigma_{zM}}{\partial h}\right) \cdot \frac{\partial A}{\partial b} = \left(\frac{\partial \sigma_{zV}}{\partial b} + \frac{\partial \sigma_{zM}}{\partial b}\right) \cdot \frac{\partial A}{\partial h} \tag{40}$$

By applying the well-known method of Lagrange multipliers to the expression (36), it is obtained that:

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g_{13}}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_{13}}{\partial b} \tag{41}$$

i.e.:

$$\frac{\partial \sigma_{xM}}{\partial h} \cdot \frac{\partial A}{\partial b} = \frac{\partial \sigma_{xM}}{\partial b} \cdot \frac{\partial A}{\partial h} \tag{42}$$

Based on the obtained expressions, it is seen that if the relations (40) and (42) are fulfilled simultaneously, then the equality (38) is also satisfied.

It is now necessary to solve the previous equations. If we start from the simplest equation (42), it is obtained that:

$$\frac{\partial N_1}{\partial h} \cdot \frac{\partial A}{\partial b} = \frac{\partial N_1}{\partial b} \cdot \frac{\partial A}{\partial h} \tag{43}$$

The partial derivatives have the following values:

$$\frac{\partial N_1}{\partial b} = \frac{\partial}{\partial b}\left(\frac{F \cdot h^4}{h^6 + Kn \cdot b^6}\right) = -6F\,\frac{Kn \cdot h^4 \cdot b^5}{(h^6 + Kn \cdot b^6)^2} \tag{44}$$

$$\frac{\partial N_1}{\partial h} = \frac{\partial}{\partial h}\left(\frac{F \cdot h^4}{h^6 + Kn \cdot b^6}\right) = -F\,\frac{2h^6 - 4Kn \cdot b^6}{(h^6 + Kn \cdot b^6)^2}\,h^3 \tag{45}$$

By replacing in (43) and using the known relation (46), [18]:

$$\frac{\partial A}{\partial b} \Big/ \frac{\partial A}{\partial h} = e \tag{46}$$

it is obtained that:

$$e \cdot k_{\sigma 3}^{\;6} - 3 \cdot Kn \cdot k_{\sigma 3} - 2 \cdot e \cdot Kn = 0 \tag{47}$$

Solving the equation (47) results in obtaining the optimum coefficient of the ratio between the height and width of the girder $k_{\sigma 3}$ in relation to the partial condition of the strength criterion.

By replacing this value in the constraint equation (36), the optimum height $h_{\sigma 3}$ in relation to the partial condition of the strength criterion is obtained:

$$h_{\sigma 3} = \sqrt{\frac{6 \cdot K_2 \cdot F \cdot k_{\sigma 3}^{\;6}}{\sigma_k \cdot (k_{\sigma 3}^{\;6} + Kn)}} \tag{48}$$

$$b_{\sigma 3} = \frac{h_{\sigma 3}}{k_{\sigma 3}} \tag{49}$$

Let us now observe the equation (40):

$$\frac{\partial \sigma_{zV}}{\partial b} \cdot \frac{\partial A}{\partial h} + \frac{\partial \sigma_{zM}}{\partial b} \cdot \frac{\partial A}{\partial h} = \frac{\partial \sigma_{zV}}{\partial h} \cdot \frac{\partial A}{\partial b} + \frac{\partial \sigma_{zM}}{\partial h} \cdot \frac{\partial A}{\partial b} \tag{50}$$

The partial derivatives have the following values:

$$\frac{\partial \sigma_{zV}}{\partial b} = -\frac{M_{cv}}{\alpha_x \cdot h \cdot A^2} \cdot \frac{\partial A}{\partial b} \tag{51}$$

$$\frac{\partial \sigma_{zV}}{\partial h} = -\frac{M_{cv}}{\alpha_x \cdot h \cdot A^2} \cdot \frac{\partial A}{\partial h} - \frac{M_{cv} + c \cdot A}{\alpha_x \cdot h^2 \cdot A} \tag{52}$$

$$\frac{\partial \sigma_{zM}}{\partial b} = -36K_3 \cdot F \cdot \frac{Kn \cdot h^4 \cdot b^5}{(h^6 + Kn \cdot b^6)^2} \tag{53}$$

$$\frac{\partial \sigma_{zM}}{\partial h} = -12K_3 \cdot F \cdot \frac{h^6 - 2 \cdot Kn \cdot b^6}{(h^6 + Kn \cdot b^6)^2}\,h^3 \tag{54}$$

Further rearrangement results in (55):

$$\frac{M_{cv} + c \cdot A}{\alpha_x \cdot h^2 \cdot A} \cdot \frac{\partial A}{\partial b} = \frac{12 \cdot K_3 \cdot F \cdot h^3}{(h^6 + Kn \cdot b^6)^2} \cdot \left[Kn \cdot h \cdot b^5 \cdot \frac{\partial A}{\partial h} - (h^6 - 2 \cdot Kn \cdot b^6) \cdot \frac{\partial A}{\partial b}\right] \tag{55}$$

The constraint equation (35) can be written in the form (56):

$$\frac{M_{cv} + c \cdot A}{\alpha_x \cdot h \cdot A} + \frac{6 \cdot K_3 \cdot F \cdot h^4}{h^6 + Kn \cdot b^6} = \sigma_k \tag{56}$$

Solving the system of nonlinear algebraic equations (56) and (55) results in obtaining the optimum height $h_{\sigma 2}$ and width $b_{\sigma 2}$ in relation to the partial condition of the strength criterion.

The principal equation (38) is now observed:

$$\frac{2(\sigma_{zV}+\sigma_{zM})-\sigma_{xM}}{(\sigma_{zV}+\sigma_{zM})-2\sigma_{xM}}\cdot\left[\left(\frac{\partial\sigma_{zV}}{\partial h}+\frac{\partial\sigma_{zM}}{\partial h}\right)\frac{\partial A}{\partial b}-\left(\frac{\partial\sigma_{zV}}{\partial b}+\frac{\partial\sigma_{zM}}{\partial b}\right)\frac{\partial A}{\partial h}\right]=\frac{\partial\sigma_{xM}}{\partial h}\frac{\partial A}{\partial b}-\frac{\partial\sigma_{xM}}{\partial b}\frac{\partial A}{\partial h} \tag{57}$$

The partial derivatives have the following values:

$$\frac{\partial\sigma_{zV}}{\partial b}=-\frac{M_{cv}}{\alpha_x\cdot h\cdot A^2}\cdot\frac{\partial A}{\partial b} \tag{58}$$

$$\frac{\partial\sigma_{zV}}{\partial h}=-\frac{M_{cv}}{\alpha_x\cdot h\cdot A^2}\cdot\frac{\partial A}{\partial h}-\frac{M_{cv}+c\cdot A}{\alpha_x\cdot h^2\cdot A} \tag{59}$$

$$\frac{\partial\sigma_{zM}}{\partial b}=-36\cdot K_3\cdot F\cdot\frac{Kn\cdot h^4\cdot b^5}{(h^6+Kn\cdot b^6)^2} \tag{60}$$

$$\frac{\partial\sigma_{zM}}{\partial h}=-12\cdot K_3\cdot F\cdot\frac{h^6-2\cdot Kn\cdot b^6}{(h^6+Kn\cdot b^6)^2}\cdot h^3 \tag{61}$$

$$\frac{\partial\sigma_{xM}}{\partial b}=-36\cdot K_2\cdot F\cdot\frac{Kn\cdot h^4\cdot b^5}{(h^6+Kn\cdot b^6)^2} \tag{62}$$

$$\frac{\partial\sigma_{xM}}{\partial h}=-12\cdot K_2\cdot F\cdot\frac{h^6-2\cdot Kn\cdot b^6}{(h^6+Kn\cdot b^6)^2}\cdot h^3 \tag{63}$$

By replacement in the previous expression, the following equation (64) is obtained:

$$\begin{aligned}&\frac{12F\cdot h^3}{(h^6+Kn\cdot b^6)^2}\cdot\left(3Kn\cdot h\cdot b^5\cdot\frac{\partial A}{\partial h}-(h^6-2Kn\cdot b^6)\cdot\frac{\partial A}{\partial b}\right)\cdot\left(K_4\frac{M_{cv}+c\cdot A}{\alpha_x\cdot h\cdot A}+2K_5\cdot\frac{6F\cdot h^4}{h^6+Kn\cdot b^6}\right)=\\&=\frac{1}{h}\cdot\frac{\partial A}{\partial b}\left[2\left(\frac{M_{cv}+c\cdot A}{\alpha_x\cdot h\cdot A}\right)^2+K_4\frac{M_{cv}+c\cdot A}{\alpha_x\cdot h\cdot A}\cdot\frac{6F\cdot h^4}{h^6+Kn\cdot b^6}\right]\end{aligned} \tag{64}$$

where:

$$K_4=2\cdot K_3-K_2 \tag{65}$$

$$K_5={K_2}^2-K_2\cdot K_3+{K_3}^2 \tag{66}$$

The constraint equation (34) can be written in the form (67):

$$\left(\frac{M_{cv}+c\cdot A}{\alpha_x\cdot h\cdot A}+\frac{6\cdot K_3\cdot F\cdot h^4}{h^6+Kn\cdot b^6}\right)^2+\left(\frac{6\cdot K_2\cdot F\cdot h^4}{h^6+Kn\cdot b^6}\right)^2-\left(\frac{M_{cv}+c\cdot A}{\alpha_x\cdot h\cdot A}+\frac{6\cdot K_3\cdot F\cdot h^4}{h^6+Kn\cdot b^6}\right)\cdot\left(\frac{6\cdot K_2\cdot F\cdot h^4}{h^6+Kn\cdot b^6}\right)={\sigma_k}^2 \tag{67}$$

Solving the system of nonlinear algebraic equations (57) and (67) results in obtaining the optimum height $h_{\sigma 1}$ and width $b_{\sigma 1}$ in relation to the partial condition of the strength criterion.

As it can be seen, there are three different solutions. In order to analyze which one is the most optimum one, it is necessary to have graphical representation of the obtained solutions in the same plane.

4. NUMERICAL REPRESENTATION OF THE RESULTS OBTAINED

The functions (14), (15) and (16) depending on *h* and *k*, read:

$$\begin{aligned}&f_{11}(h,k)=4{\alpha_x}^2{\sigma_{dop}}^2(e+k)^2(k^6+Kn)^2h^6-4c^2(e+k)^2(k^6+Kn)^2h^4-24\alpha_xK_4cF(e+k)^2k^6(k^6+Kn)h^3-\\&-4(e+k)k\left[sM_{cv}c(k^6+Kn)^2+36{\alpha_x}^2K_5F^2(e+k)k^{11}\right]h^2-12\alpha_xK_4sM_{cv}F(e+k)k^7(k^6+Kn)h-s^2{M_{cv}}^2k^2(k^6+Kn)^2\geq 0\end{aligned} \tag{68}$$

$$f_{12}(h,k)=2\alpha_x(e+k)(k^6+Kn)\sigma_{dop}h^3-2c(e+k)(k^6+Kn)h^2-12\alpha_xK_3F(e+k)k^6h-skM_{cv}(k^6+Kn)\geq 0 \tag{69}$$

$$f_{13}(h,k)=\sigma_{dop}(k^6+Kn)h^2-6K_2Fk^6\geq 0 \tag{70}$$

These functions will be presented in the *k*-*h* plane, where it is necessary to fulfil certain boundary conditions:

$$k\geq\frac{s\cdot f}{65\cdot e}\cdot\sqrt{\frac{R_e}{23.5}} \tag{71}$$

$$h\geq\frac{b_1\cdot k}{f} \tag{72}$$

The function (71) relates to the condition of stability of the top flange, whereas (72) relates to the technological possibilities of manufacturing the box section.

The optimum point in this diagram will be the lowest point that fulfils the above mentioned conditions and constraints.

This will be illustrated through the following examples.

The following diagrams (Fig. 4 – Fig. 7) will show how the curves f_{11}, f_{12} and f_{13} change depending on the classification class and selection of materials according to this criterion, where it will be adopted, for illustration, that the span is *L*=20m and the carrying capacity is *Q* = 12,5t.

The following initial data will be adopted: *e*=1.33, for S235 : *s*=210, *f*=0.88, and for S355 : *s*=170, *f*=0.87.

The diagrams (Fig. 4 and Fig. 5) show how the curves f_{11}, f_{12} and f_{13} change according to the strength criterion, for classification class 1, where it is adopted that the base material is S235 (Fig. 4) and S355 (Fig. 5).

It is seen to which extent the selection of base material influences the shapes of the curves f_{11}, f_{12} and f_{13}, which is seen from (Fig. 4 – Fig. 7).

The diagrams (Fig. 6 and Fig. 7) show how the curves f_{11}, f_{12} and f_{13} change according to the strength criterion, for classification class 2, where it is adopted that the base material is S235 (Fig. 6), i.e. S355 (Fig. 7). It is seen to which extent the selection of base material influences the shapes of the curves f_{11}, f_{12} and f_{13}, as well as the change of classification class, which is seen from these diagrams.

It is seen from the previous diagrams that in these cases the optimum point according to the strength criterion will be in the intersection of the vertical line of the function (71) and the function (69).

The results from the previous examples will be shown in Table 1. The solutions were obtained in the software package MathCad.

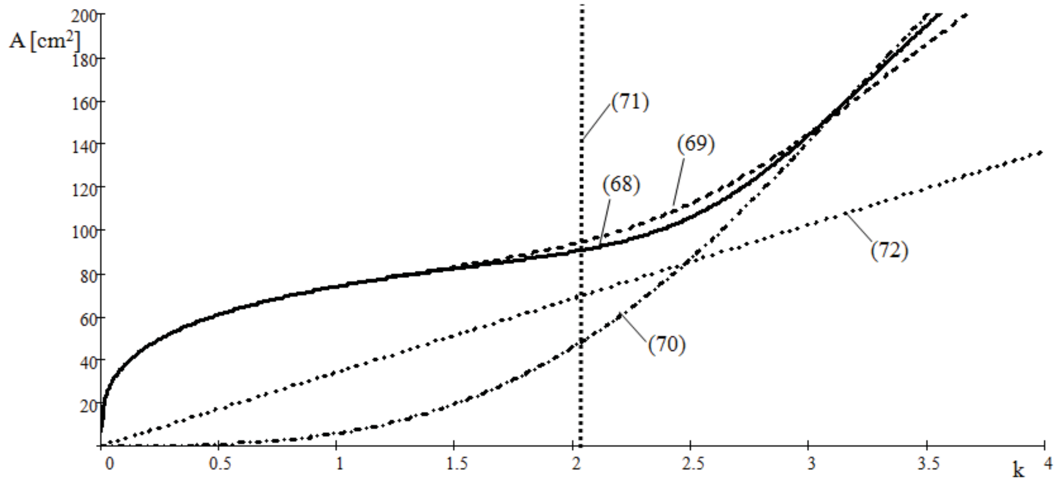

Figure 4: Comparative analysis of optimum values

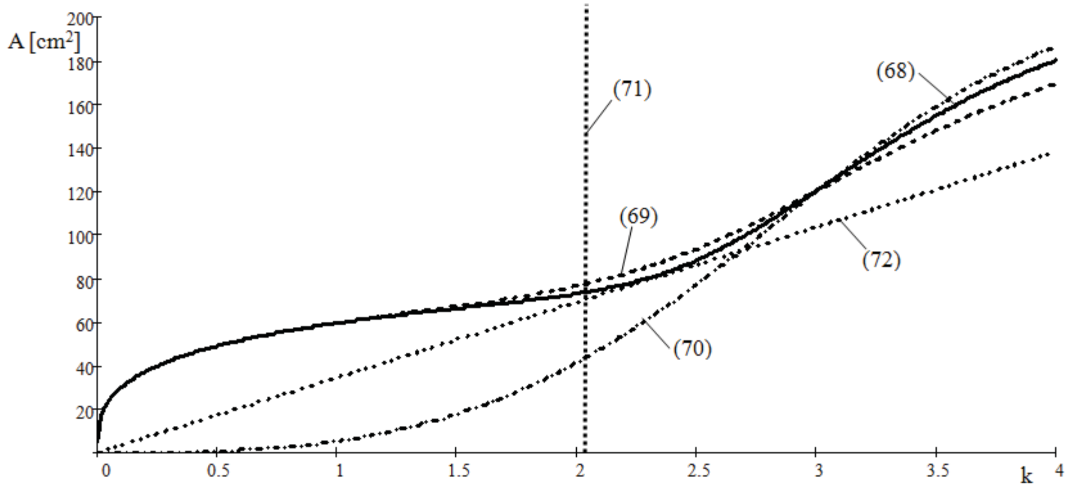

Figure 5: Comparative analysis of optimum values

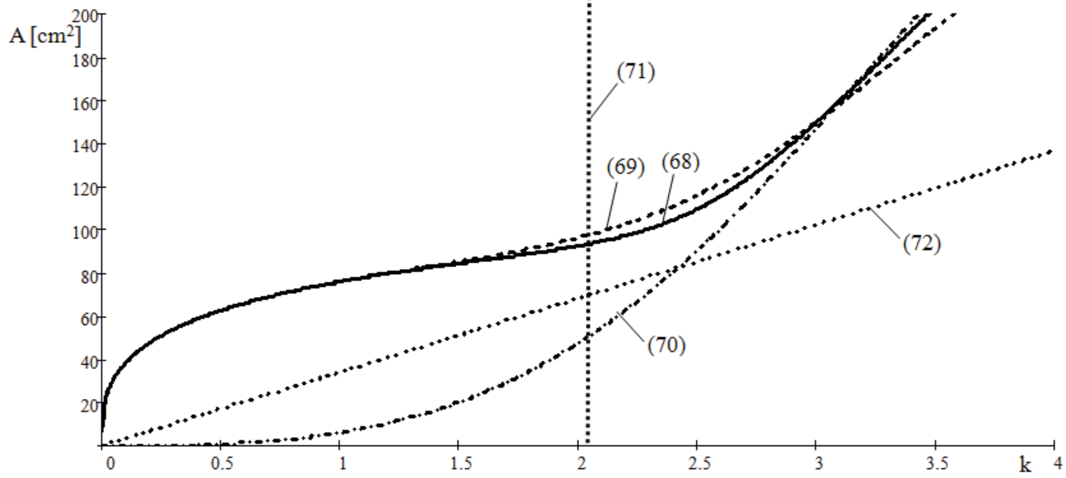

Figure 6: Comparative analysis of optimum values

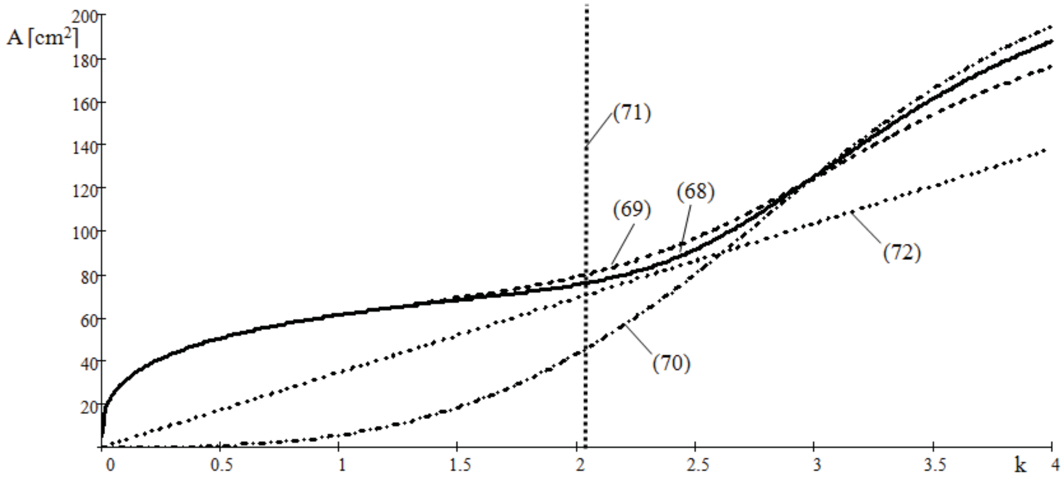

Figure 7: Comparative analysis of optimum values

Table 1: Values of optimum parameters

Material	**f$_{11}$**		**f$_{12}$**		**f$_{13}$**		**Optimum**		**Cl. class**
	k	*h(cm)*	*k*	*h(cm)*	*k*	*h(cm)*	*k*	*h(cm)*	
S235	2,821	127.05	1.518	83.20	6.184	330.03	2.133	97.26	***1***
	2.806	130.64	1.512	85.49	6.184	344.01	2.133	100.30	***2***
S355	2.685	98.70	1.457	66.36	5.441	216.62	2.098	79.00	***1***
	2.670	101.40	1.451	68.18	5.441	225.80	2.098	81.47	***2***

5. CONCLUSION

The paper defined optimum dimensions of the box section of the main girder of the bridge crane for the case of placing the rail in the middle of the top flange in an analytical form, by using the method of Lagrange multipliers, according to criterion of strength.

It was shown that the proper selection of girder height and plate thickness can considerably influence the reduction in the cross sectional area at the same time satisfying all constraint functions.

The results were obtained in explicit form, which is very favourable for discussion of solutions as well as for consideration of influences of individual geometrical parameters and their ratios. Comparison of the obtained results with certain solutions of bridge cranes shows that the obtained cross sectional areas are smaller, which verifies the optimization results.

In addition, the usage of the method of Lagrange multipliers is justified because the optimization results are obtained in analytical form, which allows getting conclusions about influences of particular parameters and further researches toward mass reduction.

The results obtained may be of great use to the engineer-designer, particularly in the first phase of the design procedure when the basic dimensions of the main girder of the bridge crane, as its most responsible part, are defined.

The conclusion is that further research should be directed toward a multicriteria analysis where it is necessary to include additional constraint functions, such as: lateral stability, local stability of plates, deflection dynamic stiffness, material fatigue, influence of manufacturing technology, optimization of the ratio of plate thicknesses, types of material, conditions of crane operation.

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