

Control in Transport and Dosing Devices with Time Delay- Method for Extracting Region of Absolute Stability

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Abstract: *In modern system of automatic control, either in its object, or in the control part of the system, there are usually certain areas of uneven dimensions through which is conducted the transport of mass or energy of most diverse fluids, solids, or signal. Delivery of energy, momentum, movement of the working body from one place to another happens with some delay. To rule out the possibility of inaccuracies in the methods of analysis and synthesis of this kind of automatic control system, it is necessary to take into account the occurrence of this delay. Influence of delay phenomena is essential for proper qualitative and quantitative description of the various processes. Typical types of delays are: transportation, technological and information. Typical representatives of system with transport delay are objects of type conveyor for coal, machinery for the production of plastic (extruders) and drugs and other devices, where is carried out transport, dosage and storage of solids. The material (granulate to produce plastics, drugs, coal...) is removed from the tanks to bunker. The conveyor belt appears as an integral part of the unit of dosing devices. Cross-section A of the bunker was unchanged over the entire height of the bunker. The task of these devices for transporting and dosing is the quantity of material to be retained in the bunker at a constant value ($h = \text{const}$) and we will develop system of automatic control for this purpose. According to earlier developed method of D – composition for time delay system with proportional regulator, we can separate the region in three-dimensional space frequency $\omega(\text{Hz})$, gain $K=1/\alpha$ and time delay constant (τ), so that adjustable parameters guarantee absolute stability of synthesized automatic control system. To simulate dynamic behavior of the developed system we will use MATLAB software package. From the transient curve we can observe stability of the system for different value of parameters (α, τ) picked up from separate region in parametric plane $\alpha-\tau$.*

Keywords: absolute stability, parametric plane, time delay system, transport and dosing devices

I. INTRODUCTION

This method which Russian scientist Neimark [7],[8] was researched is the method for testing stability of time-delay systems so-called D-composition method. All previous works about this method was given in [1]. The method for separation the region in the parameter plane, which enables closed-loop system will have absolute stability was also particularly developed and explained in [4] and this paper will continue extend last mentioned results in the sense of their application with computer science. This paper presents D-decomposition method in the area of absolute stability, developed from Neimark in order to extract region of absolute stability in two-parameters plane for special class of time – delay system for automatic control. The basis of

mathematical equations and rules for shading parametric curves remain the same as in the case of system without delay. We will discuss the case of closed-loop system with a single delay, when the adjustable parameters are non-linearly related to polynomial coefficients of quasicharacteristic equation [8].

The first methods for testing stability of time delay system in the parametric plane, came from a Russian scientist Neimark [7],[8]. Root-locus technique was employed to determine the critical open-loop gain of the closed loop system for the fixed time delay according to the works of Chu [3]. The root-locus method is generally consider to one adjustable parameter, such as the open-loop gain. The general part of Siljak [9], [10], [11] contributed the full generalization of all outstanding issues, which was successfully solved in the class of time delay systems with lumped parameters. Eisenberg [4] was able to predict the relative stability ($\xi \text{ const}$) of a system in which loop gain and some system time constants were considered as two free parameters for a fixed time delay. The establishment of a region that provides absolute stability for the time delay systems and non linear combination of adjustable parameters, including pure time delay, was considered and solved by Mikic [6].

$$W_{ok} = \frac{N(s)}{\alpha D(s)} e^{-\tau s} \quad (1)$$

so that quasicharacteristic equation has the following form:

$$f(s, e^{-\tau s}) = \alpha D(s) + N(s) e^{-\tau s} = 0 \quad (2)$$

where $K = 1/\alpha$ is proportional regulator gain, so α is a regulator parameter linearly related to polynomial coefficients of quasicharacteristic polynomial. The transport and dosing processes present devices with transport delay. The another kind of transport is technological and informational. Typical representatives of these systems are objects of type conveyor for coal, machinery for the production of plastic (extruders), followed by drugs and other devices where transport is carried out solids and its dosage and storage which mathematical model will be developed in this paper.

The material (granulate to produce plastics, drugs, coal...) is removed from the tanks to bunker. The conveyor belt appears as an integral part of the unit of dosing devices. Belt speed is constant. The amount of transporting material, at a constant belt speed, then only depends on the filling coefficient $\psi(t)$, which may vary according to the needs of consumers and gives a uniform height and thickness of material over the entire length of conveyor. Cross-section A of the bunker was unchanged over the entire height of the bunker. The task of these devices for transporting and dosing is to quantity of material to be retained in the bunker at a constant value ($h = \text{const}$) and we will developed system of automatic control for this purpose.

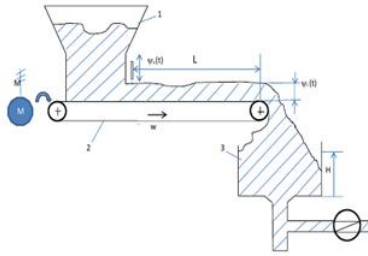


Fig.1. Functional scheme of transport and dosing device

II. EXTRACTION THE AREA OF ABSOLUTE STABILITY

The system will possess absolute stability only if all the roots of quasicharacteristic equation are within contour which cover left side of the complex plane. So this method realize the transformation left side of the complex plane on the parametric plane α - τ .

A. Decomposition curves

For ω - frequency complex variable s has the form:

$$s = \sigma + j\omega \quad (3)$$

So if (3) put into polynomials $D(s)$ and $N(s)$ and for $\sigma = 0$ we got:

$$D(\omega) = R_D(\omega) + jI_D(\omega) \quad (4)$$

$$N(\omega) = R_N(\omega) + jI_N(\omega) \quad (5)$$

So equation (2) gives a form

$$f(\omega) = R_F(\omega) + jI_F(\omega) \quad (6)$$

By substituting (4) and (5) in (2), quasicharacteristic equation will also have real and imaginary part, i.e. in polar coordinates (4) and (5) are:

$$D(\omega) = r_D(\omega)e^{j\Phi_D} \quad (7)$$

$$N(\omega) = r_N(\omega)e^{j\Phi_N} \quad (8)$$

and then substituting (7) and (8) in (2) are followed next decomposition curves:

$$\alpha = \pm \frac{r_N(\omega)}{r_D(\omega)} \quad (9)$$

$$\tau = \frac{1}{\omega} \left[\Phi_N(\omega) - \Phi_D(\omega) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2} \right] \quad (10)$$

$$k \in Z, \omega \in [-\infty, +\infty)$$

Note: The upper sign of (6) corresponds to the upper sign of (7) and the lower sign of (6) corresponds to the lower sign of (7).

B. Curve shading

Shading of decomposed curves is determined by the sign of Jacobians (as in systems without delays). For complex s (3) quasicharacteristic equation is going to be:

$$R_F(\omega) = R_N(\omega) + \alpha e^{-\tau\omega} \cdot [R_D(\omega) \cdot \cos(\tau\omega) - I_D(\omega) \cdot \sin(\tau\omega)] \quad (11)$$

$$I_F(\omega) = I_N(\omega) + \alpha e^{-\tau\omega} \cdot [I_D(\omega) \cdot \cos(\tau\omega) + R_D(\omega) \cdot \sin(\tau\omega)] \quad (12)$$

Jacobians of the system as follows:

$$J = \begin{vmatrix} \frac{\partial R_F}{\partial \tau} & \frac{\partial R_F}{\partial \alpha} \\ \frac{\partial I_F}{\partial \tau} & \frac{\partial I_F}{\partial \alpha} \end{vmatrix} = -\alpha \cdot \omega \cdot r_D^2(\omega) \quad (13)$$

C. Singular lines

Singular lines, in the case of extracting area of absolute stability, is defined for boundary cases $\omega \rightarrow \infty$ - and $\omega_n \rightarrow +\infty$, in (2), (6) and (7):

$$\alpha = \lim_{\omega \rightarrow \pm\infty} \frac{r_N(\omega)}{r_D(\omega)} \quad (14)$$

$$\alpha = \lim_{\omega \rightarrow \pm\infty} \frac{r_N(\omega)}{r_D(\omega)} \quad (15)$$

$$\tau = \lim_{\omega \rightarrow \pm\infty} \frac{1}{\omega} \left[\Phi_N(\omega) - \Phi_D(\omega) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2} \right] \quad (16)$$

$$\tau = \lim_{\omega \rightarrow a} \frac{1}{\omega} \left[\Phi_N(\omega) - \Phi_D(\omega) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2} \right] \quad (17)$$

Where a is every value of ω where (9) and (10) are not defined.

III. APPLICATION THE METHOD – TRANSPORT AND DOSING DEVICE

Application of the methods described here will be illustrated by the example of transport and dosing process. A mathematical model is developed for control systems with proportional controller which gain is $K = 1 / \alpha$ and given object (Fig. 2) for some nominal parameter values, with time delay identical to $\tau=l/w$, where is l the length of transport belt and w is the velocity of the belt. Mathematical model is developed in [1]. The open loop transfer function of feedback system is:

$$W_{ok} = \frac{0,11}{\alpha \cdot s} e^{-\tau s} \quad (18)$$

A. Synthesis of closed -loop system

We can see that we can approach the synthesis by comparing (18) i (1) according to the methods described in chapter III. Then equations (9) and (10) become:

$$\alpha = \pm \frac{0,11}{\omega} \quad (19)$$

$$\tau = \frac{1}{\omega} \left[-\text{sign}(\omega) \cdot \frac{\pi}{2} + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2} \right] \quad (20)$$

$$J = -\alpha \cdot \omega^3 \quad (21)$$

Using (19), (20) and (21) it is extracted the field of absolute stability. Singular lines and Jacobians are determined from (13), (14), (15), (16) and (17) so we get two singular lines $\alpha = 0$ and $\tau = 0$. Areas given from extraction curves (18), (19) and (20) by rules of shading which defined values of parameters for system with absolute stability are shown on Fig.2 and on Fig.5 are noted like 'The area of absolute stability'.

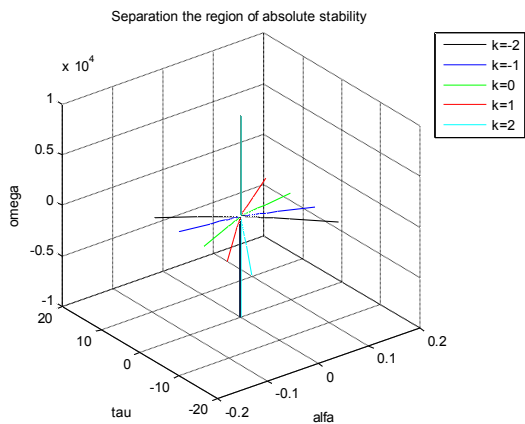


Fig. 2 Stability area for $\omega=(-10000,10000)$

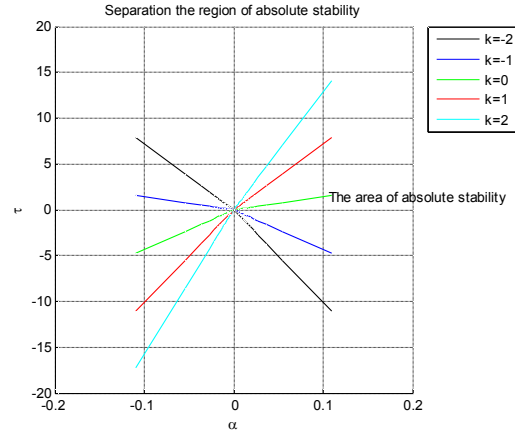


Fig. 3 Top view of Fig. 2 (α - τ plane)

It could be possible to define unstable from stable region in parametric plane α - τ which with boundaries represents region of absolute stability. In that case results obtained by this method are the same like the well-known ones. [7],[8].

B. Dynamic analysis of synthesized system

From the area of absolute stability we highlight the point which determines the controller parameters $\alpha = 0.1$ and $\tau = 2s$, and on the basis of (17) receives the open loop transfer function of the system. Simulation of the system behavior is done with MATLAB software with step function. Simulation result of step response is shown on Fig. 6 For

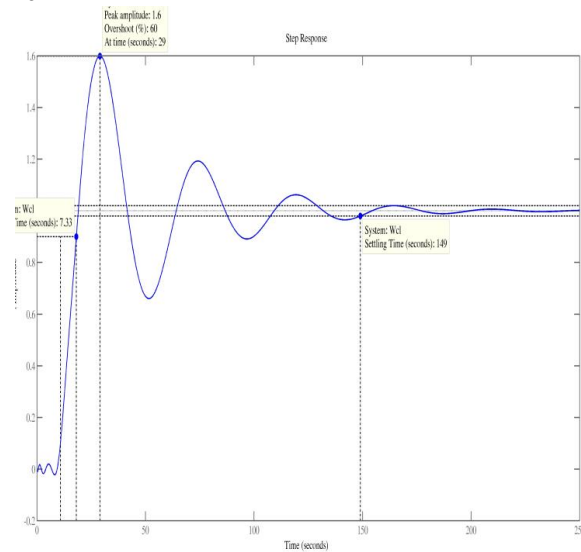


Fig. 4 Step response (transport and dosing device)

IV. CONCLUSION

New software package MATLAB enables to obtain more precocious D-decomposition method applied to adoption new principle of separation the field of absolute stability in the parametric plane of α - τ [1]. The results presenting here

overview variation and dependences of parameters in three-dimensional form(α , τ , ω) where could be possible to choose from the given area values for parameters which guarantee much better accuracy in methodology than the last obtained results [1], [2]. System behavior analysis was done from simulation of working condition by MATLAB software with step function and thus allows verification of system properties. It is possible with this method to do verification of synthesized automatic control system with the real model. The obtained experimental results can, in as much as possible points, matched with the simulation curve (Fig.4) , by changing parameter α or τ from the separation area with this method. With this method we also could separate region of relative stability- pre-defined settling time and damping factor (next articles). The section of all this area given the area in parametric plane α - τ region which guarantee absolute stability of system and required settling time and damping factor. It means that when we choose parameters α or τ from that region we can get the simulation curve with specific form. Experimental results, given in real condition, have to get curve form similar or (in best case) the same like simulation curve. This is well-known method of verification the model. So, if we have the region from which we can choose parameters α or τ which guarantee the specific form of simulation curve we can enable experimental curve have matched in as much as possible points , by changing parameters α (the gain of proportional regulator) and τ (the velocity of the belt). In this way we improving the accuracy of the working system of automatic control for transport and dosing devices.

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