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Design of PI Controllers for the Hydraulic Control System with a Long Transmission Line

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Abstract ***– This paper presents design of a PI controller for the system of a pump-controlled motor with a long transmission line. The D–decomposition method including system performances, damping and settling time was applied. The system performances are included in a new manner without the need for calculation of Chebyshev functions for every change of the damping coefficient thanks to strong sofware support.***

Key words: ***PI controller, control system, long transmission line, root locus, transfer function***

I. INTRODUCTION

Increasingly strict and wide requirements regarding displacement hydrostatic power transmitters have recently appeared in the sense of simultaneous accomplishment of high power exploitation degrees, high speed of response with the reduction of price [1-4]. This particularly refers to high power systems and systems with variable load (building and mining machines, agricultural machines, transportation machines, machine tools, etc). It is obvious that these requirements result in the need for more intense development of systems with displacement control in relation to the systems with damping control. It is obvious that these requirements result in the need for more intense development of systems with displacement control in relation to the systems with damping control. One of the main preconditions for quality and reliable operation of a high power system is the stable and quality operation of the system for automatic control of hydrostatic power transmitter, the pump-controlled motor with long transmission lines (Figure 1). The existence of a long transmission line in this system makes its dynamics rather complex because the physical values, pressure and flow, which characterize the transfer of energy along the long transmission line depend both on the time coordinate and the space coordinate. Dependence of these physical values on the space coordinate, too, conditions that in the mathematical description of the long transmission line the space distribution cannot be neglected, so that it is described by a model with distributed parameters. Models with distributed parameters are described by partial differential equations and the model obtained is of an infinitesimally high order [5-9]. In addition to mathematical modeling of the long transmission line by means of the model with distributed parameters, it is possible to describe the long transmission line by common differential equations, i.e. models with lumped parameters [1-4] because solving common differential equations makes considerably fewer difficulties in comparison with solving partial differential equations. The authors of this paper considered the problem of modelling and dynamic behaviour of such systems in a very systematic way, and the results are presented in several papers, the most significant of which are [4], [10] and [12]. Reference [10] gives a complete mathematical model of the system of a pump-controlled motor with a long transmission line by means of a model with lumped parameters where the long transmission line is divided into n equal "Π" segments. The mathematical model thus obtained is of high order but by applying the appropriate methodology its order is reduced, which considerably increases its use value. From the aspect of control, reference [10] presents design of a P controller by applying the Nyquist criterion, including system performances, damping and settling time. The P controller designed in this way, for the described system, eliminates the occurrence of oscillations of the transfer characteristic due to the existence of the long transmission line. However, design of the P controller could not eliminate the error, so that a PI controller was designed in order to solve that problem.

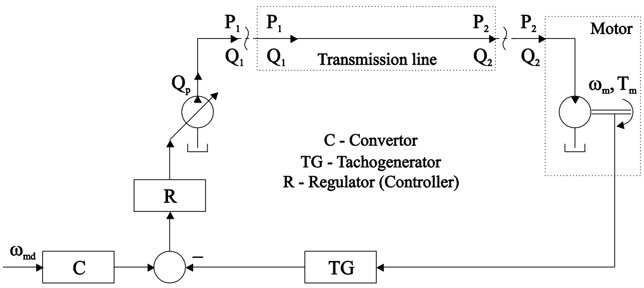

Fig. 1. Symbolic diagram of the closed automatic control system of a pump-controlled motor with a long transmission line

II. DESIGN OF A PI CONTROLLER

Design of a PI controller was performed by applying the D–decomposition method , including system performances. The open loop transfer function was determined on the basis of the block diagram shown in Figure 2.

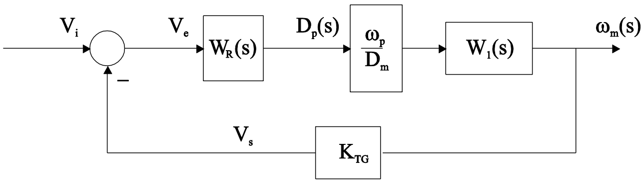

Fig. 2. Block diagram of the system

The transfer function of a part of the system W1(s) is given by Equation (1) and it is in a detailed way derived in reference [10].

$$W_1(s) = \frac{D_m \omega_m(s)}{\omega_p D_p(s)} = \frac{1}{\sum_{i=0}^{n} a_i s^i} \quad (1)$$

$$W_{ok} = K_{ok} W_R(s) W_1(s) \quad (2)$$

$$W_R(s) = k_p + \frac{k_I}{s} = \frac{k_p s + k_I}{s} \quad (3)$$

$$W_1(s) = \frac{\omega_m(s)}{D_p(s)} = \frac{1}{\sum_{i=0}^{n} a_i s^i} \quad (4)$$

$$W_{ok} = K_{ok} \frac{k_p s + k_I}{s \sum_{i=0}^{n} a_i s^i} = K_{ok} \frac{k_p s + k_I}{\sum_{i=0}^{n} a_i s^{i+1}} \quad (5)$$

$$f(s) = 1 + W_{ok}(s) = 0 \quad (6)$$

$$f(s) = \sum_{i=0}^{n} a_i s^{i+1} + K_{ok} k_p s + K_{ok} k_I \quad (7)$$

$$f(s) = a_n s^{n+1} + a_{n-1} s^n + \ldots + a_1 s^2 + a_0 s + K_{ok} k_p s + K_{ok} k_I \quad (8)$$

$$f(s) = a_n s^{n+1} + a_{n-1} s^n + \ldots + a_1 s^2 + \alpha s + \beta \quad (9)$$

$$\alpha = K_{ok}(a_0 + k_p) \qquad \beta = K_{ok} k_I \quad (10)$$

$$f(s) = R(s) + \alpha M(s) + \beta Q(s) \quad (11)$$

$$R(s) = \sum_{i=1}^{n} a_i s^{i+1} \quad (12)$$

$$R(j\omega) = R_1(\omega) + jR_2(\omega) \quad (13)$$

$$R_1(\omega) = -a_9 \omega^{10} + a_7 \omega^8 - a_5 \omega^6 + a_3 \omega^4 - a_1 \omega^2 \quad (14)$$

$$R_2(\omega) = -a_{10} \omega^{11} + a_8 \omega^9 - a_6 \omega^7 + a_4 \omega^5 - a_2 \omega^3 \quad (15)$$

$$M(j\omega) = M_1(\omega) + jM_2(\omega) \quad (16)$$

$$M_1(\omega) = 0 \qquad M_2(\omega) = \omega \quad (17)$$

$$Q(j\omega) = Q_1(\omega) + jQ_2(\omega) \quad (18)$$

$$Q_1(\omega) = 1 \qquad Q_2(\omega) = 0 \quad (19)$$

$$\Delta(\omega) = \begin{vmatrix} M_1(\omega) & Q_1(\omega) \\ M_2(\omega) & Q_2(\omega) \end{vmatrix} = -\omega$$

$$\Delta(\omega) < 0; \qquad \omega > 0 \quad (20)$$

$$\alpha(\omega) = \frac{Q_1(\omega) R_2(\omega) - R_1(\omega) Q_2(\omega)}{\Delta(\omega)} = -\frac{R_2(\omega)}{\omega} \quad (21)$$

$$\beta(\omega) = \frac{M_2(\omega) R_1(\omega) - R_2(\omega) M_1(\omega)}{\Delta(\omega)} = -R_1(\omega) \quad (22)$$

$$\beta(\omega) = \frac{a_9 \omega^8 - a_7 \omega^6 + a_5 \omega^4 - a_3 \omega^2 + a_1}{a_{10} \omega^8 - a_8 \omega^6 + a_6 \omega^4 - a_4 \omega^2 + a_2} \alpha(\omega) \quad (23)$$

$$k_I = \frac{a_9 \omega^8 - a_7 \omega^6 + a_5 \omega^4 - a_3 \omega^2 + a_1}{a_{10} \omega^8 - a_8 \omega^6 + a_6 \omega^4 - a_4 \omega^2 + a_2} (a_0 + k_p) \quad (24)$$

On the basis of the obtained Equation (24), Figure 3 presents the area of stable operation of the system. Selection of the parameters Kp and Ki below the curve in the area of stable operation of the system defines the transfer function of the PI controller which guarantees stable operation of the system. However, the figure does not include system performances, damping and settling time, which will guarantee both stable and quality operation of the described system.

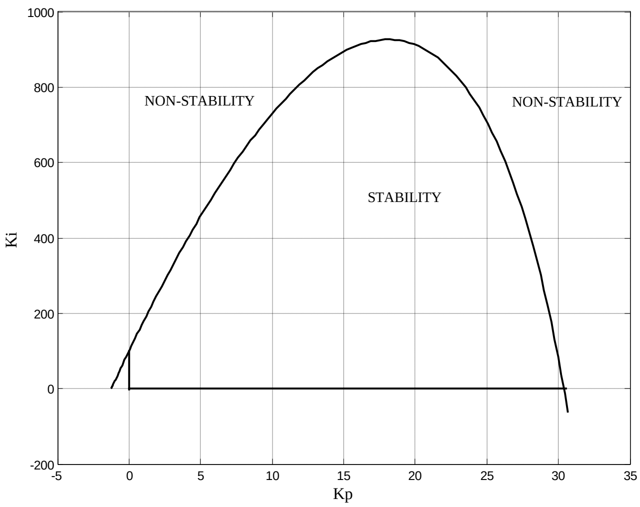

Fig. 3. Area of stability for selection of parameters of the PI controller

System performances are here included in a completely new manner by the root locus method, which was in a detailed way described in design of the P controller [10]. Thanks to strong Matlab support, a program was written by which diagrams of the root locus of the closed loop in the network of system performances, damping and settling time are obtained. The hodograph of the root locus presented in Figure 4 allows determination of the gain Kp for all values of the damping coefficient from the interval $0 \le \xi \le 1$, without using the analytical expression for the relationship between the damping coefficient and the transfer function given in [11].

$$W(\omega_{n,\xi}) = \frac{\sum_{k=0}^{n} (-1)^k a_k \omega_n^k T_k(\xi) + j\sqrt{1-\xi^2} \sum_{k=0}^{n} (-1)^{k+1} a_k \omega_n^k U_k(\xi)}{\sum_{k=0}^{m} (-1)^k b_k \omega_n^k T_k(\xi) + j\sqrt{1-\xi^2} \sum_{k=0}^{m} (-1)^{k+1} b_k \omega_n^k U_k(\xi)} \quad (25)$$

On the basis of Equation (25) it is possible to determine the gain Kp for the strictly defined damping value $\xi = \text{const.}$, and every change of damping requires repeated calculation of the Chebyshev functions of the first and second kinds $T(\xi)$ and $U(\xi)$. By applying Matlab, and on the basis of the hodograph of the root locus in the network of damping coefficients $0 \le \xi \le 1$, the gain Kp, overshoot and frequency for the requested damping coefficient ξ are obtained without counting the Chebyshev functions $T(\xi)$ and $U(\xi)$. By selecting the gain Kp<10.6, from the area $0.6 < \xi < 1$, for a certain length of the transmission line and by combining it with the gain Ki on the basis of Figure 3, the transfer function of the controller which now includes system performances is obtained.

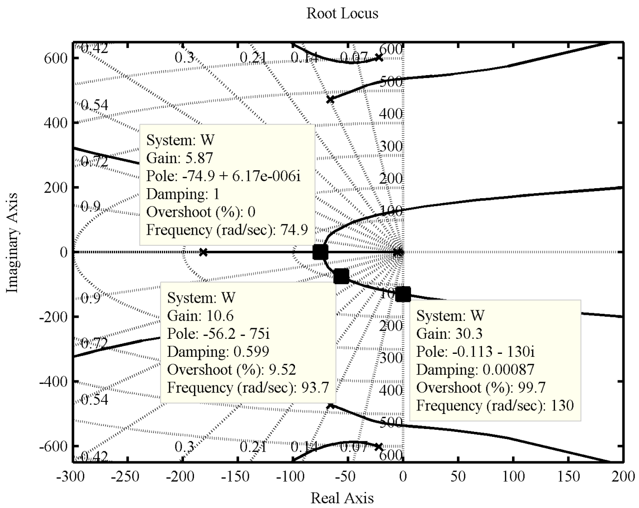

Fig. 4. Root locus of the closed loop

III. DYNAMIC BEHAVIOUR OF THE SYSTEM CONTROLLED BY THE PI CONTROLLER

On the basis of performed synthesis of the transfer function of the PI controller, which includes system performances, a program was written in Matlab and it was the basis for simulations of dynamic behaviour of the system. Figure 5 shows system responses for three different values of paramaters of the gains Kp and Ki.

The comparative response of the system controlled by the P controller and the PI controller for the gain Kp = 10, which guarantees the quality operation of the system, is presented in Figure 6.

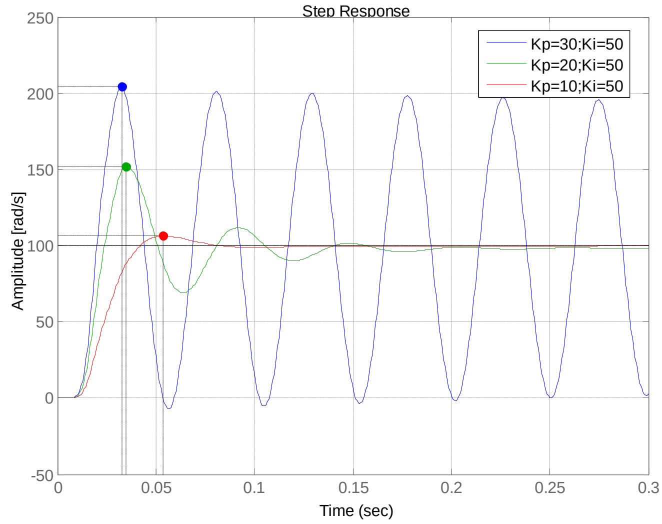

Fig. 5. System response at the transmission line length of l=16m, the line being controlled by the PI controller

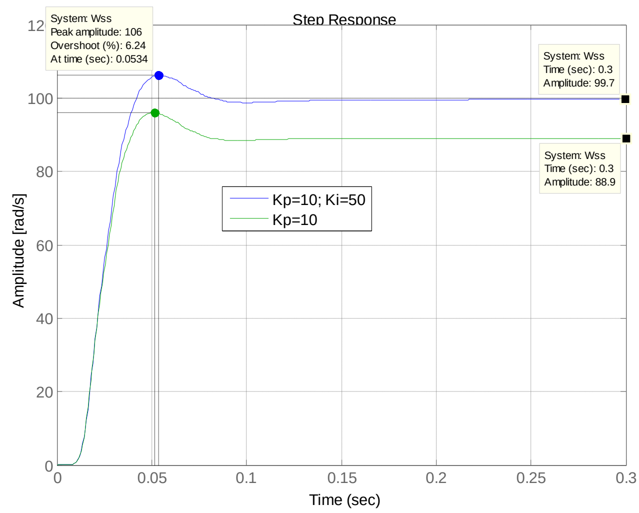

Fig. 6. Comparative response of the system at the transmission line length of l=16m, the line being controlled by P and PI controllers

Selection of these values of gain for the transmission line length of 16 m results in obtaining the overshoot value of 6.24% in the system response, where the settling time is 70 ms, and the static error is reduced to 0 because of the introduction of the PI controller.

IV. CONCLUSION

The PI controller is proposed for regulating hydraulic systems with long lines. By using the possibilities offered by computers and software the graphical method is used in the design of PI controllers. This type of controllers frequently satisfies practical needs because the feedback loop system is fast, and the static error is reduced to 0. The proposed method allows easy design of PI controllers in significant modifications of lengths of the transmission line. The system described in this paper provides significant energy savings.

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REFERENCES

[1] Watton J., “Fluid Power Systems”, Prentice Hall, 1989.
[2] Watton J., “The dynamic performance of an electrohydraulic servovalve /motor system with transmission line effects”, ASME Journal Dynamic Systems, Measurement and Control, 109, pp. 14-18, 1987.
[3] Watton J., Tadmori M., ”A comparison of techniques for the analysis of transmission line dynamics in electrohydraulic control systems”, Journal of Applied Mathematical Modelling, 12, pp. 457-466. 2001.
[4] N.N. Nedić, LJ.M. Dubonjić, “The Stability and Response of the Electrohydraulic Valve Controlled Hydromotor with Long Transmission Flow Line”, VIII International SAUM Conference, Belgrade, pp. 186-193, 2004.
[5] Stecki J.S., Davis D.C., “Fluid transmission lines – distributed parameter models” Part 1: a review of the state of the art. Proc IMechE, 200, Part A, pp. 215-228, 1986.
[6] Stecki J.S., Davis D.C., “Fluid transmission lines – distributed parameter models” Part 2: comparison of models. Proc IMechE, 200, Part A, pp. 229-236, 1986
[7] Hullender D.A., Healey A.J., “Rational Polynomial approximation for fluid transmission models” Fluid Transmission line dynamics, published by ASME, pp. 33-56, 1981.
[8] Hsue C.Y., Huulender, D.A., “Modal approximations for the fluid dynamics of hydraulic and pneumatic transmission lines”, Fluid Transmission line dynamics, published by ASME, pp. 51-77, 1983.
[9] Hullender D.A., Woods R. Hsue C., “Time Domain Simulation of fluid Transmission Lines using Minimum Order State Variable Models” Fluid Transmission Line Dynamics, published by ASME, pp. 78-97, 1983.
[10] Nedić N., Filipović V., Dubonjić LJ., “Design of Controllers With Fixed Order for Hydraulic Control System With a Long Transmission Line”, FME Transactions (2010) Vol. 38 No. 2, pp. 79-86, 2010.
[11] M. R. Stojić, D.D. Šiljak, “Generalization of Hurwitz, Nyquist and Mikhailov Stability Criteria”, IEEE Transactions on Automatic Control AC-10, pp. 250-255, 1965.
[12] Lj. M. Dubonjić, “Dynamical analysis of electro-hydraulic control systems with long transmission line”, Master thesis, Faculty of Mechanical Engineering, Kraljevo, 2002 (in Serbian).
[13] Ho M.T., Data A., Battacharya, “Control system design using low order controllers: Constant Gain, PI and PID”, Proceedings of the American control Conference, pp. 571-578, Albuquerque, USA,1997.