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OPTIMIZATION OF THE BOX SECTION OF THE MAIN GIRDERS OF THE BRIDGE CRANE BY USING THE METHOD OF LAGRANGE MULTIPLIERS

The paper considers the problem of optimization of the box section of the main girder of the bridge crane for the case of placing the rail above the web plate. Reduction of the girder mass is set as the objective function. The method of Lagrange multiplier was used as the methodology for approximate determination of optimum dependences of geometrical parameters of the box section. The criterion of strength were applied as the constraint function. The analysis of the optimization results and the solutions was the basis for recommendations which are significant for designers during construction of cranes.

INTRODUCTION

The main task in the process of designing the carrying structure of the bridge crane is determination of optimum dimensions of the main girder box section. The mass of the main girder has the largest share in the total mass of the bridge crane, so it is very important to perform its optimization in order to reduce the total costs of manufacturing the whole carrying structure. That is the reason why the selection of the optimum shape and geometrical parameters which influence the reduction of mass and costs of manufacturing is the subject of research of a lot of authors ([2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [13], [14], [15] i [16]).

The analysis of cost structure for manufacturing metal structures made in [2], showed that the participation of material costs in the total costs is the largest (30-73) %, and that the other costs are lower**.**

Having in mind all the above mentioned results and conclusions, the aim of this paper is to define optimum values of geometrical parameters of the box girder cross-section that will lead to the reduction of its mass.

MATEMATHICAL FORMULATION OF THE OPTIMIZATION PROBLEM

The task of optimization is to define geometrical parameters of the cross section of the girder as well as their mutual relations, which result in its minimum area. Minimization of the mass corresponds to minimization of the volume, i.e. the area of the cross section of the girder, where the given boundary conditions must be satisfied.

The optimization problem defined in this way can be given the following general mathematical formulation.

If $\vec{x}=\left[x_1, x_2, ..., x_n\right]$ is vector of the given, and $\vec{y}=\left[y_1, y_2, ..., y_m\right]$ is vector of variable parameters, then the objective function is expressed as $F=F(\vec{x},\vec{y})$. Observed parameters have to satisfy the limitation equations also.

$$g_s(\vec{x},\vec{y}) \le 0, \; s=1,...,m. \tag{1}$$

The optimization task can be formulated in the following way: at defined vector of known parameters $\vec{x}$, there should be determined the responding values of variable parameters $\vec{y}_o=\vec{y}_o(\vec{x})$, where the objective function realizes the minimum

$$F_o=\min_{\vec{y}} F(\vec{x},\vec{y})=F_o(\vec{x}). \tag{2}$$

In this paper, variable parameters vector is $\vec{y}=\left[b,h\right]$, and the given parameters vector

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$\vec{x}=[Q,\ M_{cv},M_{ch},c,\ L,k_a,e_k,G_k...]$, where:
- b i h - the height and width of the girder,
- Q - the carrying capacity of the crane,
- M_{cv} i M_{ch} - the bending moments in the vertical and horizontal planes,
- c - the coefficient of influence of the dead weight of the girder on the bending moment
- L- the span of the crane,
- k_a - the dynamic coefficient of crane load in the horizontal plane, [12],
- e_k - the distance between the cab and the crane runway,
- G_k - the mass of the crane cab.

The paper observes the following limitation:

$$g=\sigma_{\max}-\sigma_k\le 0, \tag{3}$$

where:

$\sigma_{\max}$ - the maximum equivalent stress,

σ_k - the permissible stress.

The Lagrange function is defined in the following way:

$$\Phi=A+\lambda\cdot g, \tag{4}$$

where the following must be fulfilled:

$$\frac{\partial\Phi}{\partial b}=0;\frac{\partial A}{\partial b}+\lambda\cdot\frac{\partial g}{\partial b}=0, \tag{5}$$

$$\frac{\partial\Phi}{\partial h}=0;\frac{\partial A}{\partial h}+\lambda\cdot\frac{\partial g}{\partial h}=0, \tag{6}$$

$$\frac{\partial\Phi}{\partial\lambda}=0;\Rightarrow g=0. \tag{7}$$

OBJECTIVE FUNCTION

The objective function is represented by the area of the cross section of the box girder (Fig. 1). The paper treats two optimization parameters *(h, b).* The wall thicknesses t_1 and t_2 (Fig. 1) are not treated as optimization parameters for the purpose of simplification of the procedure. Their values were adopted in accordance with the recommendations of crane manufacturers [1].

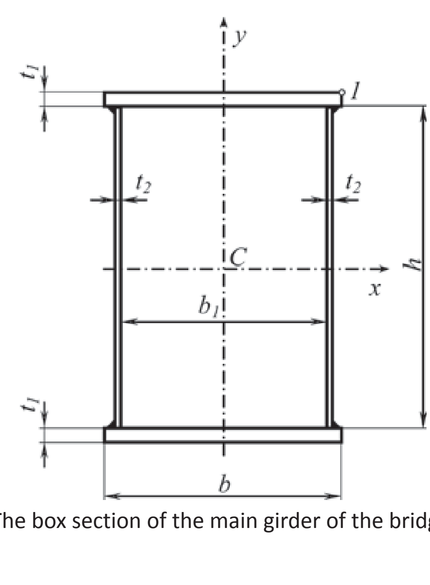

Fig. 1. The box section of the main girder of the bridge crane

The area of the cross section, i.e. the objective function, is:

$$A(h,b) = f(h,b) = 2\cdot(e\cdot b\cdot h + h^2)/s, \tag{8}$$

where:

$e = t_1/t_2$ - the ratio between thicknesses of plates at the flange and at the web,

$s = h/t_2$ - the ratio between the height and thickness of the plate at the web,

$k = h/b$ - the ratio between the height and width of the girder.

To know the optimal value of the ratio between the height and width of the girder *k* is of particular significance for the designer, especially in the initial design phase.

The expressions for the moments of inertia around the *x* and *y* axes are:

$$I_x = \frac{1}{6}\cdot\frac{h^4}{s} + \frac{1}{2}\cdot e\cdot b\cdot\frac{(s+e)^2}{s^3}\cdot h^3, \tag{9}$$

$$I_y = \frac{1}{6}\cdot e\cdot\frac{h}{s}\cdot b^3 + \frac{1}{2}\cdot\frac{h^2}{s}\cdot\frac{(f\cdot b\cdot s+h)^2}{s^2}, \tag{10}$$

where:

$f = b_1/b < 1$ - the ratio between the distance of web plates and the width of flange plates of the box girder.

Since the expressions for the moments of inertia (I_x, I_y) and the section moduli (W_x, W_y) are complex, it is common to take approximate values of expressions by neglecting the members of the lower order ([8], [9], [14], [15] i [16]):

$$I_x = \beta_x^2\cdot h^2\cdot A,\; W_x = \alpha_x\cdot h\cdot A, \tag{11}$$

$$I_y = \beta_y^2\cdot b^2\cdot A,\; W_y = \alpha_y\cdot b\cdot A, \tag{12}$$

where:

β_x, β_y - the dimensionless coefficient of the moment of inertia for the *x* and *y* - axes,

α_x, α_y - the dimensionless coefficient of the resistance moment of inertia for the *x* and *y* - axes.

The coefficients β_x and α_x are obtained from the conditions of equality of the equation (9) and the expression (11):

$$\beta_x = \frac{1}{2\cdot s}\cdot\sqrt{\frac{k\cdot s^2 + 3\cdot e\cdot(s+e)^2}{3\cdot(e+k)}},\; \alpha_x = \frac{2\cdot s}{s+2\cdot e}\cdot\beta_x^2. \tag{13}$$

Using the fact that $s \gg e$ and $s \gg k$ the coefficients with the form β_x and α_x can be simplified:

$$\beta`_x \cong \frac{1}{2}\cdot\sqrt{\frac{k+3\cdot e}{3\cdot(e+k)}},\; \alpha`_x \cong \frac{k+3\cdot e}{6\cdot(e+k)}. \tag{14}$$

By repeating the procedure for the moment of inertia and the section moduli for the *y* - axis, the following values of coefficients are obtained:

$$\beta_y = \frac{1}{2\cdot s}\cdot\sqrt{\frac{e\cdot s^2 + 3\cdot k\cdot(f\cdot s+k)^2}{3\cdot(e+k)}},\; \alpha_y = 2\cdot\beta_y^2, \tag{15}$$

i.e. in a simpler form:

$$\beta`_y \approx \frac{1}{2}\cdot\sqrt{\frac{3\cdot k\cdot f^2+e}{3\cdot(e+k)}},\; \alpha`_y = \frac{3\cdot k\cdot f^2+e}{6\cdot(e+k)}. \tag{16}$$

CONSTRAINT FUNCTION

The maximum equivalent stress which occurs in the main girder of the bridge crane is at point 1 (Fig. 1).

The constraint function according to this criterion is:

$$\sigma_{\max} = \sigma_{zV1} + \sigma_{zH1} = \frac{M_{VI}}{W_x} + \frac{M_{HI}}{W_y} \le \sigma_k , \tag{17}$$

$$\sigma_k = f_y / \nu_1 , \tag{18}$$

where:

f_y - the minimum yield stress of the plate material,

ν_1 - the factored load coefficient for load case 1.

The constraint function according to this criterion is:

$$g = g(h,b) = \frac{M_{cv} + c \cdot A}{\alpha_x \cdot h \cdot A} + \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b \cdot A} - \sigma_k \le 0 . \tag{19}$$

By eliminating the parameter λ from equations (5) and (6), we get:

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g}{\partial b} . \tag{20}$$

The corresponding partial derivatives are:

$$\begin{aligned} \frac{\partial g}{\partial b} &= -\left[\frac{M_{cv}}{\alpha_x \cdot h} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial b} + \frac{M_{ch}}{\alpha_y \cdot b} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial b} + \frac{M_{ch}}{\alpha_y} \cdot \frac{1}{A} \cdot \frac{1}{b^2} + \frac{k_a \cdot c}{\alpha_y} \cdot \frac{1}{b^2} \right], \\ \frac{\partial g}{\partial h} &= -\left[\frac{M_{cv}}{\alpha_x \cdot h} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial h} + \frac{M_{cv}}{\alpha_x} \cdot \frac{1}{A} \cdot \frac{1}{h^2} + \frac{c}{\alpha_x} \cdot \frac{1}{h^2} + \frac{M_{ch}}{\alpha_y \cdot b} \cdot \frac{1}{A^2} \cdot \frac{\partial A}{\partial h} \right]. \end{aligned} \tag{21}$$

By replacing the expression (21) in (20), after rearrangement, the following relation is obtained:

$$\frac{M_{cv} + c \cdot A}{\alpha_x \cdot h^2 \cdot A} \cdot \frac{\partial A}{\partial b} = \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b^2 \cdot A} \cdot \frac{\partial A}{\partial h} , \tag{22}$$

$$\frac{\partial A}{\partial b} = 2 \cdot e \cdot h / s, \quad \frac{\partial A}{\partial h} = 2 \cdot (e \cdot b + 2 \cdot h) / s. \tag{23}$$

Using (19), we get:

$$\frac{M_{cv} + c \cdot A}{\alpha_x \cdot h \cdot A} + \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b \cdot A} = \sigma_k . \tag{24}$$

Using relations (5) and (23), we get the parameter value λ :

$$\lambda = \frac{A^2}{\left(\frac{M_{cv}}{\alpha_x \cdot h} + \frac{M_{ch}}{\alpha_y \cdot b} + \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b^2 \cdot \frac{\partial A}{\partial b}} \cdot A \right)} . \tag{25}$$

From the equations system (22) and (24), it is necessary to find the optimal dimensions h and b by the given criterion, i.e. their relation k .

As the net weight moment is considerably smaller than the moment caused by active load, it can be written:

$$\frac{M_{cv} + c \cdot A}{M_{ch} + k_a \cdot c \cdot A} \approx \frac{M_{cv}}{M_{ch}} , \tag{26}$$

where now it is necessary to analyze this relation.

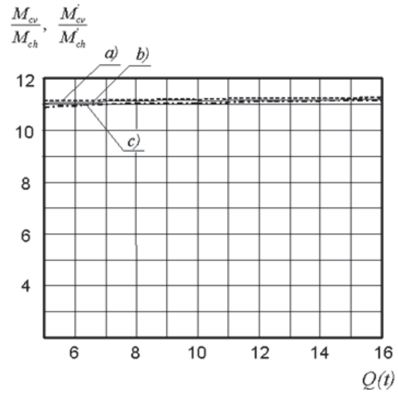

Fig. 2. Approximation of the ratio between bending moments in the vertical and horizontal planes

As $e_k \approx 2,5\,m$, [12] is notably smaller than the crane span L, it is shown in diagram (Fig. 2) that the cabin influence is not of great significance for this analysis, where the driving class 2 is observed.

Now it can be written:

$$\frac{M_{cv}}{M_{ch}} \approx \frac{M`_{cv}}{M`_{ch}} = \frac{1}{k_a} \cdot \frac{R \cdot (L-e_1)^2}{R_h \cdot (L-e_1)^2} = \frac{1}{k_a} \cdot \frac{\psi \cdot Q + m_k}{Q + m_k} = \frac{\psi \cdot Q + m_o + K \cdot Q^\alpha}{Q + m_o + K \cdot Q^\alpha}, \tag{27}$$

where:

K - the coefficient of influence of the classification class on the mass of the trolley,

ψ - the dynamic coefficient of the influence of load oscillation in the vertical plane,

α - the coefficient of influence of the load mass on the mass of the trolley,

m_o - the assumed mass of the trolley in the first approximation.

As we can see (Fig. 3), this relation depends both on capacity and driving class.

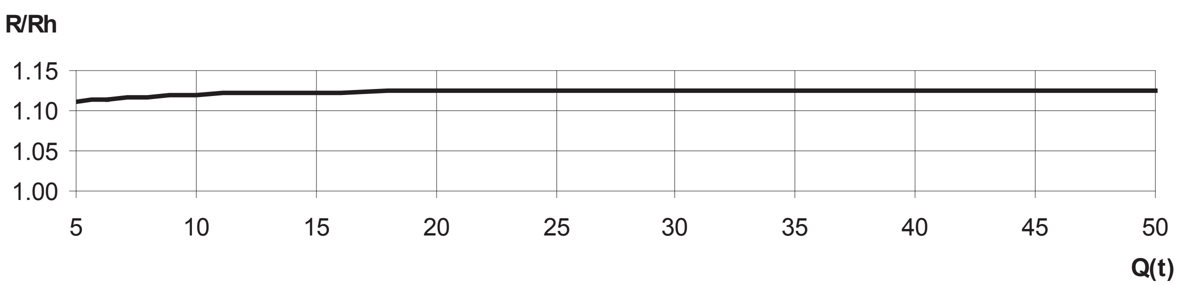

Fig. 3. The relationship between the load in the vertical and horizontal planes

If the relation (27) is shown by the functions:

$$\frac{M_{cv}}{M_{ch}} = \frac{1}{k_a} \cdot \frac{f_1(Q)}{f_2(Q)} = \frac{1}{k_a} \cdot \frac{\psi \cdot Q + m_o + K \cdot Q^\alpha}{Q + m_o + K \cdot Q^\alpha} = \frac{1}{k_a} \cdot c_1, \tag{28}$$

$$c_1 = \frac{R}{R_h} = \frac{f_1(Q)}{f_2(Q)} = \frac{\psi \cdot Q + m_o + K \cdot Q^\alpha}{Q + m_o + K \cdot Q^\alpha}, \tag{29}$$

and if it develops (29) into Taylor series, the expression (28) becomes,

$$\frac{M_{cv}}{M_{ch}} = \frac{1}{k_a} \cdot \frac{f_1(Q)}{f_2(Q)} \approx \frac{1}{k_a} \cdot \frac{f_1(Q_o) + (Q - Q_o) \cdot f_1`(Q_o)}{f_2(Q_o) + (Q - Q_o) \cdot f_2`(Q_o)}. \tag{30}$$

Now the expression (22) can be written in the following way:

$$\frac{M_{cv} + c \cdot A}{M_{ch} + k_a \cdot c \cdot A} \cdot \frac{\partial A}{\partial b} \approx \frac{M_{cv}}{M_{ch}} \cdot \frac{\partial A}{\partial b} = \frac{\alpha_x \cdot h^2}{\alpha_y \cdot b^2} \cdot \frac{\partial A}{\partial h}. \tag{31}$$

Corresponding partial extracts of the objective function (8), by the variables *b* and *h* read:

$$\frac{\partial A}{\partial b}=2\cdot e\cdot t_2,\ \frac{\partial A}{\partial h}=2\cdot t_2\,. \tag{32}$$

Replacing relations (32) into (31), we finally get:

$$k_{op}=\sqrt{\frac{e\cdot\alpha_y}{\alpha_x}\cdot\frac{M_{cv}}{M_{ch}}}=\sqrt{\frac{e\cdot\alpha_y}{\alpha_x}\cdot\frac{c_1}{k_a}}\,. \tag{33}$$

When the relation k_{op} is known, from the limitation equation (19) the girder height is determined. Objective function can be written in the following way.

$$A=\frac{2}{s}\cdot(\frac{e}{k}+1)\cdot h^2\,. \tag{34}$$

From relations (19) and (34) we get:

$$h_{op}{}^3-\frac{\left(\frac{c}{\alpha_x}+\frac{k_a\cdot c}{\alpha_y}\cdot k_{op}\right)}{\sigma_k}\cdot h_{op}{}^2-\frac{s}{2}\cdot\frac{\left(\frac{M_{cv}}{\alpha_x}+\frac{M_{ch}}{\alpha_y}\cdot k_{op}\right)}{\sigma_k\cdot(e/k_{op}+1)}=0\,. \tag{35}$$

By solving the equation (35), we come to the wanted solution of optimal height. Optimal width is obtained by the expression:

$$b_{op}=h_{op}/k_{op}\,. \tag{36}$$

Limitation function can also be written in the following form:

$$A_{op}=A(h)\geq\frac{\frac{M_{cv}}{\alpha_x}+\frac{M_{ch}}{\alpha_y}\cdot k_{op}}{\sigma_k\cdot h-\frac{c}{\alpha_x}-\frac{k_a\cdot c}{\alpha_y}\cdot k_{op}}\,. \tag{37}$$

NUMERICAL REPRESENTATION OF THE RESULTS OBTAINED

Using the expression (33) the optimum value of the parameter *k* according to the criterion of permissible stress is obtained as a function of the *Q* and k_a (Tab.1).

Using the expressions (8), (25), (35) and (36) the optimum values of the parameters *h*, *b*, *A* and λ according to the criterion of permissible stress are shown in Tab.2.

Table 1

Q (t) / k_a	5	6,3	8	10	12,5	16
0,085	4,67	4,68	4,68	4,69	4,69	4,70
0,1	4,30	4,31	4,32	4,32	4,33	4,33
0,115	4,01	4,02	4,03	4,03	4,03	4,04

Table 2

	S235	S275	S355
h (cm)	117,9	108,8	95
b (cm)	27,2	25,1	21,9
A (cm^2)	173,1	158,8	138,9
λ	5,85	4,57	3,08

The expression (37) represents the objective function obtained from the constraint function according to the criterion of permissible stress and together with the objective function (34) it can be graphically represented. At the intersection of these curves, on the abscissa, there is an optimum height *h* for the constraint function according to the criterion of permissible stress. Figure 4 shows how the position of the intersection point changes depending on the selection of material (solid line for S235, dash line for S275 and dot line for S355).

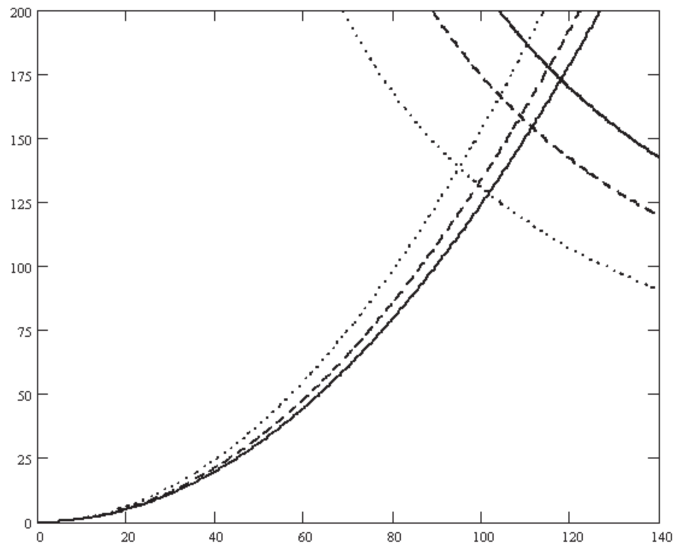

Fig. 4. Optimum values of the girder height and the objective function according to the strength

One of the main parameters figuring in the objective function (8) is slenderness s, defined as ([12]):

$$160 \cdot \sqrt{23,5/f_y} < s \le 265 \cdot \sqrt{23,5/f_y}\ . \tag{38}$$

For initial analysis, the medium values can be adopted, so that for $\mathrm{S235}$ we take the value $s = 210$. Other parameter values, at this stage, are: $e = 1,33,\ f = 0,85,\ k_a = 0,1,\ e_k = 2,3\,m,\ G_k = 15\,kN$. To perform the analysis, it is necessary to perceive the recommendations listed in the standard, but also those given by the crane producers [1]. Recommendation of Serbian crane producers is that the minimum width value b_1 is $b_1 > 30\,cm$, from which we get:

$$k \le f \cdot h/30\,, \tag{39}$$

while the sheet metal stability condition of the upper belt, with appropriate transformations, is defined as:

$$k \ge \frac{s \cdot f}{65 \cdot e} \cdot \sqrt{23,5/f_y}\ . \tag{40}$$

If we equate the expressions (7) and (37), we get the parameter dependency k, by hardness criterion:

$$k = F\left(s, e, h, M_{cv}, M_{ch}, \alpha_x, \alpha_y\right). \tag{41}$$

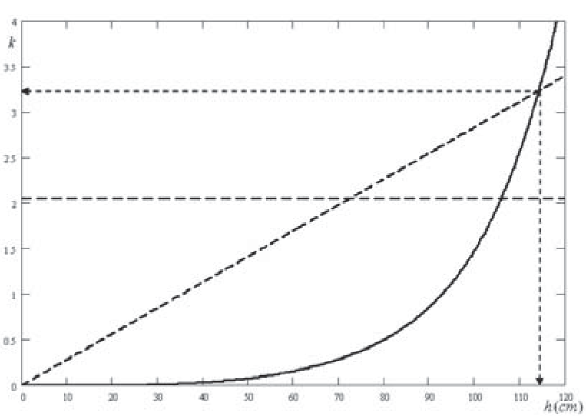

Fig. 5. Determination of the optimum value of the parameter *k* for the crane span *L=18m* and the carrying capacity *Q=16t*

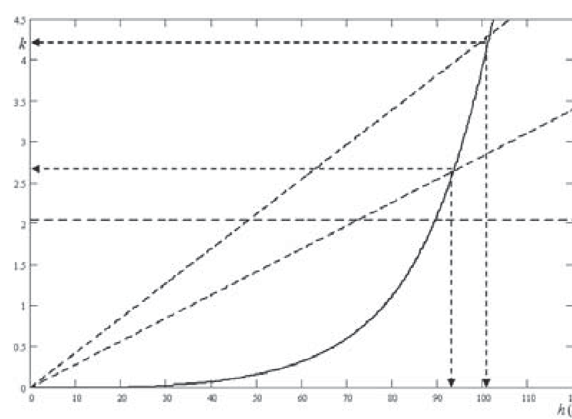

Fig. 6. Determination of the optimum value of the parameter *k* for the crane span *L=15m* and the carrying capacity *Q=12,5t*

In diagrams (Fig. 5 and Fig. 6) are shown the obtained optimal geometric parameters for characteristic capacities and spans of two pillar bridge cranes. Thus performed procedure enables fast and effective determining of optimal value of parameter *k* by critical function. Width value b_1 does not influence the optimization procedure, but it influences the obtained values of optimization parameter *k* (Fig. 6).

CONCLUSION

The paper defined optimum dimensions of the box section of the main girder of the bridge crane in an analytical form, by using the method of Lagrange multipliers according to permissible stress. The objective function is the minimum mass, i.e. the minimum area of the cross section. The results obtained may be of great use to the engineer-designer, particularly in the first phase of the design procedure when the basic dimensions of the main girder of the bridge crane, as its most responsible part, are defined.

Justification of applying the method of Lagrange multipliers was also shown because the optimization results were obtained in an analytical form, which allows drawing conclusions on the influence of certain parameters and directions of further research concerning the reduction of mass.

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