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A Comparative Study of Discrete and Modal Approximation of Hydraulic Transmission Lines

D. Pršić +, N. Nedić +, and Lj. Dubonjić +

+ University of Kragujevac, Faculty of Mechanical Engineering Kraljevo, Dositejeva 19, 36000 Kraljevo, Serbia
Phone: (381) 36 383-377, Fax: (381) 36 383-378, E-Mail: {prsic.d, nedic.n, dubonjic.lj}@mfkv.kg.ac.rs

Abstract ***- Analysis of dynamics of the fluid transmission line in the time domain requires considerable simplification of models which are used in the frequency domain. The paper compares the results of simulation of two such approximations. The first one is based on spatial discretization of a pipeline into segments with lumped parameters. Electro-hydraulic analogy is used for development of the model. The second approach is based on modal approximation. Each single-resonance mode of pipeline dynamics is described by the second order transfer function. Viscosity effect is described by the linear friction model. The bond graph technique is used in both cases for presentation of the model. Out of four causal possibilities, the case with mixed boundary conditions - flow rate and pressure as inputs is observed. The influence of the number of segments/modes and frequency of the input signal on the system response is analyzed.***

Key words: ***fluid transmission line, discrete model, modal approximation, bond graph, simulation***

I. INTRODUCTION

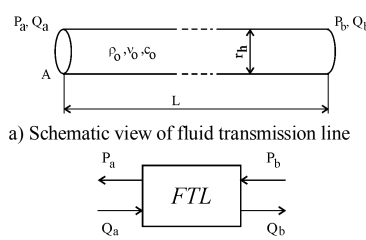
In a lot of applications there is a need for modelling fluid transmission lines (FTL) and fluid transmission networks. In fluid power control systems, reasons for spatial distribution of parameters can be different [1]. Due to constructionally reasons, inefficient volumes usually occur between the pump and the valve, or between the valve and the actuator. The dynamics of these long transmission lines may lead to undesired oscillations. Therefore, the analysis of dynamics of the whole system must also take into account the dynamics of lines.

Physical variables which describe these systems depend not only on the time coordinate but on the spatial coordinate as well. Hence, they are described with partial differential equations, i.e. with infinite-order models in the complex domain. For frequency domain analysis, such models can be used without any simplifications for different boundary conditions. However, certain simplifications are necessary for time domain analysis and simulation. Numerous approximation techniques are encountered in literature: method of characteristics, direct numerical methods, discrete methods, quasi-method of characteristics [1-3]. This paper presents the results of simulation of two models: one is discrete, based on the division of non-homogenous fields ($p(x,t), Q(x,t)$) into segments with homogeneous fields ($p(t), Q(t)$) of all physical variables [4], and the other is obtained by modal approximation [5,6]. The first model uses electro-hydraulic analogy, and the second one is obtained by using rational transfer functions with analytical determination of the modal coefficients.

Depending on the adopted assumptions, viscous and heat transfer effects, pipeline models range from lossless, through linear up to dissipative models [1-3]. This paper uses the linear friction model of the rigid, circular pipe with laminar, Hagen-Poiseuille flow, without heat transfer.

In order to easily notice the effects of involving dynamics of the transmission line in hydraulic control systems it is desirable to use the modular model of a single pipeline section which can easily be included or omitted from the model of the entire system. That is why the bond graph is used for presentation of both models [7]. The bond graph allows connection of pipeline models with the models of other components in fluid power control systems.

For one-dimensional distributed parameter models of a single pipeline, schematically shown in Fig. 1a, there are four causal possibilities: symmetric boundary conditions with pressure inputs (P_a, P_b) or flow rate inputs (Q_a, Q_b) and mixed boundary conditions with (P_a, Q_b) or (P_b, Q_a) as inputs. The last causal combination (P_b, Q_a - inputs, Fig. 1b) is used in this paper. This combination corresponds to long transmission lines between the valve and the actuator.



a) Schematic view of fluid transmission line

b) Models causality

Fig.1. Fluid transmission line

II. DISCRETE MODEL APPROXIMATION

In this approach, the pipeline is observed as a cascade network of lumped elements (Fig. 2), where dynamics of each of them is described by common (linear or nonlinear) differential equations. The number of lumped elements depends on the frequency band of interest. The length of each segment should be much smaller than the shortest wavelength of interest.

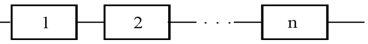


Fig.2. A cascaded network representing a transmission line

Practically, we need to do a model of a one segment and then serially connect those models into the model of long transmission line. Electro-hydraulic analogy is used for modelling. The segment is described by the so-called π-circuit or T- circuit [1,8]. In both cases, the same elements are used: the capacitor (fluid compressibility), the coil (fluid inertia) and the resistor (friction). The difference is in the way of connecting those elements. It should be noted that when the segment cascade is made, regardless of whether we use a π or a T model, there are blocks inside the model that periodically repeat (Fig. 3) [4]:

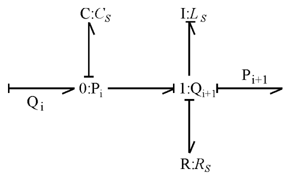
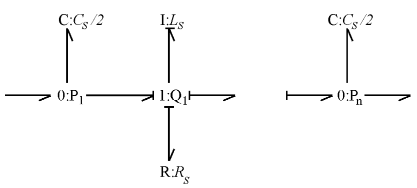


Fig.3. Segment's bond graph model that repeat periodically

The parameters C_S, I_S, R_S are the segment capacitance, inertance and resistance respectively.

The models of end elements (1-st and n-th elements) in the cascade in Fig. 2 depend on the model causality, i.e. the environment in which the pipeline model is used. The π interface presented in Fig. 4 is used for the causality shown in Fig. 1b.



1-st element n-th element

Fig.4. π - interface of FTL discrete model

The model shown in Fig. 3 and Fig. 4 serves for simulation of the model obtained by the discrete method of approximation.

III. MODAL APPROXIMATION

The transient processes of single transmission line ca be modeled with so called four-pole equation that relates signals at the ends of pipelines in the complex domain [1,2]:

$$\begin{bmatrix} Q_b(s) \\ P_a(s) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\cosh\Gamma(s)} & \dfrac{-\sinh\Gamma(s)}{Z_c(s)\cosh\Gamma(s)} \\ \dfrac{Z_c\sinh\Gamma(s)}{\cosh\Gamma(s)} & \dfrac{1}{\cosh\Gamma(s)} \end{bmatrix} \begin{bmatrix} Q_a(s) \\ P_b(s) \end{bmatrix} \quad (1)$$

where $\Gamma(s)$ and $Z_c(s)$ are the propagation operator and line characteristic impedance, respectively. Depending on geometry of the pipe cross section and included heat transfer and distributed viscosity effects, functions $\Gamma(s)$ and $Z_c(s)$ have different forms. For the case of a linear friction model we have [9]:

$$\Gamma(s) = \frac{Ls}{c_0}\sqrt{1+\frac{B}{s}}\ ; \quad Z_c(s) = \frac{\rho_0\, c_0}{A}\sqrt{1+\frac{B}{s}} \quad (2)$$

where L is length of pipe and B is the friction coefficient.

Since the model (1) represents an infinite-order model, it is not suitable for analysis of nonlinear systems in the time domain. Using modal approximation [5,6], the system (1) can be described as the finite dimensional linear system with transfer functions in quadratic modal forms:

$$\begin{bmatrix} P_a(s) \\ Q_b(s) \end{bmatrix} = \sum_{i=1}^{n} \begin{bmatrix} P_{ai}(s) \\ Q_{bi}(s) \end{bmatrix} = \sum_{i=1}^{n} T_i(\bar{s}) \begin{bmatrix} P_b(s) \\ Q_a(s) \end{bmatrix} \quad (3)$$

where $T_i(s)$ is transfer function for the i-th mode. For the linear friction model and causality presented in Fig. 1b $T_i(\bar{s})$ can be expressed as:

$$T_i(\bar{s}) = \frac{1}{\Delta(\bar{s})} \begin{bmatrix} (-1)^{i+1}\dfrac{2\lambda_{ci}}{D_n} & \dfrac{2Z_0}{D_n}(\bar{s}+8) \\ -\dfrac{2}{Z_0 D_n}\bar{s} & (-1)^{i+1}\dfrac{2\lambda_{ci}}{D_n} \end{bmatrix} \quad (4)$$

where

$$\Delta(\bar{s}) = \bar{s}^2 + 8\bar{s} + \lambda_{ci}^2 \quad (5)$$

$\bar{s}$ is normalized Laplace operator defined and λ_{ci} is root indices.

This parameter represents the normalized undamped natural frequency of the blocked line for losses or the linear friction model. D_n is the dissipation number and Z_0 is the pipeline impedance constant [9].

Using partitioned flow and pressure Q_{bi} and P_{ai} as state variables, modal transfer function can be transformed into modal, second order state space equation of the form:

$$\begin{bmatrix} C\dot{P}_{ai} \\ I\dot{Q}_{bi} \end{bmatrix} = \begin{bmatrix} 0 & -a_i \\ a_i & -R \end{bmatrix} \begin{bmatrix} P_{ai} \\ Q_{bi} \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} P_b \\ Q_a \end{bmatrix} \quad (6)$$

where

$$a_i = (-1)^{i+1}(i-1/2)\pi/2 \quad (7)$$

C, I, R are pipeline lumped capacitance, inertance and resistance respectively.

Based on the equation (6), the model of the i-th mode can be presented by using the bond graph (Fig. 5).

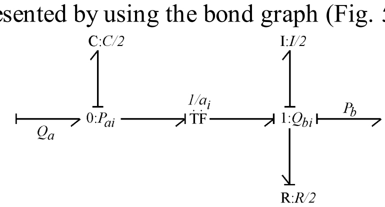


Fig. 5. The modal bond graph representation of the i-th mode for the pipeline

Correction of steady state parameters for the model presented in the previous figure is necessary because of the finite number of modes included in the model. In this paper the following residual matrix coefficient is used for truncated modes with natural frequencies well above the input frequencies [9]:

$$M_{ss,res} = M_{ss} - \sum_{i=1}^{n} M_{ssi} \tag{8}$$

where is M_{ss} steady state coefficient matrix and M_{ssi} is the transfer function for the *i*-th mode at the steady state.

Parallel connection of the modal models presented in Fig. 5 and introduction of the correction given by the relation (8) lead to the modal model of long transmission line shown in Fig. 6.

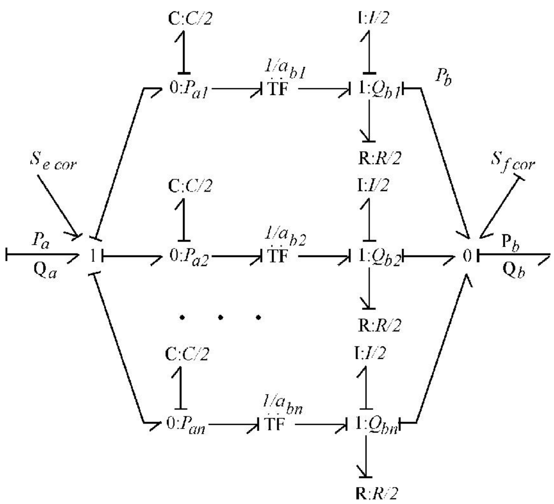


Fig. 6. The modal bond graph representation of the pipeline with steady state correction

The 1-junction and the 0-junction are used for connection of the modal models. Correction of steady state parameters was performed by means of two modulated source elements. The source $S_{e\,cor}$ serves for correction of the steady state value of pressure P_a. The source $S_{f\,cor}$ serves for correction of the steady state value of flow rate Q_b. The model presented in Fig. 6 serves for simulation of the model obtained by modal approximation.

IV SIMULATION

The parameters at which the simulation was performed are:

$r_h = 5x10^{-3} m$; $L = 16 m$; $\rho_0 = 860 kg/m^3$;

$v_0 = 9.7x10^{-5} m^2/s$; $\beta_e = 1.4x10^9 Pa$; $Q_0 = 3.510^{-4} m^3/s$,

Before the beginning of simulation initial conditions for all segments/modes have to be set up. It is assumed that the system is in steady state before the beginning of the transition process. Input flow rate is changing according to:

$$Q(t) = Q_0 \sin[\omega(t - 0.01)]\, h(t - 0.01) \tag{9}$$

The results of simulation of discrete and modal models for different numbers of segments/modes and for two frequencies ($\omega = 50 rad/s$, $\omega = 500 rad/s$) are presented in Fig. 7.

Both methods show that at low frequencies the pipeline dynamics can be modelled by using one segment/mode or it can even be neglected. At high frequencies, in addition to the change of gain, the phase lag increases. The results of simulation differ at high frequencies and a small number of segments/modes. With the increased number of segments/modes, those differences disappear even at high frequencies.

V CONCLUSION

The paper presents the results of simulation of two methods of fluid transmission line modelling. Both methods practically show similar results at low and high frequencies. At high frequencies, it is necessary to involve a higher number of segments/modes. Nevertheless, the modal method has the advantage due to easy maintenance and higher flexibility of the model. For example, in the discrete method, introduction of a new segment requires changes in the models of previous segments. Another advantage refers to the speed of simulation. Modal models have faster execution at least an order of magnitude than discrete models.

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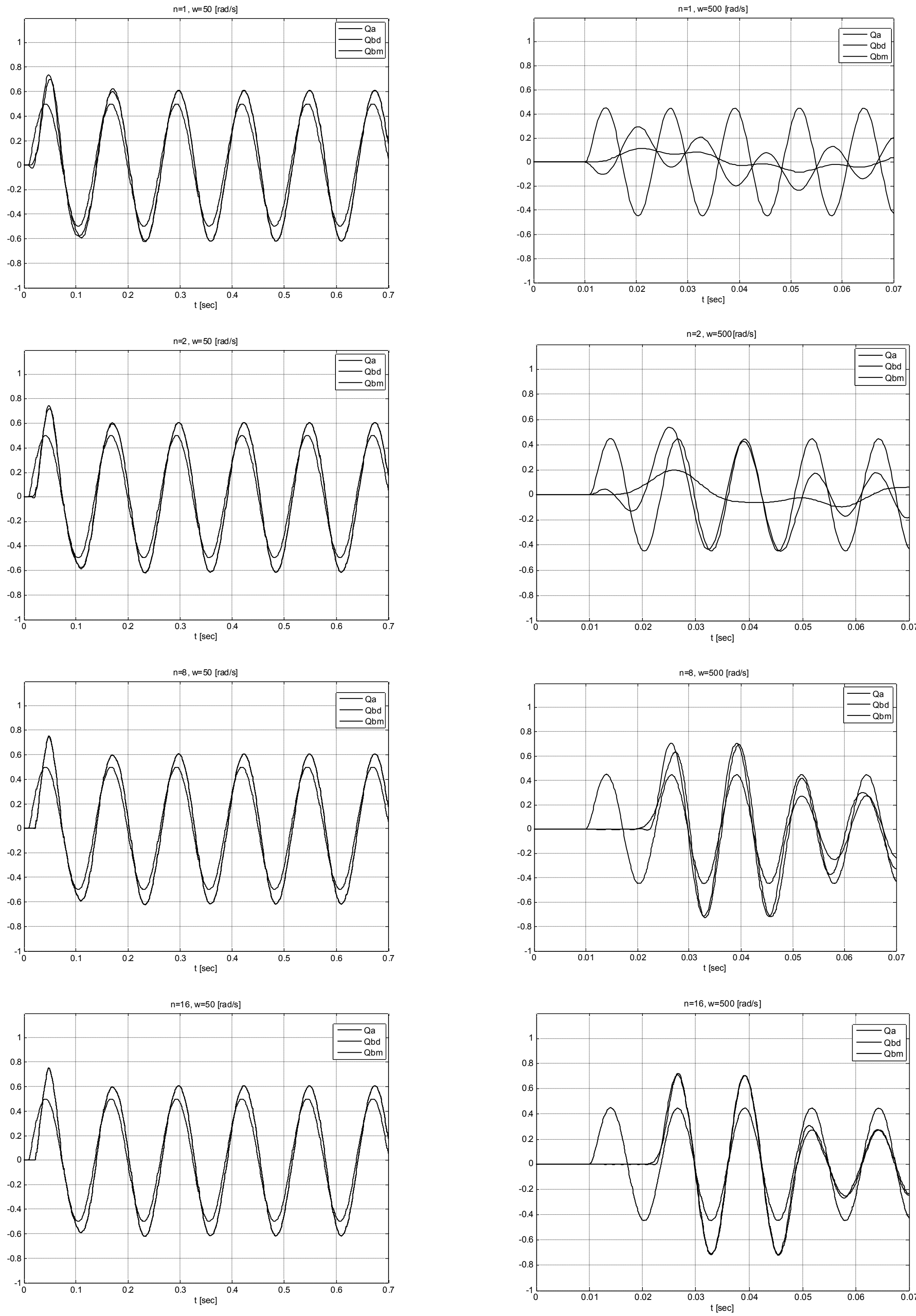

a) Discrete model

b) Modal model

Fig. 7. Simulation results for models with different number of segments