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Original scientific paper

OPTIMIZATION OF THE BOX SECTION OF THE MAIN GIRDER OF THE DOUBLE BEAM BRIDGE CRANE ACCORDING TO THE CRITERIA OF LATERAL STABILITY AND LOCAL STABILITY OF PLATES

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Abstract: *The paper considers the problem of optimization of the box section of the main girder of the double beam bridge crane for the case of placing the rail above the web plate. Reduction of the girder mass is set as the objective function. Research on the mutual dependence of geometric parameters of cross section, and their ratio, using the method of Lagrange multiplier as characteristic criteria is done. Also, the paper shows an algorithm for selection of the optimal criteria and determining the optimal geometric parameters of cross section. The criteria of lateral stability and local stability of plates were applied as the contraint functions. The obtained results of optimization of geometrical parameters were verified on numerical examples.*

Key words: *box section, double beam bridge crane, optimization, lateral stability, local stability of plates*

1. INTRODUCTION

The goal in the process of designing the carrying structure of the double beam bridge crane is determination of optimum dimensions of rectangular box cross-section of the main girder. The main girder is the most responsible part of the double beam bridge crane and therefore it is necessary, during optimization, to affect the increase in its carrying capacity with simultaneous reduction of its mass. The mass of the main girder has the largest share in the total mass of the bridge crane.
That is the reason why the selection of the optimum shape and geometrical parameters, which influence the reduction of mass and costs of manufacturing, is the subject of research of many authors regardless of whether they deal specifically with cranes or carrying structures in general ([1-14]). Most authors set permissible stress or two constraint functions: permissible stress and permissible deflection as the constraint function. The criterion of lateral stability has lately been increasingly applied as the constraint function ([7], [11], [12] and [13]). Having in mind all the above mentioned results and conclusions, the aim of this paper is to define optimum values of rectangular box cross-section of the main girder that will lead to the reduction of its mass.

2. MATHEMATICAL FORMULATION OF THE OPTIMIZATION PROBLEM

The task of optimization is to define geometrical parameters of the cross section of the girder as well as their mutual relations, which result in its minimum area. Minimization of the mass corresponds to minimization of the volume, i.e. the area of the cross section of the girder, where the given boundary conditions must be satisfied. The area of the cross section primarily depends on: height and width of the girder, thickness of plates and their mutual relations.
The optimization problem defined in this way can be given the following general mathematical formulation:
minimize $f(\boldsymbol{X})$ subject to $g(\boldsymbol{X}) \le 0$, where:
$f(\boldsymbol{X})$ the objective function ,
$g(\boldsymbol{X}) \le 0$ the constraint function,
$\boldsymbol{X} = \{x_1, ..., x_D\}^T$ represents the design vector made of D design variables. Design variables are the values that should be defined during the optimization procedure.
In this paper optimization for the criterion of lateral stability (3.1) and the criterion of local stability of plates was performed (3.2):

$$g_1 = \sigma_{r1} - \sigma_{k1} \le 0\,, \tag{1.1}$$

$$g_2 = \sigma_{r2} - \sigma_{k2} \le 0\,, \tag{1.2}$$

where:
σ_{r1}, σ_{k1} - the calculation and permissible stresses in lateral buckling of the girder,
σ_{r2}, σ_{k2} - the calculation and permissible stresses local stability of plates.
The Lagrange function is defined in the following way:

$$\Phi = A + \lambda_1 \cdot g_1 + \lambda_2 \cdot g_2\,, \tag{2}$$

$$\frac{\partial \Phi}{\partial b} = 0; \frac{\partial A}{\partial b} + \lambda_1 \cdot \frac{\partial g_1}{\partial b} + \lambda_2 \cdot \frac{\partial g_2}{\partial b} = 0\,, \tag{3.1}$$

$$\frac{\partial \Phi}{\partial h} = 0; \frac{\partial A}{\partial h} + \lambda_1 \cdot \frac{\partial g_1}{\partial h} + \lambda_2 \cdot \frac{\partial g_2}{\partial h} = 0\,, \tag{3.2}$$

$$\frac{\partial \Phi}{\partial \lambda_1} = 0; \Rightarrow g_1 = 0\,,\ \frac{\partial \Phi}{\partial \lambda_2} = 0; \Rightarrow g_2 = 0\,, \tag{3.3}$$

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$$\lambda_1 \cdot \left(\frac{\partial g_1}{\partial b} \cdot \frac{\partial A}{\partial h} - \frac{\partial g_1}{\partial h} \cdot \frac{\partial A}{\partial b} \right) + \lambda_2 \cdot \left(\frac{\partial g_2}{\partial b} \cdot \frac{\partial A}{\partial h} - \frac{\partial g_2}{\partial h} \cdot \frac{\partial A}{\partial b} \right) = 0$$

Since $\lambda_1, \lambda_2 \neq 0$, it is obtained:

1. $$\frac{\partial A}{\partial b} \cdot \frac{\partial g_1}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_1}{\partial b} \ \wedge \ g_1 = 0 \tag{4}$$

2. $$\frac{\partial A}{\partial b} \cdot \frac{\partial g_2}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_2}{\partial b} \ \wedge \ g_2 = 0 \tag{5}$$

3. OBJECTIVE AND CONSTRAINT FUNCTIONS

3.1. Objective function

The objective function is represented by the area of the cross section of the box girder. The paper treats two optimization parameters *(h, b)*. The wall thicknesses t_1 and t_2 (Fig. 1) are not treated as optimization parameters for the purpose of simplification of the procedure. Their values were adopted in accordance with the recommendations of crane manufacturers [14].

The vector of the given parameters is:

$$\vec{x} = \left(M_{cv}, M_{ch}, Q, L, \sigma_o, G_k, E, k_a, ... \right) \tag{6}$$

where:

M_{cv} and M_{ch} are the bending moments in the vertical and horizontal planes, Q – the carrying capacity of the crane,, L- the span of the crane, G_k - the mass of the crane cab, E - the module of elasticity of the main girder of the crane, k_a – the dynamic coefficient of crane load in the horizontal plane [10].

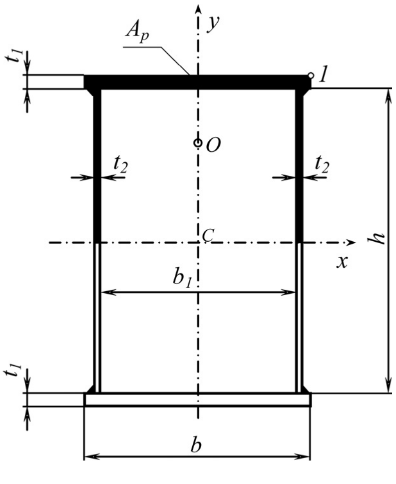

Fig.1. The box section of the main girder of the bridge crane

The area of the cross section, i.e. the objective function, is:

$$A(h,b) = f(h,b) = 2 \cdot (e \cdot b \cdot h + h^2) / s \ , \tag{7}$$

where:

$e = t_1 / t_2$ - the ratio between thicknesses of plates at the flange and at the web,

$s = h / t_2$ - the ratio between the height and thickness of the plate at the web,

$k = h / b$ - the ratio between the height and width of the girder.

To know the optimal value of the ratio between the height and width of the girder k is of particular significance for the designer, especially in the initial design phase so that its determination is the subject of research in a large number of papers ([3], [5], [6] and [12]).

The expressions for the moments of inertia around the x and y axes are:

$$I_x = \frac{1}{6} \cdot \frac{h^4}{s} + \frac{1}{2} \cdot e \cdot b \cdot \frac{(s+e)^2}{s^3} \cdot h^3 \ , \tag{8}$$

$$I_y = \frac{1}{6} \cdot e \cdot \frac{h}{s} \cdot b^3 + \frac{1}{2} \cdot \frac{h^2}{s} \cdot \frac{(f \cdot b \cdot s + h)^2}{s^2} \ , \tag{9}$$

where:

$f = b_1 / b < 1$ - the ratio between the distance of web plates and the width of flange plates of the box girder.

Since the expressions for the moments of inertia *(I_x, I_y)* and the moments of resistance *(W_x, W_y)* are complex, it is common to take approximate values of expressions by neglecting the members of the lower order ([5], [6] and [12]):

$$I_x = \beta_x^2 \cdot h^2 \cdot A \ , \ W_x = \alpha_x \cdot h \cdot A \ , \tag{10}$$

$$I_y = \beta_y^2 \cdot b^2 \cdot A \ ; \ W_y = \alpha_y \cdot b \cdot A \ , \tag{11}$$

where:

β_x, β_y - the dimensionless coefficient of the moment of inertia for the x and y – axes,

α_x, α_y - the dimensionless coefficient of the resistance moment of inertia for the x and y – axes.

The coefficients β_x and α_x are obtained from the conditions of equality of the equation (8) and the expression (10):

$$\beta_x = \frac{1}{2 \cdot s} \cdot \sqrt{\frac{k \cdot s^2 + 3 \cdot e \cdot (s+e)^2}{3 \cdot (e+k)}} \ ,$$

$$\alpha_x = \frac{2 \cdot s}{s + 2 \cdot e} \cdot \beta_x^2 \ . \tag{12}$$

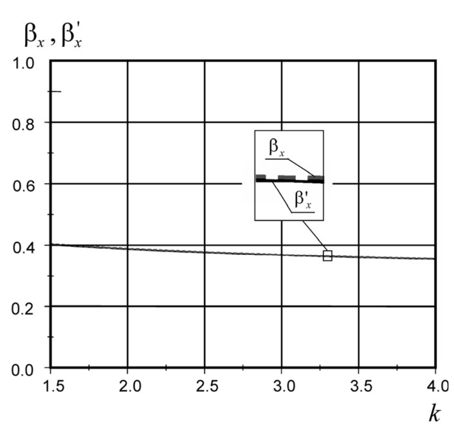

Fig.2. Approximation of the coefficient of the moment of inertia around the x-axis

Using the fact that $s \gg e$ and $s \gg k$ the coefficients with the form β_x and α_x can be simplified:

$$\beta`_x \cong \frac{1}{2}\cdot\sqrt{\frac{k+3\cdot e}{3\cdot(e+k)}}\,,\ \alpha`_x \cong \frac{k+3\cdot e}{6\cdot(e+k)}\,. \tag{13}$$

This approximation can be graphically represented (Figure 2), where it is seen that deviations are negligible in the considered range of parameters *k*.

By repeating the procedure for the moment of inertia and the moment of resistance for the *y* – axis, the following values of coefficients are obtained:

$$\beta`_y \approx \frac{1}{2}\cdot\sqrt{\frac{3\cdot k\cdot f^2+e}{3\cdot(e+k)}}\,,\ \alpha`_y = \frac{3\cdot k\cdot f^2+e}{6\cdot(e+k)}\,. \tag{14}$$

3.2. Constraint functions

3.2.1. The criterion of lateral stability

Testing of the box girder stability against lateral buckling was carried out in compliance with the Serbian standards of the group [8]. In accordance with the standards [8], the compression zone of the box girder is observed as an independent bar which is controlled against buckling due to the equivalent force arising from the bending moment of the girder (Figure 3).

The pressure force D of the imagined bar acts at point "*o*" whose section is the hatched contour (A_p).The pressure force along the girder is variable, and the defined length of buckling is ([9], [10]): $l_i = 0{,}63\cdot L$, whereas the equivalent pressure force along the girder is:

$$D = \int_{A_p}\sigma_x\cdot dA = \frac{M_{cv}\cdot S_{px}}{I_x}\,. \tag{15}$$

This criterion is fulfilled if the condition of lateral stability is satisfied:

$$g_1 = \sigma_{r1} - \sigma_{k1} = \frac{D}{A_p}\cdot\frac{1}{\chi} + 0{,}9\cdot\frac{M_{ch}}{W_y} - \sigma_{k1} \le 0\,, \tag{16}$$

where:

$\sigma_{k1} = 0{,}76\cdot f_y$ - critical stress, f_y - yield strength.

The buckling coefficient χ has the following values [10]:

$\chi = 1$, if the relative slenderness of the bar is $\bar{\lambda} \le 0{,}2$, i.e.

$$\chi = \frac{2}{\beta+\sqrt{\beta^2-4\cdot\bar{\lambda}}}\,, \tag{17}$$

if the relative slenderness of the bar is $\bar{\lambda} > 0{,}2$ (according to recommendations it is always greater than 0,2). For the box cross section of the main girder of the crane, the coefficient β is:

$$\beta = 1 + 0{,}489\cdot(\bar{\lambda}-0{,}2) + \bar{\lambda}^2\,. \tag{18}$$

The relative slenderness of the bar ($\bar{\lambda}$) is calculated by the expression:

$$\bar{\lambda} = \frac{0{,}63\cdot L}{\beta\cdot b\cdot\lambda_v}\,, \tag{19}$$

which, after some transformation, can be written as:

$$\frac{1}{\chi`} = a\cdot\bar{\lambda}^2 + g\cdot\bar{\lambda} + d\,. \tag{20}$$

This approximation resulted in negligible deviations (Figure 3). By using the relations (16), (19) and (20), it is obtained that:

$$\frac{D}{A_p}\cdot\frac{1}{\chi} = \frac{2\cdot\gamma_x}{\beta_x^2\cdot h\cdot A}\cdot\left(M_{cv}+c\cdot A\right)\cdot\frac{f(b)}{\beta_y^2\cdot b^2}\,, \tag{21}$$

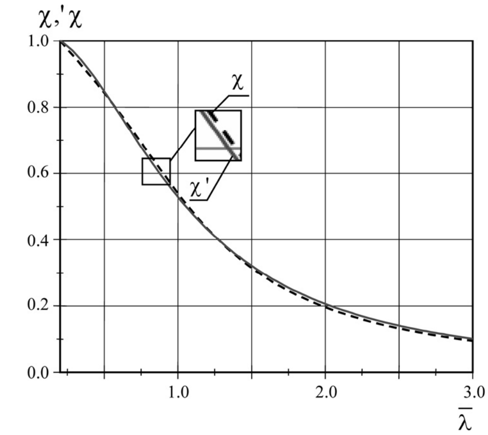

Fig.3. Presentation of approximation of the buckling coefficient

In order to apply the method of Lagrange multipliers, for the criterion of lateral stability, it is necessary to find the corresponding partial derivatives (4) by using the relation (7) and (16):

$$\frac{0{,}9}{4}\cdot\frac{\beta_x^2}{\gamma_x}\cdot\frac{M_{ch}}{M_{cv}} = \frac{1}{h^2}\cdot(e\cdot a\cdot m^2 + e\cdot g\cdot m\cdot\beta_y\cdot b) + \frac{1}{h^2}\cdot(e\cdot d\cdot\beta_y^2\cdot b^2 - 2\cdot a\cdot m^2\cdot k - g\cdot m\cdot\beta_y\cdot h)\,. \tag{22}$$

This equation is, with the corresponding transformations and neglecting higher order members, reduced to the form:

$$F(k,e) = \frac{(k+3e)}{(2e+k)} + 0{,}442\frac{1}{k} - 10{,}08\frac{1}{k^2} - 0{,}442 = 0\,. \tag{23}$$

From this expression, the optimum value of parameter *k*, according to the criterion of lateral stability as a function of the value of parameter *e*, can be obtained. By using the obtained dependences from the constraint function according to the criterion of lateral stability, the objective function can be written in the following form:

$$A_1(h) \ge \frac{K_1\cdot M_{cv}\cdot k_1^2\cdot f(h) + K_2\cdot M_{ch}\cdot k_1\cdot h^2}{\sigma_k\cdot h^3 - K_1\cdot c\cdot k_1^2\cdot f(h) - K_2\cdot k_a\cdot c\cdot k_1\cdot h^2}\,, \tag{24}$$

3.2.2. The criterion of local buckling of plates

Testing of the box girder stability was carried out in accordance with the standards group [13]. According to this standard, it is necessary to check the stability of the flange plate with the width b_1 and the thickness t_1 (Figure 4), the stability of the web plate above the longitudinal stiffener (length *a*, height h_1 and thickness t_2 – Figure 4) as well as the stability of the web plate under the longitudinal stiffener (length *a*, height h_2 and thickness t_2 – Figure 4).

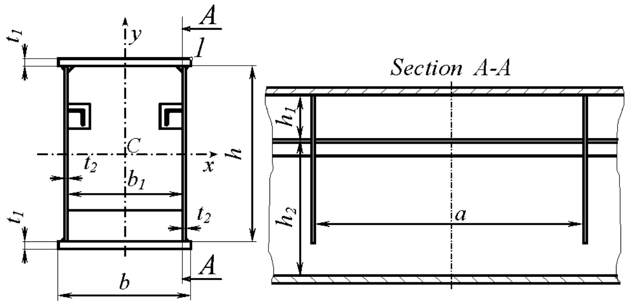

Fig.4. Elements of the box profile relevant for testing of the local stability

Testing of the stability of the flange plate segment (Figure 4) subjected to the action of normal compressive stress in the *x* direction was carried out.

This criterion is fulfilled if the following condition is satisfied:

$$g_2 = \frac{M_{cv} + c \cdot A}{\alpha_x \cdot h \cdot A} + f \cdot \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b \cdot A} - \sigma_{k2} \le 0\,, \qquad (25)$$

where:

$\sigma_{k2} = \kappa_x \cdot f_y / (\gamma_m \cdot \nu_1)$ - critical stress,

$\gamma_m = 1{,}1$ - the general resistance factor,

$\nu_1 = 1{,}5$ - the factored load coefficient for load case 1

κ_x - a reduction factor according to Equation:

$$\kappa_x = c_e \cdot \left(\frac{1}{\lambda_x} - \frac{0{,}22}{\lambda_x^{\,2}} \right) \le 1 \text{ for } \lambda_x > 0{,}673\,, \qquad (26)$$

$\kappa_x = 1$ for $\lambda_x \le 0{,}673$,

where:

λ_x - the non-dimensional plate slenderness,

$$\lambda_x = \sqrt{\frac{f_{yk}}{K\sigma \cdot \sigma_e}}\,, \qquad (27)$$

$$c_e = 1{,}25 - 0{,}12 \cdot \psi_e\,,\ c_e \le 1{,}25\,, \qquad (28)$$

ψ_e - the edge stress ratio of the plate, relative to the maximum compressive stress,

$K\sigma$ - a buckling factor according to [13],

σ_e - a reference stress according to Equation:

$$\sigma_e = \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot (t_1 / b_1)^2 \,. \qquad (29)$$

$K\sigma$ depends on the ratio between the plate sides $\alpha_e = a / b_1$. Now, this relation is to be analyzed.

In the cross section *I* the vertical diaphragms are placed at the distance of $2h$, so that this ratio takes the value:

$\alpha_e = a / b_1 = 2 \cdot h / (f \cdot b) > 1$.

Also, it is necessary to analyze koefficient ψ_e. It takes following value:

$$\psi_e = \frac{\sigma_2}{\sigma_1} = \frac{\dfrac{c_1}{k_a} \cdot \dfrac{\alpha_y}{\alpha_x} - f \cdot k}{\dfrac{c_1}{k_a} \cdot \dfrac{\alpha_y}{\alpha_x} + f \cdot k}\,, \qquad (30)$$

where:

$\nu_1 = 1{,}5$ - the factored load coefficient for load case 1,

σ_1, σ_2 - the stresses due to the factored load.

For average values, this ratio can also be approximately written by the expression $\psi_p \approx 0{,}83 - 0{,}06 \cdot k$. By using this approximation, with appropriate transformation, it can be obtained:

$$K\sigma_p = \frac{8{,}2}{1{,}88 - 0{,}06 \cdot k}\,. \qquad (31)$$

$$\lambda_{xp} \approx \frac{Ko}{\sqrt{K\sigma_p}} \cdot \frac{s \cdot f}{e \cdot k}\,, \qquad (27.1)$$

where:

$Ko = \dfrac{1}{\pi} \cdot \sqrt{\dfrac{12 \cdot (1 - \nu^2) \cdot f_y}{E}}$ - constant.

In Figure 5 it is shown that the factor κ_x takes the value 1 for the defined range of the ratio *k*, classification class 2m/M5 and girder material S235JRG2.

In order to apply the method of Lagrange multipliers, it is necessary to find the corresponding partial derivatives (5), in accordance with the expressions (7) and (25):

After some transformation, it is obtained:

$$k_2 = \sqrt{\frac{e \cdot \alpha_y}{f \cdot \alpha_x} \cdot \frac{c_1}{k_a}}\,. \qquad (32)$$

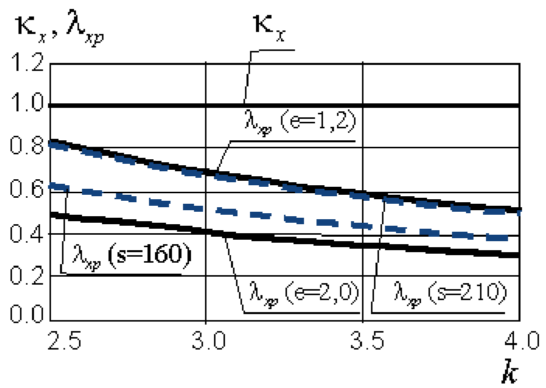

Fig.5. Change of the coefficients λ_{xp} and κ_x as the function of the parameter k

From this expression, we can get the optimum value of the parameter *k* according to the criterion of lateral stability of the flange plate. Using the obtained dependencies from the constraint function according to the criterion of stability of the flange plate, the constraint function can be written in the following form:

$$A_2(h) \ge \frac{M_{cv} / \alpha_x + f \cdot M_{ch} / \alpha_y \cdot k_{pl}}{\sigma_{k2} \cdot h - c / \alpha_x - f \cdot k_a \cdot c / \alpha_y \cdot k_{pl}}\,. \qquad (33)$$

Testing of the local stability of the web plate in area 1 and area 2, whose dimensions are given in Fig. 5, was carried out.

Besides the normal stresses in *x* direction, there is normal stress in *y* direction too, due to the action of wheel pressure.

The case when, in addition to vertical stiffeners at midspan, a row of horizontal stiffeners is also placed at the distance of $(0,25\div 0,33)\cdot h$ was considered, according to crane manufacturers. The areas 1 and 2 are analyzed.

Area 1: The criterion of stability of the web plate in area 1 is fulfilled if the following condition is satisfied:

$$\left(\frac{|\sigma_{Sd1,x}|}{f_{b,Rd1,x}}\right)^{e_{1x}}+\left(\frac{|\sigma_{Sd1,y}|}{f_{b,Rd1,y}}\right)^{e_{1y}}-\left(\kappa_{1x}\cdot\kappa_{1y}\right)^{6}\cdot\left(\frac{|\sigma_{Sd1,x}\cdot\sigma_{Sd1,y}|}{f_{b,Rd1,x}\cdot f_{b,Rd1,y}}\right)\leq 1, \quad (34.1)$$

where:

$|\sigma_{Sd1,x}|=\dfrac{M_{cv}+c\cdot A}{\alpha_x\cdot h\cdot A}+f\cdot\dfrac{M_{ch}+k_a\cdot c\cdot A}{\alpha_y\cdot b\cdot A}$ - the highest value of the compressive stress in the x direction at point 1,

$|\sigma_{Sd1,y}|=\nu_1\cdot\dfrac{\gamma\cdot F_1}{t_2\cdot l_{1r}}$ - the highest value of the compressive stress in the y direction at point 1,

$f_{b,Rd1,x}=\kappa_{1x}\cdot f_y/\gamma_m$, $f_{b,Rd1,y}=\kappa_{1y}\cdot f_y/\gamma_m$ - the limit design compressive stresses,

$e_{1x}=1+\kappa_{1x}^4$, $e_{1y}=1+\kappa_{1y}^4$ - coefficients,

κ_{1x} - a reduction factor for area 1.

There is the same analogy for coefficients λ_{1x}, c_{1e}, as well as for ψ_{1e} and $K\sigma_1$. The y direction case will be analyzed later on.

A reference stress σ_{1e} is obtained according to equation:

$$\sigma_{1e}=\frac{\pi^2\cdot E}{12\cdot(1-\nu^2)}\cdot\left(t_2/h_1\right)^2=\frac{\pi^2\cdot E}{12\cdot(1-\nu^2)}\cdot\left(4/s\right)^2. \quad (35.1)$$

Since there is:

$$\psi_{1p}\approx 0,54+0,015\cdot k, \quad (36.1)$$

it can be noted:

$$K\sigma_{1p}=\frac{8,2}{1,59+0,015\cdot k}, \quad (37.1)$$

$$\lambda_{1xp}\approx\frac{Ko}{\sqrt{K\sigma_{1p}}}\cdot(s/4). \quad (38.1)$$

Fig. 6 shows that for average and expected parameters values, the value of coefficient is $\kappa_{1x}=1$, wherein classification class is 2 and material is S235JRG2. Incresing the slenderness s leads to some lower values.

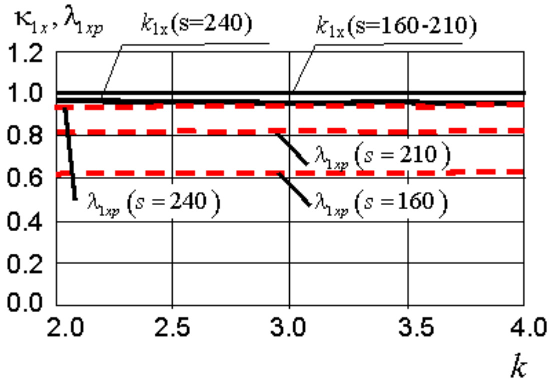

Fig.6. Buckling coefficient

Further on, load in y direction is analyzed:

F_1 - the highest wheel pressure,

$l_{1r}=12,15+1,4\cdot e$ - the effective distribution length [13],

κ_{1y} - a reduction factor for area 1:

$$\kappa_{1y}=1,13\cdot\left(\frac{1}{\lambda_{1y}}-\frac{0,22}{\lambda_{1y}^{\ 2}}\right)\leq 1 \text{ for } \lambda_{1y}>0,831, \quad (39)$$

$\kappa_{1y}=1$ for $\lambda_{1y}\leq 0,831$,

where:

λ_{1y} - the non-dimensional plate slenderness for area 1 according to Equation:

$$\lambda_{1y}=\sqrt{\frac{f_y}{K\sigma_{1y}\cdot\sigma_{1e}\cdot a/c_{1r}}}=\frac{Ko}{\sqrt{K\sigma_{1y}\cdot a/c_{1r}}}\cdot(s/4), \quad (40)$$

$K\sigma_{1y}\approx 0,5$ -a buckling factor for area 1 according to [13],

c_{1r} - the width over which the transverse load is distributed (corresponds to l_{1r}).

Fig. 7 shows that for average and expected parameters values, the value of coefficient is $\kappa_{1y}=1$, wherein classification class is 2 and material is S235JRG2.

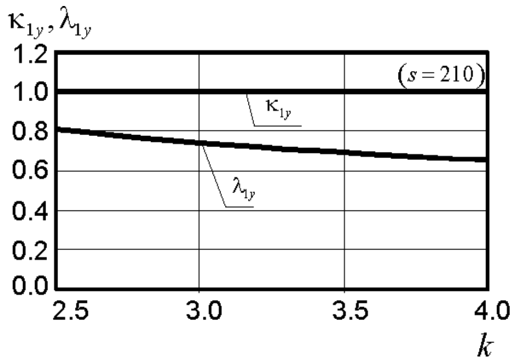

Fig.7. Buckling coefficient

On the basis of the obtained values the relation (34.1) can be written:

$$\sqrt{|\sigma_{Sd1,x}|^2+|\sigma_{Sd1,y}|^2-|\sigma_{Sd1,x}\cdot\sigma_{Sd1,y}|}\leq f_y/\gamma_m. \quad (34.1.1)$$

To maintain this relation the following condition must be fulfilled:

$|\sigma_{Sd1,x}|\geq|\sigma_{Sd1,y}|$, or the left side must be greater then the right side, whereby this dependences are given in relation to bridge crane span.

Similar procedure is applied for area 2.

This criterion is satisfied if the local stability condition is fulfilled:

$$\left(\frac{|\sigma_{Sd2,x}|}{f_{b,Rd2,x}}\right)^{e_{2x}}+\left(\frac{|\sigma_{Sd2,y}|}{f_{b,Rd2,y}}\right)^{e_{2y}}-\left(\kappa_{2x}\cdot\kappa_{2y}\right)^{6}\cdot\left(\frac{|\sigma_{Sd2,x}\cdot\sigma_{Sd2,y}|}{f_{b,Rd2,x}\cdot f_{b,Rd2,y}}\right)\leq 1, \quad (34.2)$$

where:

$|\sigma_{Sd2,x}|=\dfrac{M_{cv}+c\cdot A}{2\cdot\alpha_x\cdot h\cdot A}+f\cdot\dfrac{M_{ch}+k_a\cdot c\cdot A}{\alpha_y\cdot b\cdot A}$ - the maximum compressive stress in point 2 in x direction,

$\left|\sigma_{Sd2,y}\right| = \nu_1 \cdot \dfrac{\gamma \cdot F_1}{t_2 \cdot l_{2r}}$ - the maximum compressive stress in point 2 in *y* direction,

$f_{b,Rd2,x} = \kappa_{2x} \cdot f_y / \gamma_m$, $f_{b,Rd2,y} = \kappa_{2y} \cdot f_y / \gamma_m$ - the critical compressive stresses,

$e_{2x} = 1 + \kappa_{2x}^4$, $e_{2y} = 1 + \kappa_y^4$ - coefficients,

κ_{2x} - a reduction factor for area 2.

The same analogy exists for λ_{2x}, as well as for ψ_{2e} and $K\sigma_2$. Analysis for *y* direction will be done later on.

$c_{2e} = 1,25$.

A reference stress σ_{2e} is obtained according to equation:

$$\sigma_{2e} = \frac{\pi^2 \cdot E}{12 \cdot (1-\nu^2)} \cdot \left(\frac{4}{3 \cdot s}\right)^2 . \qquad (35.2)$$

Since there is:

$$\psi_{2p} \approx -\left(0,6 + 0,01 \cdot k\right), \qquad (36.2)$$

it can be noted

$$K\sigma_{2p} = 15,1 + 1,8 \cdot k + 0,0978 \cdot k^2 , \qquad (37.2)$$

$$\lambda_{2xp} \approx \frac{Ko}{\sqrt{K\sigma_{2p}}} \cdot (3 \cdot s / 4) . \qquad (38.2)$$

Fig. 8 shows that for average and expected parameters values, the value of coefficient κ_{2x} varies, wherein classification class is 2 and material is S235JRG2. Incresing the slenderness *s* leads to much lower values.

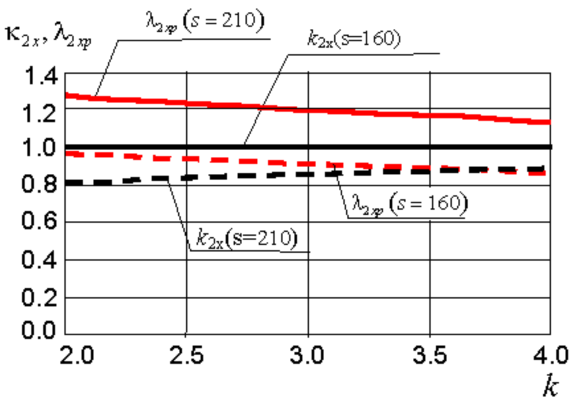

Fig.8. Buckling coefficient

Further on, load in y direction is analyzed:

$l_{2r} = 12,15 + 2 \cdot e \cdot h / s + h / 2$ - the effective distribution length [13],

κ_{2y} - a reduction factor for area 2.

There exists the same analogy for coefficient λ_{2y} as for $K\sigma_{2y}$:

$$\lambda_{2y} = \sqrt{\frac{f_y}{K\sigma_{2y} \cdot \sigma_{2e} \cdot a / c_{2r}}} = \frac{Ko}{\sqrt{K\sigma_{2y} \cdot a / c_{2r}}} \cdot \frac{3 \cdot s}{4} , \qquad (41)$$

$K\sigma_{2y} \approx 1,2$ - buckling coefficient [13],

c_{2r} - the width over which the transverse load is distributed (corresponds to l_{2r}).

Fig. 9 shows that for average and expected parameters values, the value of coefficient κ_{2y} varies, wherein classification class is 2 and material is S235JRG2.

Based on the obtained values the relation (34.2) can be written:

$$\left(\left|\sigma_{Sd2,x}\right| / f_{b,Rd2,x}\right)^{e_{2x}} + \left(\left|\sigma_{Sd2,y}\right| / f_{b,Rd2,y}\right)^{e_{2y}} \le 1 , \qquad (34.2.1)$$

since $\left(\kappa_{2x} \cdot \kappa_{2y}\right)^6 \ll 1$.

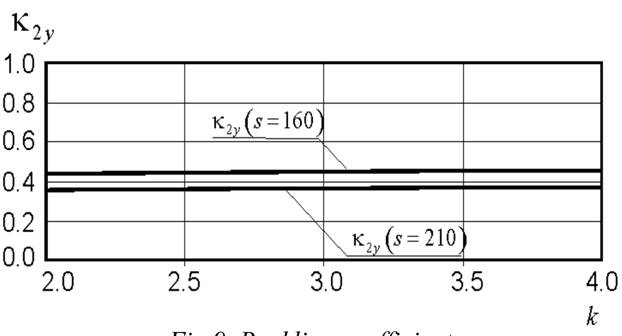

Fig.9. Buckling coefficient

To do analysis, the ratio between the maximum compressive stress in this area and the compressive stress that occurred.

$\left|\sigma_{Sd2,x}\right| = \psi_2 \cdot \left|\sigma_{Sd\max,x}\right| \approx (0,52 + 0,009 \cdot k) \cdot \left|\sigma_{Sd\max,x}\right|$.

Now (34.2.1) becomes as follows:

$$\left(\psi_2 / \kappa_{2x}\right)^{1+\kappa_{2x}^4} + \left(\left|\sigma_{Sd2,y}\right| \cdot \gamma_m / (\kappa_{2y} \cdot f_y)\right)^{1+\kappa_{2y}^4} \le 1 . \qquad (34.2.2)$$

Fig. 10 shows that for average parameters values and with load changing, this relation varies, where it was considered $s = 160$.

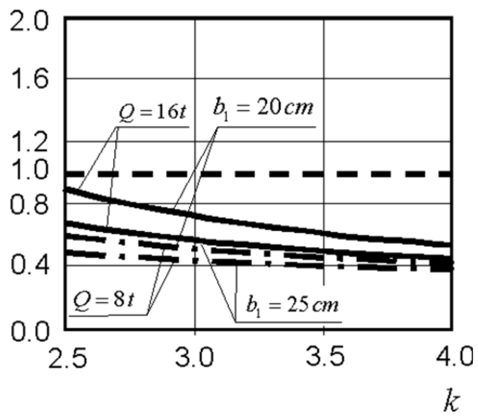

Fig.10. The results

Based on all mentioned above, it is possible to adopt such parameters that make the local stability criterion fulfilled. The area 1 is critical one for the investigation of vertical plates local stability, while the highest stress is analogous to flange plate stress. Factors κ_x and κ_{1x} take value 1 for expected values of relation *k* and slenderness s. According to this criterion, the same curve or constraint (g_2) is taken into account, as obtained in previous expression. In addition, according to this criterion, the optimal value for k is determined out of expression (32), just as flange plate local stability. The objective function is defined by relation (33).

4. NUMERICAL REPRESENTATION OF THE RESULTS OBTAINED

Using the expression (23) the optimum value of the parameter k_1 according to the criterion of lateral stability is obtained. Optimum values of the parameter k_1 as a function of the coefficient e, while $k_a = 0{,}1$, are presented in Table 1.

Table 1.

e	1,2	1,3	1,4	1,5
k_1	3,35	3,33	3,31	3,30

The expression (24) represents the objective function obtained from the constraint function according to the criterion of lateral stability and together with the objective function (7) it can be graphically represented. Fig. 11 shows how the position of the intersection point changes depending on the selection of material, where adopted values are *L=20 m* and *Q=12.5 t*. Using the expression (32) the optimum value of the parameter k according to the criterion of lateral stability is obtained. Optimum values of the parameter k_2 as a function of the coefficient e, while $k_a = 0{,}1$, are presented in Table 2.

Table 2.

e	1,2	1,3	1,4	1,5
k_2	4,20	4,30	4,40	4,55

The expression (33) represents the objective function obtained from the constraint function according to the criterion of local stability and together with the objective function (7) it can be graphically represented. Fig. 12 shows how the position of the intersection point changes depending on the selection of material, where adopted values are *L=20 m* and *Q=12.5 t*. In order to perform a comparative analysis of optimization results, it is necessary to define the initial parameters of cranes, which refer to their geometrical characteristics, classification class and carrying capacity. These are the data, which the designer receives from the investor as the project task.

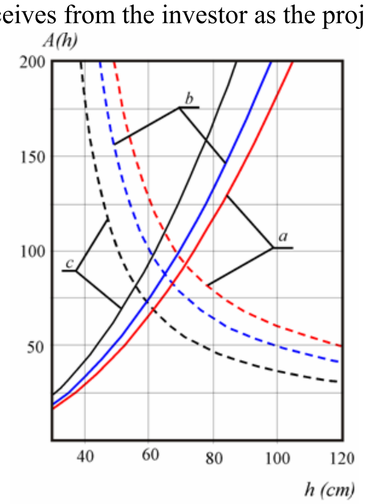

Fig.11. Optimum values of the girder height and the objective function according to the criterion of lateral stability a) S235JRG2 b) S275JR c) S355JR

One of the main parameters that exist in the objective function (7) is slenderness s, which is defined as follows ([10]):

$$160 \cdot \sqrt{23{,}5 / f_y} < s \le 265 \cdot \sqrt{23{,}5 / f_y}\,. \tag{42}$$

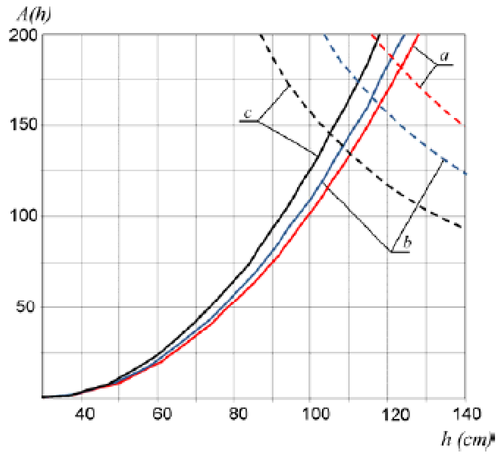

Fig.12. Optimum values of the girder height and the objective function according to the criterion of local stability a) S235JRG2 b) S275JR c) S355JR

For starting the analysis, values in the middle of the range can be taken, so that for S235JRG2 it is $s = 210$. Other parameters' values in this phase are:

$$\begin{aligned} &e = 1{,}33,\ f = 0{,}85,\ \psi = 1{,}15,\ k_a = 0{,}1, \\ &e_k = 2{,}3\,m,\ G_k = 15\,kN. \end{aligned} \tag{43}$$

The analysis was performed for the classification class 2m/M5 (FEM 9.511/ISO 4301-1), which is, according to the Serbian standarad most frequently used in practice. The following values hold for it:

$$\gamma = 1{,}05,\ \ \alpha = 1{,}20,\ \ K = 0{,}08,\ \ m_o = 1{,}20\,. \tag{44}$$

The analysis was performed for steel S235JRG2. In order to perform a comparative analysis, it is necessary to take into consideration the recommendations specified in the standard as well as those given by crane manufacturers [14]. Serbian crane manufacturers recommend that the minimum value of the width b_1 should be $b_1 > 20\,cm$, wherefrom it is obtained that:

$$k \le f \cdot h / 20, \tag{45}$$

while stability condition of top flange plate, after proper transformation, which reads:

$$k \ge \frac{s \cdot f}{65 \cdot e} \cdot \sqrt{\frac{23.5}{f_y}}\,. \tag{46}$$

If the expressions (7) and (24) are made equal, the dependence of the parameter k_1 according to the criterion of lateral stability is obtained:

$$k_1 = F\left(s, e, h, K_1, K_2, M_{cv}, M_{ch}, f_y, c, k_a\right). \tag{47}$$

If the expressions (7) and (33) are made equal, the dependence of the parameter k_1 according to the criterion of local stability is obtained:

$$k_2 = F\left(s, e, f, h, Q, M_{cv}, M_{ch}, \alpha_x, \alpha_y, k_a, f_y, \nu, E\right). \tag{48}$$

Figures 13-14 present optimum geometric parameters' values that are obtained for characteristic load capacities and spans of double-girder bridge cranes.
This procedure enables quick and efficient determination of optimum value of parameter *k,* according to critical function.

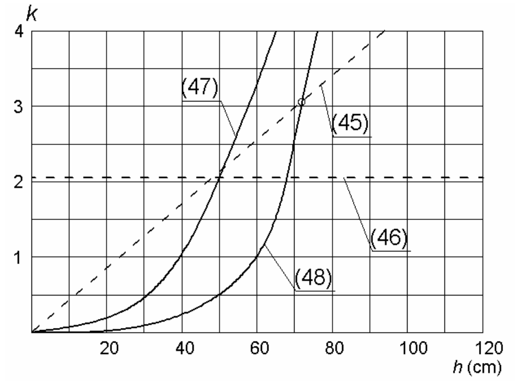

Fig.13. Multicriteria determination of the optimum value of the parameter k for the crane span L=12 m and the carrying capacity Q=5 t

Width b_1 value have no influence onto the optimization procedure, but has the impact on obtained results of the optimized parameter *k* (Fig. 14).

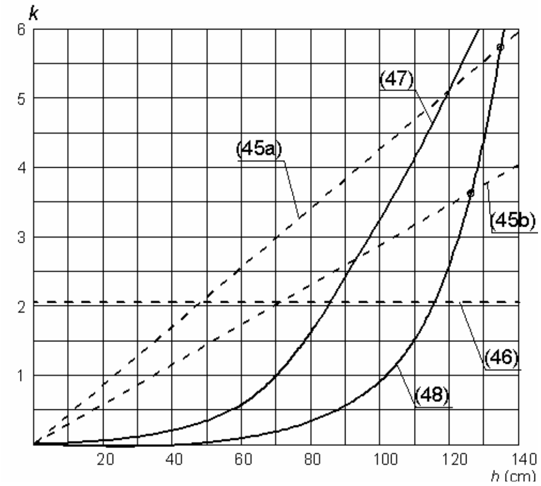

Fig.14. Multicriteria determination of the optimum value of the parameter k for the crane span L=20 m and the carrying capacity Q=16 t a) b_1=20 cm b) b_1=30 cm

5. CONCLUSION

The paper defined optimum dimensions of the box section of the main girder of the bridge crane in an analytical form, by using the method of Lagrange multipliers, according to lateral stability and local stability of plates criteria. [13]. The objective function is the minimum mass, i.e. the minimum area of the cross section, where the given constraints are satisfied: lateral and local stability, manufacturing technology and top (compressed) flange plate stability. It can be observed that greater hights of box girder cross section are obtained according to local stability criterion [13] (consequently greater cross section areas) in comparison with those which are calculated due to lateral stability criterion, which is taken from national standarad. So, the local stability of plates is the primary criterion [13]. The results obtained may be of great use to the engineer-designer, particularly in the first phase of the design procedure when the basic dimensions of the main girder of the bridge crane, as its most responsible part, are defined. In addition, the usage of the method of Lagrange multipliers is justified because the optimization results are obtained in analytical form, which allows getting conclusions about influences of particular parameters and further researches toward mass reduction.

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