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Parametric Methods in Analysis and Synthesis of Controlled Time Delay System– Circulating Reservoir for Mixing Liquids

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***Abstract**: Synthesis of time delay systems in parametric plane is based on the D-decomposition method, developed from Neimark [8]. In this case this method has a lot of specifics regarding to the eneeds of extraction the region in parametric plane which ensures the system having predefined damping factor [1], [2].This paper presents and investigates the further expansion of last obtained results and develops the methods for synthesis and analysis of closed-loop system with proportional regulator for circulating reservoir for mixing liquids. Now it would be possible to separate the region in three-dimensional space(undamped frequency ω_n (Hz), loop gain (K=1/α) and time delay constant (τ)) which adjustable parameters guarantee damping factor of controlled system will have a priori defined value .*

***Keywords** : time delay system , relative stability , parametric plane, damping factor, circulating reservoir for mixing liquids*

I. INTRODUCTION

The first methods for testing stability of time delay system in the parametric plane, came from a Russian scientist Neimark [7],[8]. According to the works of Chu [3] root-locus technique was employed to determine the critical open-loop gain of the closed loop system for the fixed time delay. The root-locus method is generally consider to one adjustable parameter, such as the open-loop gain. The main part of Siljak [9], [10] contributed the full generalization of all outstanding issues, which was successfully solved in the class of time delay systems with lumped parameters. Eisenberg [4] was able to predict the relative stability ($\xi \neq$ const) of a system in which loop gain and some system time constants were considered as two free parameters for a fixed time delay. The establishment of a region that provides pre-settling time for the time delay systems and non linear combination of adjustable parameters, including pure time delay , was considered and solved by Loo [5]. The application Mitrovic`s method in the analysis and synthesis of feedback control systems with delays is given by Mikic [6]. The method for extracting the region in the parameter plane, which enables closed- loop system will have pre-defined damping factor was also particularly developed and explained [1], [2] and this paper will continue extend last mentioned results and their application.
The basis of mathematical equations and rules for shading parametric curves remain the same as in the case of-system without delay.We will discuss the case of closed-loop system with a single delay, when the adjustable parameters are non-linearly related to polynomial coefficients of quasicharacteristic equation [8] .

$$W_{ok} = \frac{N(s)}{\alpha D(s)} e^{-\tau s} \tag{1}$$

so that quasicharacteristic equation has the following form:

$$f(s, e^{-\tau s}) = \alpha D(s) + N(s) e^{-\tau s} = 0 \tag{2}$$

where K = 1/α is proportional regulator gain, so α is a regulator parameter linearly related to polynomial coefficients of quasicharacteristic polynomial. Pure time delay is τ, which in the case of circulating reservoir for mixing liquids (Fig.1) introduced into the control parts of the object, through valves and pipes to reservoir, as described in the definition of a mathematical model of this system [1].This system belongs to the class of time delay system, where the adjustable time for transport delay of liquids are existed.

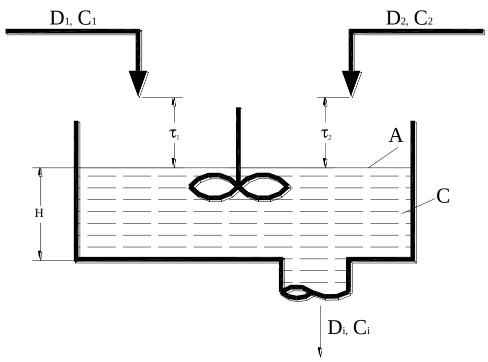

Fig.1.Functional sheme of circulating reservoir gor mixing liquids

II. EXTRACTION THE AREA OF PRE-DEFINED DAMPING FACTOR

The system will possess an appropriate damping factor only if all the roots of quasicharacteristic equation are within this contour shown on Fig.2.

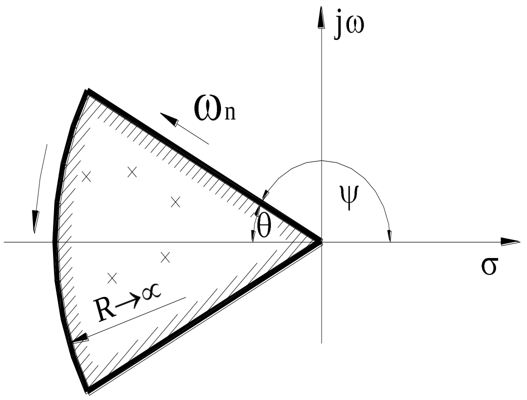

Fig.2.The method is transforming this contour from complex plane to parametric plane τ-α

A. Decomposition curves

For ω_n-undamped frequency and ξ – damping factor,complex variable s has the form:

$$s = \omega_n e^{j\theta} = -\omega_n\xi + j\omega_n\sqrt{1-\xi^2}, \xi = -\cos\theta \tag{3}$$

and for $T_k$andU_k, which areChebishev's polynomials,its degrees are given in the form:

$$s^k = \omega_n^k T_k(-\xi) + j\omega_n^k\sqrt{1-\xi^2}U_k(-\xi) \tag{4}$$

By substituting (4) in (2), quasicharacteristic equation will also have real and imaginary part, i.e. in polar coordinates:

$$D(\omega_n,\xi) = r_D(\omega_n,\xi)e^{j\Phi_D};$$
$$N(\omega_n,\xi) = r_N(\omega_n,\xi)e^{j\Phi_N} \tag{5}$$

and then substituting (5) in (1) from (2) are followed next decomposition curves:

$$\alpha = \pm\frac{r_N(\omega_n,\xi)}{r_D(\omega_n,\xi)}e^{\tau\omega_n\xi} \tag{6}$$

$$\tau = \frac{1}{\omega_n\sqrt{1-\xi^2}}\left[\Phi_N(\omega_n,\xi) - \Phi_D(\omega_n,\xi) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right] \tag{7}$$

$$k \in Z, \omega_n \in 0,+\infty$$

Note:The upper sign of (6) correspondents to the upper sign of (7) and the lower sign of (6) correspondents to the lower sign of (7).

B.Curve shading

Shading of decomposed curves is determined by the sign of Jacobians (as in systems without delays). For complex s (3) quasicharacteristic equation is going to be:

$$f(\omega_n,\xi) = R_F(\omega_n,\xi,\tau,\alpha) + jI_F(\omega_n,\xi,\tau,\alpha)$$

$$R_F(\omega_n,\xi) = R_N(\omega_n,\xi) + \alpha e^{-\tau\omega_n\xi}\cdot\begin{bmatrix} R_D(\omega_n,\xi)\cdot\cos(\tau\omega_n\sqrt{1-\xi^2}) - \\ -I_D(\omega_n,\xi)\cdot\sin(\tau\omega_n\sqrt{1-\xi^2}) \end{bmatrix} \tag{8}$$

$$I_F(\omega_n,\xi) = I_N(\omega_n,\xi) + \alpha e^{-\tau\omega_n\xi}\cdot\begin{bmatrix} I_D(\omega_n,\xi)\cdot\cos(\tau\omega_n\sqrt{1-\xi^2}) + \\ +R_D(\omega_n,\xi)\cdot\sin(\tau\omega_n\sqrt{1-\xi^2}) \end{bmatrix} \tag{9}$$

Jacobians of the system as follows:

$$J = \begin{vmatrix} \frac{\partial R_F}{\partial\tau} & \frac{\partial R_F}{\partial\alpha} \\ \frac{\partial I_F}{\partial\tau} & \frac{\partial I_F}{\partial\alpha} \end{vmatrix} = \alpha\cdot\omega_n\sqrt{1-\xi^2}\cdot e^{-2\tau\omega_n\xi}\cdot r_D^2(\omega_n,\xi) \tag{10}$$

C.Singular lines

Singular lines, in the case of extracting area of pre-defined damping factor, is defined for boundary cases $\omega_n \to 0+$ and $\omega_n \to +\infty$, in (2), (6) and (7):

$$\alpha = \pm\frac{N(0)}{D(0)}e^{\lim\limits_{\omega_n\to 0+}\frac{\xi}{\sqrt{1-\xi^2}}\cdot\left[\Phi_N(\omega_n,\xi) - \Phi_D(\omega_n,\xi) + 2k\pi + \frac{\pi}{2} \pm \frac{\pi}{2}\right]} \tag{11}$$

$$\alpha = \pm\lim_{s\to+\infty} e^{\frac{k\pi\xi}{\sqrt{1-\xi^2}}}\frac{N(s)}{D(s)} \quad (12) \qquad \text{and } \tau = 0 \quad (13)$$

A special case for equations of decomposed curves is ξ = 1 and derived from (2) for s =- ω_n, and the curve of constant damping factor becomes:

$$\alpha = -\frac{N(-\omega_n)}{D(-\omega_n)}e^{\tau\omega_n} \tag{14}$$

Singular lines in that case are:

$$\alpha = -\frac{N(0)}{D(0)} \quad (15) \qquad \alpha = \pm\lim_{s\to\infty}\frac{N(s)}{D(s)} \quad (16)$$

The area between the lines (12) and (16) shoud be excluded from parametric plane. It could be emphasized that the method of shading singular lines is the same as for systems without delay, except for $\omega_n \to 0+$ and $\omega_n \to +\infty$, when also could be occurred singular lines for $\omega_n = \omega_n$ * which figure zero values in the denominator in the equation (7). Singularly lines in those cases are received by the merging points of curves break, which are getting for $\omega_n = \omega_n$ * substituted in (6) and (7). The procedure of selection periodicity factor k = k * is defined by procedure of selection the area of absolute stability for required automatic control system.[2]. Thus the first of all it is necessary to define area of absolute stability[2],[4],[7] in order to obtain corresponding period k = k*and with that value of k* we are getting (6) and (7) which extract the area of pre-defined damping factor in parametric three-dimensional space. That space shows the boundary from undamped to damped region and definethe area for parameters values α and τ (for pre-defined ξ). For a more detailed mathematical proof of how we choose factors periodicity see in [2].

III. APPLICATION THE METHOD – CIRCULATING RESEVOIR FOR MIXING LIQUIDS

Application of the methods described here will be illustrated by the example of circulating reservoir for mixing two liquids. A mathematical model is developed for control systems with proportional controller which gain is K = 1 / α and given object (Fig. 2) for some nominal parameter values, with time delay identical for both liquid flows and pre-defined damping factor ξ = 0,5. The open loop transfer of feedback system is:

$$W_{ok} = \frac{(2{,}31\cdot 10^{-4}s + 1{,}34\cdot 10^{-7})\cdot e^{-\tau s}}{\alpha\cdot(s^{2}+17{,}3\cdot 10^{-4}\cdot s + 61{,}5\cdot 10^{-8})} \qquad (17)$$

A. *Synthesis of closed - loop system*

We can see that we can approach the synthesis by comparing (17) i (1) according to the methods described in chapter III. Then equations (6), (7) and (10) become:

$$\alpha = \pm\sqrt{\frac{5{,}34\cdot 10^{-8}\omega_n^2 - 6{,}205\cdot 10^{-11}\omega_n\xi + 1{,}80\cdot 10^{-14}}{(2\cdot\omega_n^2\xi^2 - \omega_n^2 - 17{,}3\cdot 10^{-4}\omega_n\xi + 61{,}5\cdot 10^{-8})^2 + (17{,}3\cdot 10^{-4}\cdot\omega_n\sqrt{1-\xi^2} - 2\cdot\omega_n^2\xi\sqrt{1-\xi^2})^2}}\, e^{\tau\omega_n\xi} \qquad (18)$$

$$\tau = \frac{1}{\omega_n\sqrt{1-\xi^2}}\left[\begin{array}{l} arctg\dfrac{2{,}31\cdot 10^{-4}\omega_n\sqrt{1-\xi^2}}{-2{,}31\cdot 10^{-4}\omega_n\xi + 1{,}34\cdot 10^{-7}} - \\ -arctg\dfrac{17{,}3\cdot 10^{-4}\omega_n\sqrt{1-\xi^2} - 2\omega_n^2\sqrt{1-\xi^2}}{2\omega_n^2\xi^2 - \omega_n^2 - 17{,}3\cdot 10^{-4}\omega_n\sqrt{1-\xi^2} + 61{,}5\cdot 10^{-8}} \\ +2k\pi + \dfrac{\pi}{2} \pm \dfrac{\pi}{2} \end{array}\right] \qquad (19)$$

$$J = -\alpha\omega_n\sqrt{1-\xi^2}\, e^{-2\tau\omega_n\xi}\cdot\left\{\begin{array}{l} 2\omega_n^2\xi^2 - \omega_n^2 - 17{,}3\cdot 10^{-4}\omega_n\xi + 61{,}5\cdot 10^{-8}\ {}^2 + \\ +(17{,}3\cdot 10^{-4}\omega_n\sqrt{1-\xi^2} - \\ -2\omega_n^2\xi\sqrt{1-\xi^2})^2 \end{array}\right\} \qquad (20)$$

Using (18), (19) and (20) for ξ = 0.5 it is extracted the field of constant damping factor of that value. Singular lines and Jacobians are determined from (10), (11), (12) and (13) so we get two singular lines α = 0 and τ = 0, where according to (15) and (16) line α = 0 should be excluded from the extracting field. Areas given from extraction curves (18), (19) and (20) by rules of shading which defined values of parameters for system with damping factor ξ = 0.5 are shown on Fig.4(α > 0) and Fig.5 (α < 0) and noted like dumped.

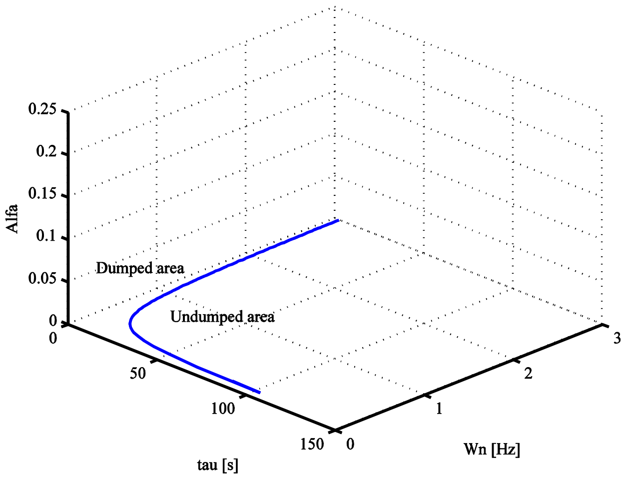

Fig. 4.Dumped area for α> 0

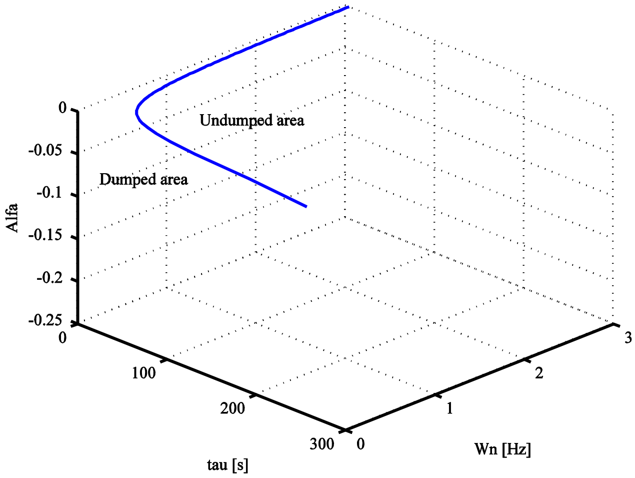

Fig. 5.Dumped area for α< 0

It could be possible to define undapmed from damped region for 3D space consisting of parametric plane α-τ and the a axis ω_n in an undamped frequency for damping factor ξ=0,5 which for ξ =0 represents the boundary of region of absolute stability. In that case results obtained by this method are the same like the well- known ones. [7],[8].

B. *Dynamic analysis of synthesized system*

From the ξ = 0.5 we highlight the point which determines the controller parameters α = 1/120 and τ = 25s, and on the basis of (17) receives the open loop transfer function of the system. Simulation of the system behavior is done with

MATLAB software with step function.
Simulation result of step response is shown on Fig. 6 for

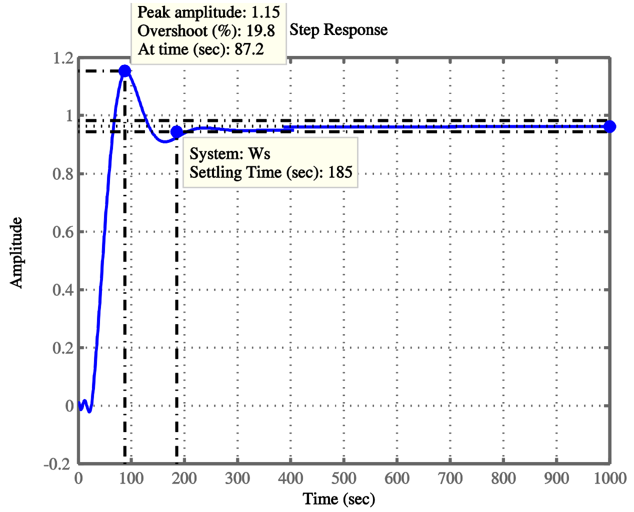

Fig. 6.Step response for feedback system (circulating reservoir for mixing liquids)

IV.CONCLUSION

New software package MATLAB enables to obtain more precocious D-decomposition method applied to adoption new principle of separation the field of pre-specified damping factor in the parametric plane of α-τ [1]. The results presenting here overview variation and dependences of parameters in three-dimensional form(α, τ, ω_n) where could be possible to choose from the given space curve the values for parameters which guarantee much better accuracy in methodology than the last obtained results [1], [2]. System behavior analysis was done from simulation of working condition by MATLAB software with step function and thus allows verification of system properties. It is also improving in process of applying method described in this paper.

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