XI International SAUM Conference
on Systems, Automatic Control and Measurements
Niš, Serbia, November 14th-16th, 2012

Simulation the Transport and Dosing Device Behaviour Like object in Automatic Control System with Pre - defined Relative Stability

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Abstract: ***Typical representatives of transport and dosing processes, where is presented transport delay are objects of type conveyor for coal, machinery for the production of plastic (extruders) and drugs and other devices where is carried out transport of solids and its dosage and storage . The task of these devices for transporting and dosing is the quantity of material to be retained in the bunker at a constant value (h = const) through transport belt from tank to bunker. In this paper we will develop system of automatic control for this purpose, which will have pre – defined relative stability (settling time and damping factor). Mathematical model of this control systems i.e. their open loop transfer functions belongs to the class of system for which required methods were first developed from Loo and Neimark. This paper presents and investigates the further expansion of last obtained results and applied those methods for synthesis automatic control system with proportional regulator for transport and dosing devices. This D-decomposition method in the area of relative stability separates constant settling time area in parametric plane which ensures the system having predefined settling time. We also will use another method that has a lot of specifics regarding to the needs of extraction the region in parametric plane which ensures the system having predefined damping factor. Now it would be possible to separate the regions in three-dimensional space- frequency ω (Hz), loop gain (K=1/α) and time delay constant (τ), where adjustable parameters guarantee settling time Ts and damping factor (ξ) of controlled system will have a priori defined value . Useful of this researches is that checking of obtained results can be made with MATLAB software package, so the simulation of dynamic behavior of the system can define settling time Ts and damping factor ξ from the transient curve.***

Keywords : ***time delay system, transport and dosing devices , damping factor, parametric plane, relative stability, settling time***

I. INTRODUCTION

The establishment of a region that provides pre-settling time for the time delay systems and non linear combination of adjustable parameters, including pure time delay , was considered and solved by Loo [5]. The application Mitrovic`s method in the analysis and synthesis of feedback control systems with delays is given by Mikic [6]. The method for extracting the region in the parameter plane, which enables closed- loop system will have pre-defined relative stability(settling time and damping factor) was also particularly developed and explained [1], [2] and this paper will continue extend last mentioned results and their application.

The basis of mathematical equations and rules for shading parametric curves remain the same as in the case of system without delay. We will discuss the case of closed-loop system with a single delay, when the adjustable parameters are non-linearly related to polynomial coefficients of quasicharacteristic equation [8] .

$$W_{ok} = \frac{N(s)}{\alpha D(s)} e^{-\tau s} \tag{1}$$

so that quasicharacteristic equation has the following form:

$$f(s, e^{-\tau s}) = \alpha D(s) + N(s) e^{-\tau s} = 0 \tag{2}$$

where K = 1/α is proportional regulator gain, so α is a regulator parameter linearly related to polynomial coefficients of quasicharacteristic polynomial. Pure time delay is τ, which in the case of transport and dosing device (Fig.1) introduced into the control parts of the object, through valves and pipes to reservoir, as described in the definition of a mathematical model of this system [1].This system belongs to the class of time delay system, where is the adjustable time for transport delay by changing the velocity of transport belt.

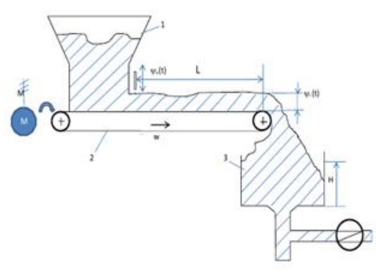

Fig. 1.Functional sheme of transport and dosing device

II. EXTRACTION THE AREA OF PRE-DEFINED SETTLING TIME-RELATIVE STABILITY

The system will have pre-defined settling time Ts, if

$$Ts = \frac{1}{\sigma_M} \ln \frac{\delta}{\Delta}, \qquad \Delta > 0, \qquad \delta < \Delta, \tag{3}$$

where is

σ_M – characteristic indicator of system
σ_M –max {Re s_k}, k=1,2…..n

A. Decomposition curves

So, let's see how is looking step response of system who has two complex poles on line $\sigma=\sigma_M$.
For classical linear system with transfer function:

$$W(s)=\frac{\omega_n^2}{(s+\varsigma\cdot\omega_n-j\cdot\omega_n\sqrt{1-\varsigma^2})\cdot(s+\varsigma\cdot\omega_n+j\cdot\omega_n\sqrt{1-\varsigma^2})} \quad (4)$$

Then step response is

$$g(\omega_n\cdot t)=\left\{1-\frac{e^{-|\sigma_M|\cdot t}}{\sqrt{1-\varsigma^2}}\cdot\sin(\omega_n t\sqrt{1-\varsigma^2}-\theta)\right\}\cdot h(t) \quad (5)$$

$$\theta=arctg\frac{\sqrt{1-\varsigma^2}}{\varsigma} \quad (6)$$

The maximum allowable deviation is

$$\varepsilon_m=\frac{e^{-|\sigma_M|\cdot t}}{\sqrt{1-\varsigma^2}} \quad (7)$$

Since we transform the whole line $\sigma=\sigma_M$ ($\zeta=0$), and for adoptable ε_m=2%

$$T_s=\frac{3{,}91}{|\sigma_M|_{\varepsilon_m=2\%}} \quad (8)$$

If complex poles have |Re s_k|>|σ_M|, then Ts have maximum value and this is the reason why is σ_M indicator the length of duration of transient. Time delay systems can whit approximation be treated like linear system, so the last mentioned for linear systems can use also for time delay system.
The system will possess an appropriate settling time only if all the roots of quasicharacteristic equation are within this contour shown on Fig.2
The method is transforming this contour from complex plane to parametric plane τ-α

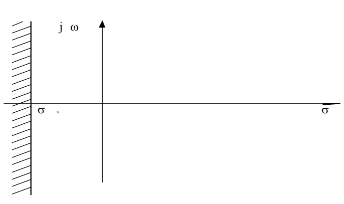

Fig. 2 Conture in complex plane

System will have required Ts if every roots of their quiazicharacteristic equation will be left from the line σ= -|σ_M|, It is necessary that line transform with equation (2) to plane α-τ., which represents D-composition method in order the automatic control time delay system have required settling time.
Complex s has a form

$$s=-|\sigma_M|+j\cdot\omega \quad (9)$$

If (9) change in D(s) and N(s) we got:

$$N(\sigma_M,\omega)=R_N(\sigma_M,\omega)+j\cdot I_N(\sigma_M,\omega) \quad (10)$$

$$D(\sigma_M,\omega)=R_D(\sigma_M,\omega)+j\cdot I_D(\sigma_M,\omega) \quad (11)$$

So equation (2) gives a form

$$f(\sigma_M,\omega)=R_f(\sigma_M,\omega)+j\cdot I_f(\sigma_M,\omega) \quad (12)$$

By substituting (10) and (11) in (2), quasicharacteristic equation will also have real and imaginary part.

$$R_F(\sigma_M,\omega)=\alpha\cdot e^{-\tau\cdot|\sigma_M|}\begin{bmatrix}R_D(\sigma_M,\omega)\cos(\tau\cdot\omega)-\\ I_D(\sigma_M,\omega)\cdot\sin(\omega\cdot\tau)\end{bmatrix}+R_N(\sigma_M,\omega) \quad (13)$$

$$I_F(\sigma_M,\omega)=\alpha\cdot e^{-\tau\cdot|\sigma_M|}\begin{bmatrix}R_D(\sigma_M,\omega)\cos(\tau\cdot\omega)\\ +I_D(\sigma_M,\omega)\cdot\sin(\omega\cdot\tau)\end{bmatrix}+I_N(\sigma_M,\omega) \quad (14)$$

In polar coordinates:

$$D(\sigma_M,\omega)=r_D(\sigma_M,\omega)e^{j\Phi_D(\sigma_M,\omega)}$$

$$N(\sigma_M,\omega=r_N(\sigma_M,\omega)e^{j\Phi_N(\sigma_M,\omega)} \quad (15)$$

and then substituting (15) in (1) from (2) are followed next decomposition curves:

$$\alpha=\pm\frac{r_N(\sigma_M,\omega)}{r_D(\sigma_M,\omega)}e^{|\sigma_M|\cdot\tau(\sigma_M,\omega)} \quad (16)$$

$$\tau=\frac{1}{\omega}\left[\Phi_N(\sigma_M,\omega)-\Phi_D(\sigma_M,\omega)+2k\pi+\frac{\pi}{2}\pm\frac{\pi}{2}\right] \quad (17)$$

$$k\in Z,\omega\in[-\infty,+\infty) \quad (18)$$

Note: The upper sign of (16) correspondents to the upper sign of (17) and the lower sign of (16) correspondents to the lower sign of (17).

B. Curve shading

Shading of decomposed curves is determined by the sign of Jacobians (as in systems without delays). For complex s (3) quasicharacteristic equation is going to be:

Jacobians of the system as follows:

$$J=\begin{vmatrix}\frac{\partial R_F}{\partial\tau} & \frac{\partial R_F}{\partial\alpha}\\ \frac{\partial I_F}{\partial\tau} & \frac{\partial I_F}{\partial\alpha}\end{vmatrix}=-\alpha\cdot\omega\cdot e^{-2|\sigma_M|\cdot\tau}\cdot r_D^2(\sigma_M,\omega) \quad (19)$$

C.Singular lines

Singular lines, in the case of extracting area of pre-defined damping factor, is defined for boundary cases $\omega_n \to 0+$ and $\omega_n \to +\infty$, in (2), (16) and (17) follows :

$$\alpha = \pm \lim_{\omega \to \pm\omega} \frac{r_N(\sigma_M, \omega)}{r_D(\sigma_M, \omega)} e^{-|\sigma_M| \cdot \tau(\sigma_M, \omega)} \tag{20}$$

$$\tau = \lim_{\omega \to \pm\infty} \frac{1}{\omega} \left[-arctg \frac{\omega}{-|\sigma_M|} + (2k+1)\pi \right] \tag{21}$$

for ω=a, where a is every value of ω in which the expression (15) and (16) are not defined

$$\alpha = \pm \lim_{\omega \to a} \frac{r_N(\sigma_M, \omega)}{r_D(\sigma_M, \omega)} e^{-|\sigma_M| \cdot \tau(\sigma_M, \omega)} \tag{22}$$

$$\tau = \lim_{\omega \to a} \frac{1}{\omega} \left[-arctg \frac{\omega}{-|\sigma_M|} + (2k+1)\pi \right] \tag{23}$$

Expressions ((20) ,(21) ,(22) and (23) are singular lines.

The procedure of selection area of pre-defined settling time is defined by procedure of selection the area of absolute stability for required automatic control system.[4]. Thus the first of all it is necessary to define area of absolute stability described in first paper [4], [6], [9].Then we find the area of pre settling time and singular lines for every $k \in Z, \omega \in [-\infty, +\infty)$

Area of pre-settling time is obtained from the section of area of pre settling time according to decomposition curves drawn in parametric space(α, τ, ω) e according to curves (16) and (17 , rules of shading and singular lines and the area of absolute stability. Parametric plane α-τ is top view of that figure in 3D space.

$$S = \bigcap_{k \to -\infty}^{k \to +\infty} S_k \cap S \text{ absolute stability} \tag{24}$$

III. APPLICATION THE METHOD – TRANSPORTING AND DOSING PROCES

Mathematical model is developed in [1]. The open loop transfer function of feedback system is:

$$W_{ok} = \frac{0.11}{\alpha \cdot s} e^{-\tau s} \tag{25}$$

A. Synthesis of controlled-loop system

We can see that we can approach the synthesis by comparing (25) and (1) according to the methods described in chapter II. Then equations (16), (17) ,(18) and (19) become:

$$\alpha = \frac{0,11}{\sqrt{\sigma_M^2 + \omega^2}} e^{|\sigma_M| \cdot \tau} \tag{26}$$

$$\tau = \frac{1}{\omega} \left[-arctg \frac{\omega}{-|\sigma_M|} + (2k+1)\pi \right] \tag{27}$$

$$J = -\alpha \cdot \omega \cdot e^{-2|\sigma_M| \cdot \tau} \cdot \sqrt{(\sigma_M^2 + \omega^2)}$$
(28)

From equation

$$T_s = \frac{3,91}{|\sigma_M|_{\varepsilon_m = 2\%}} \tag{29}$$

If we adopt pre-settling time Ts=150s

$$|\sigma_M| = 0,0261 \tag{30}$$

Using (26) and (27) for Ts=150s we extract the field of constant settling time of that value. Singular lines and Jacobians are determined from (20), (21) ,(22) and (23) so we get three singular lines α = 0 , τ = 0 and τ→∞.If we look (26) for ω=0 we have line equation

$$\alpha = \frac{0,11}{|\sigma_M|} e^{|\sigma_M| \cdot \tau} \tag{31}$$

to which is transform the double pole $s_{1/2} = -|\sigma_M|$ from the complex plane i.e. the point $-|\sigma_M|$ from the couture (Fig.2) into α-τ plane.

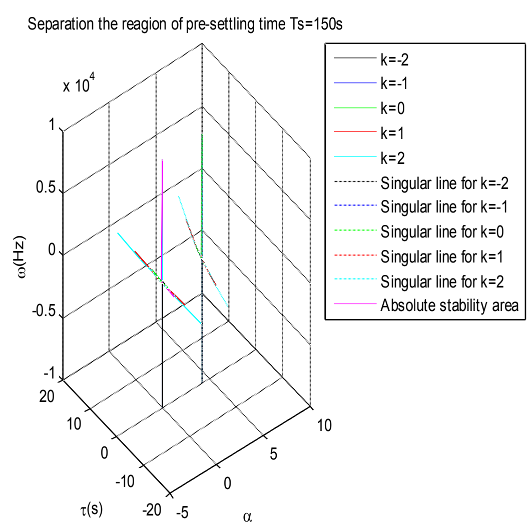

Fig. 3 Area of pre-settling time Ts=150s in parametric space(α, τ, ω)

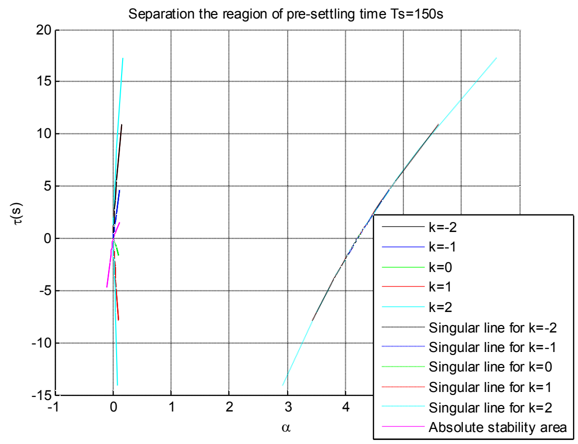

Fig. 4 Top view of Fig.4 (α-τ plane)

We see that boundary of the region consists of singular lines and decomposition curves. If we exclude singular lines and show curves only for k=-1 and absolute stability line the figure is:

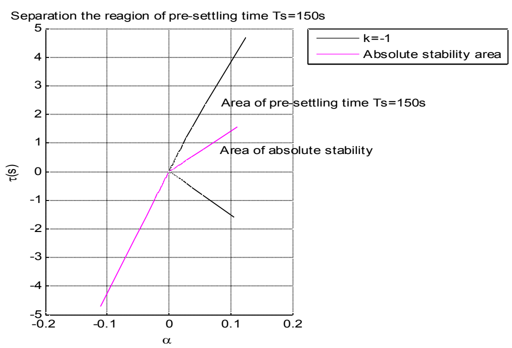

Fig. 5 Separation the region of pre–settling time Ts=150s for $\omega\in(-10^4,10^4)$

According to (24) we can separate the region of pre-settling time Ts=150s (magenta color) in α - τ plane which boundaries are the curve for absolute stability, singular line for k=-1(in process of separation area Ts) and line τ=0.

From the quantity of computer memory it depends the completeness of definition of area. Here is taken $\omega\in(-10^4,10^4)$.

B. Dynamic analysis of synthesized system

For Ts=150s we highlight the point which determines the controller parameters α = 0.1 and τ = 2s, and on the basis of (23) receives the open loop transfer function of the system. Simulation of the system behavior is done with MATLAB software with step function.

Simulation result of step response is shown on Fig. 6

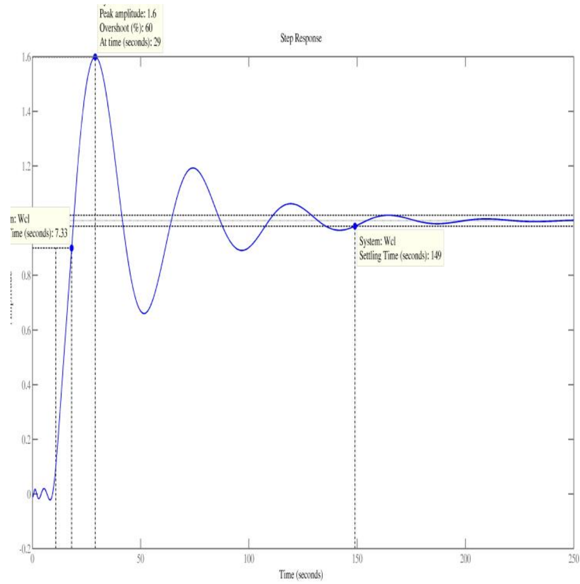

Fig. 6. Step response (transport and dosing device) for Ts=150s

IV.CONCLUSION

New software package MATLAB enables to obtain more precocious D-decomposition method applied to adoption new principle of separation the field of pre-specified settling time in the parametric plane of α-τ [1]. It is possible with this method to do verification of synthesized automatic control system with the real model. The obtained experimental results can, in as much as possible points, matched with the simulation curve (Fig.6) , by changing parameter α or τ (described in first article) from the separation area with this method.

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