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Projektovanje PD regulatora za sisteme sa velikim vremenskim kašnjenjem

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PID regulatori su najčešće korišćeni regulatori u industiji, sa udelom od 95% od svih korišćenih regulatora. Od toga najveći broj otpada na regulatore PI tipa, dok su regulatori PD tipa manje zastupljeni. U ovom radu je opisano projektovanje PD regulatora metodom D – dekompozicije, kao i primena ovog tipa regulatora na sisteme sa kašnjenjem.

Rezultati prikazani u ovom radu pokazuju da PD regulator dobijen predloženom metodom daje dosta dobre rezultate, i da diferencijalno dejstvo minimizira efekte vremenskog kašnjenja i postiže stabilnost sistema sa velikim vremenskim kašnjenjem.

Ključne reči: PD regulator, D-dekompozicija, sistemi sa kašnjenjem, stabilnost sistema, relativna stabilnost.

1. UVOD

Regulatori PID tipa su najčešće korišćeni regulatori u industriji, sa udelom od 95% od svih korišćenih regulatora. Od toga najveći broj otpada na PI regulatore. [1] Razlog za najčešće korišćenje PI regulatora u industriji je njegova relativno jednostavna struktura koja se lako može implementirati u praksi. Manje zastupljeni regulatori su regulatori PD tipa. Ovi regulatori su, takođe, relativno jednostavni, ali im je posvećeno dosta manje pažnje. Zato će u ovom radu biti više reči upravo o ovom tipu regulatora.

Metoda koja omogućava projektovanje PID regulatora visokog reda je D – dekompozicija. Metod D – dekompozicije razvio je Neimark [2-3]. Značajno proširenje metode D – dekompozicije dao je Mitrović uspostavivši čvrstu vezu vrednosti procesa iskazanim kroz odgovarajući stepen relativne stabilnosti sistema. [4] Svoje potpuno uopštenje metod D – dekompozicije je doživeo u algebarskoj metodi koju je razvio Šiljak [5-7]. Za efikasno tumačenje rezultata dobijenih D – dekompozicijom neophodna je i odgovarajuća grafička interpretacija koja zahteva odgovarajuću softversku podršku. Savremena teorija upravljanja omogućava da se za procese visokog reda projektuju regulatori čiji će red biti jednak redu procesa. [8-11] Interpretacija takvih regulatora u industriji je veoma složena i skupa. Zato se nameće potreba projektovanja regulatora niskog reda, koji su najčešće prisutni u industrijskoj praksi, za upravljanje procesima visokog reda [12-17].

Sistemi sa kašnjenjem su sistemi u kojima postoji vremensko kašnjenje između ulaza ili upravljanja i ispoljavanja efekata tih dejstava na sistem. Ona su ili posledica kašnjenja svojstvenih komponentama sistema ili namernog uvođenja kašnjenja radi lakšeg upravljanja sistemom [18].

Procesi sa vremenskim kašnjenjem zahtevaju veću pažnju prilikom odabira regulatora. U ovom radu se primenom PD regulatora ispituje stabilnost sistema sa kašnjenjem. Za projektovanje PD regulatora koristi se metoda D – dekompozicije. Osnovna pretpostavka ove metode polazi od zahteva da se u parametarskoj ravni odredi skup svih vrednosti podešljivih parametara za koje će razmatrani sistem, dat svojom karakterističnom jednačinom, biti stabilan [15]. Primenom ove metode dobijaju se parametarske jednačine na osnovu kojih je napisan program u Matlab-u. Uz pomoć ovog programa vrši se analiza predložene metode projektovanja PD regulatora za sisteme sa kašnjenjem.

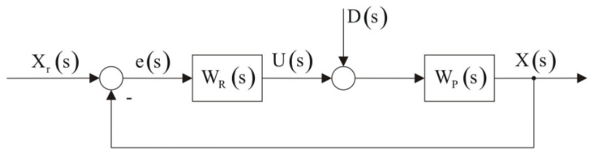
2. MATEMATIČKI MODEL SISTEMA AUTOMATSKOG REGULISANJA SA KAŠNJENJEM

Prenosna funkcija PD regulatora je:

$$W_R = K_p + K_d \cdot s \tag{1}$$

Prenosna funkcija procesa prestavljena je u obliku:

$$W_P = \frac{N(s)}{M(s)} e^{-\tau s} = \frac{\sum_{k=0}^{m} b_k s^k}{\sum_{k=0}^{n} a_k s^k} e^{-\tau s}, m \le n \tag{2}$$



Slika 1: Sistem automatskog regulisanja

Karakteristična jednačina sistema automatskog regulisanja sa slike 1 određena je jednačinom:

$$f(s) = 1 + W_R(s) \cdot W_P(s) = 0 \tag{3}$$

$$f(s) = 1 + \left(K_p + K_d \cdot s\right) \cdot \frac{N(s)}{M(s)} \cdot e^{-\tau s} \tag{4}$$

$$f(s) = M(s) \cdot e^{\tau s} + \left(K_d \cdot s + K_p\right) \cdot N(s) = 0 \tag{5}$$

$$f_1(s) = M(s) \cdot e^{\tau s} = \sum_{k=0}^{n} a_k s^k \cdot e^{\tau s} \tag{6}$$

Povezujući izraze (5) i (6) dobija se konačan izraz za karakterističnu jednačinu sistema:

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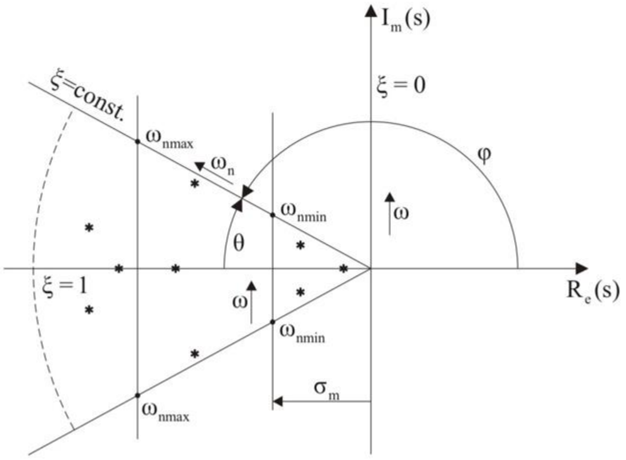
$$f(s) = f_1(s) + \left(K_d \cdot s + K_p\right) \cdot N(s) = 0 \tag{7}$$

Potrebno je u pogodnoj formi izraziti kompleksi broj s i preko njega uspostaviti vezu između stepena prigušenja ξ i promenljivih parametara regulatora K_p i K_d sadržanih u karakterističnoj jednačini (7). Na ovaj način preslikavamo oblast iz kompleksne ravni s ispod prave ξ=*const*, slika 2, u oblast odgovarajućeg koeficijenta prigušenja, predstavljenog krivom ξ=*const,* u parametarskoj ravni podešljivih parametara regulatora.



Slika 2: Oblast sa zahtevanim vremenom smirenja i relativnom stabilnošću [15]

Pošto je:

$$s = -\omega_n \xi + j\omega_n \sqrt{1-\xi^2}, 0 \le \xi \le 1 \tag{8}$$

element čistog kašnjenja može se izraziti u obliku:

$$e^{\tau s} = e^{-\tau\xi\omega_n}\left(\cos \tau\omega_n \sqrt{1-\xi^2} + j \sin \tau\omega_n \sqrt{1-\xi^2}\right) \tag{9}$$

$$e^{\tau s} = e^{-p}\left(\cos q + j \sin q\right) \tag{10}$$

pri čemu su:

$$p = \tau\xi\omega_n, q = \tau\omega_n \sqrt{1-\xi^2} \tag{11}$$

Ako je s dato jednačinom (8), tada je s^k određeno izrazom:

$$s^k = \omega_n^k \left(T_k(-\xi) + j\sqrt{1-\xi^2}U_k(-\xi)\right) \tag{12}$$

odnosno:

$$s^k = \omega_n^k \left((-1)^k T_k(\xi) + j\sqrt{1-\xi^2}(-1)^{k+1} U_k(\xi)\right) \tag{13}$$

gde su T_k i U_k Čebiševljeve funkcije prve i druge vrste za koje važe sledeće rekurentne jednačine:

$$T_{k+1}(\xi) = 2\xi T_k(\xi) - T_{k-1}(\xi) \tag{14}$$

$$U_{k+1}(\xi) = 2\xi U_k(\xi) - U_{k-1}(\xi) \tag{15}$$

$$T_0 = 1, T_1 = \xi, U_0 = 0, U_1 = 1 \tag{16}$$

Povezujući jednačine (7) i (8) dobija se:

$$f_1(\xi,\omega_n) + \left[K_d\left(-\xi\omega_n + j\omega_n\sqrt{1-\xi^2}\right) + K_p\right] \cdot N(\xi,\omega_n) = 0 \tag{17}$$

Povezujući sada jednačine (6), (10) i (13) dobija se:

$$f_1(\xi,\omega_n) + \left[K_d\left(-\xi\omega_n + j\omega_n\sqrt{1-\xi^2}\right) + K_p\right] \cdot N(\xi,\omega_n) = 0 \tag{18}$$

$$\begin{aligned} f_1(\xi,\omega_n) = {} & \left[\alpha(\xi,\omega_n)\cos q - \beta(\xi,\omega_n)\sin q\right]e^{-p} + \\ & + j\left[\alpha(\xi,\omega_n)\sin q + \beta(\xi,\omega_n)\cos q\right]e^{-p} \end{aligned} \tag{19}$$

pri čemu su:

$$\alpha(\xi,\omega_n) = \sum_{k=0}^{n} a_k (-1)^k \omega_n^k T_k(\xi) \tag{20}$$

$$\beta(\xi,\omega_n) = \sqrt{1-\xi^2}\sum_{k=0}^{n} a_k (-1)^{k+1} \omega_n^k U_k(\xi) \tag{21}$$

Izraz za $N(\xi,\omega_n)$ može se napisati u sledećem oblicima:

$$N(\xi,\omega_n) = \gamma(\xi,\omega_n) + j\delta(\xi,\omega_n) \tag{22}$$

$$N(\xi,\omega_n) = \sum_{k=0}^{m} b_k \omega_n^k \left((-1)^k T_k(\xi) + j\sqrt{1-\xi^2}(-1)^{k+1} U_k(\xi)\right) \tag{23}$$

Izjednačavajući izraze (22) i (23), i izdvajajući realni i imaginarni deo, dobija se:

$$\gamma(\xi,\omega_n) = \sum_{k=0}^{m} b_k \omega_n^k (-1)^k T_k(\xi) \tag{24}$$

$$\delta(\xi,\omega_n) = \sqrt{1-\xi^2}\sum_{k=0}^{m} b_k \omega_n^k (-1)^{k+1} U_k(\xi) \tag{25}$$

Povezujući jednačinu (17) sa jednačinama (18) – (25), i izdvajajući realni i imaginarni deo, dobijamo sledeći sistem jednačina:

$$\begin{aligned} & K_d\left[\xi\omega_n\gamma(\xi,\omega_n) + \sqrt{1-\xi^2}\omega_n\delta(\xi,\omega_n)\right] - \\ & - K_p\gamma(\xi,\omega_n) = \left(\alpha\cos q - \beta\sin q\right)e^{-p} \end{aligned} \tag{26}$$

$$\begin{aligned} & K_d\left[\xi\omega_n\delta(\xi,\omega_n) - \sqrt{1-\xi^2}\omega_n\gamma(\xi,\omega_n)\right] - \\ & - K_p\delta(\xi,\omega_n) = \left(\alpha\sin q + \beta\cos q\right)e^{-p} \end{aligned} \tag{27}$$

Rešavanjem Sistema jednačina (26), (27) pri $\omega_n \ne 0, 0 \le \xi < 1$, dobijamo parametre PD regulatora K_p i K_d. Iz ovog sistema jednačina za $\omega_n = 0$ i $\omega_n = \infty$ definišu se singularne prave koje su opisane jednačinama:

$$K_p(\xi, 0) = 0 \tag{28}$$

$$K_d(\xi, \infty) = 0 \tag{29}$$

3. PRIMENA PREDLOŽENE METODE PROJEKTOVANJA PD REGULATORA NA PRIMERIMA REGULISANJA PROCESA SA KAŠNJENJEM

Primenom softverskog paketa Matlab napisan je program za određivanje parametara PD regulatora na osnovu sistema jednačina (26), (27) i singularnh pravih (28), (29).

Ovaj program automatski crta krivu zavisnosti K_d kao funkciju proporcionalnog pojačanja K_p u parametarskoj ravni za zahtevani stepen prigušenja zatvorenog kola sistema automatskog regulisanja.

Da bismo ilustrovali predloženu metodu projektovanja PD regulatora za procese sa kašnjenjem izabrana su tri primera:

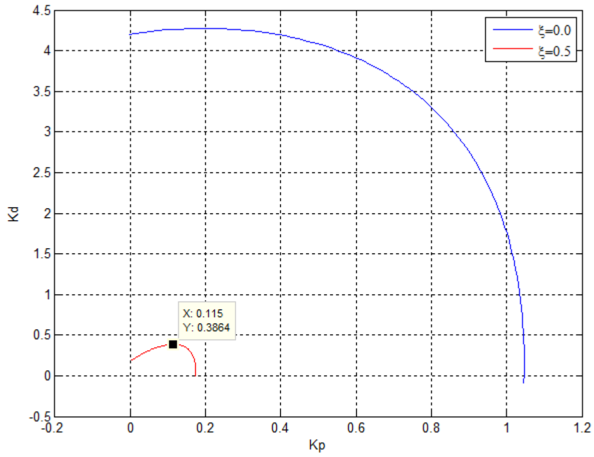
$$W_{P1}(s) = \frac{1}{(s+1)^3} e^{-15s} \tag{30}$$

$$W_{P2}(s) = \frac{1}{(s^2+s+1)(s+3)} e^{-s} \tag{31}$$

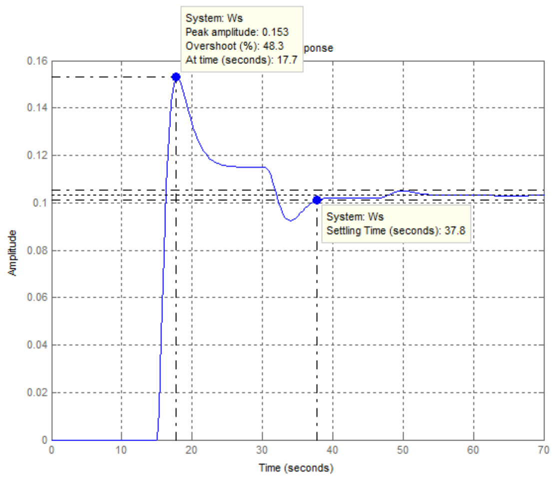
$$W_{P3}(s) = e^{-s} \tag{32}$$

Primer prenosne funkcije procesa $W_{P1}(s)$ je karakterističan sa aspekta relativno velikog vremena kašnjenja. Kod ovog procesa je odnos između vremena kašnjenja i dominantne vremenske konstante 15/3. Kod procesa opisanog prenosnom funkcijom $W_{P3}(s)$ ovaj odnos je još ekstremniji i teorijski teži beskonačno.

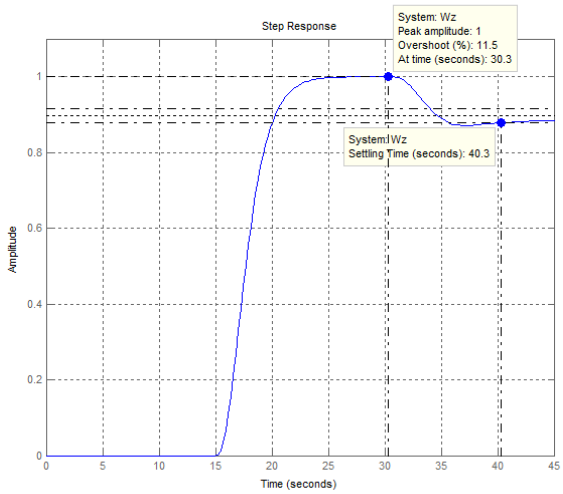
Na slici 3 prikazana je parametarska ravan za prenosnu funkciju procesa $W_{P1}(s)$ sa granicama apsolutne i relativne stabilnosti za zahtevani stepen prigušenja zatvorenog kola sistema automatskog regulisanja koji iznosi $\xi = 0,5$. Granica stabilnosti obeležena je sa $\xi = 0,0$. Vrednosti proporcionalnog pojačanja K_p i odgovarajućeg diferencijalnog pojačanja K_d za vrednost stepena prigušenja $\xi = 0,5$ očitavaju se sa slike 3.



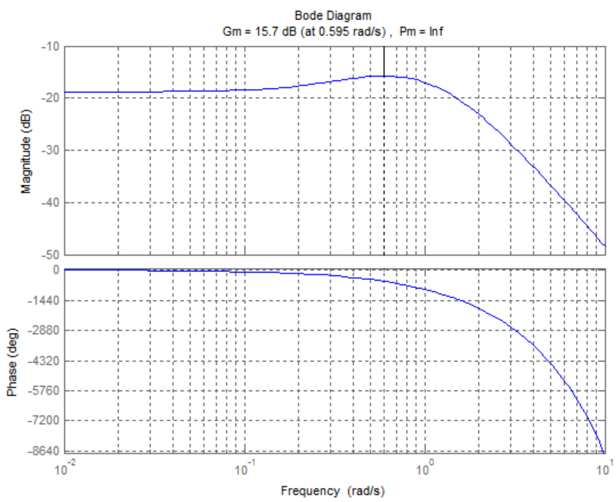
Slika 3: Parametarska ravan za različite vrednosti stepena prigušenja zatvorenog kola za prenosnu funkciju $W_{P1}(s)$



Slika 4: Odziv sistema regulisanja za vrednost stepena prigušenja $\xi = 0,5$ za prenosnu funkciju $W_{P1}(s)$



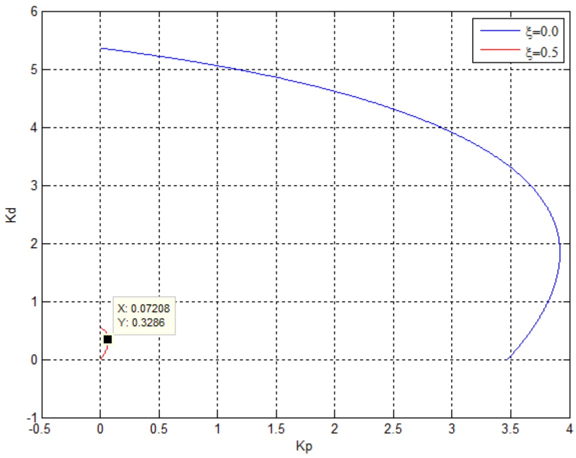
Slika 5: Odziv sistema regulisanja za vrednost stepena prigušenja $\xi = 0,5$ pri potiskivanju opterećenja poremećaja za prenosnu funkciju $W_{P1}(s)$



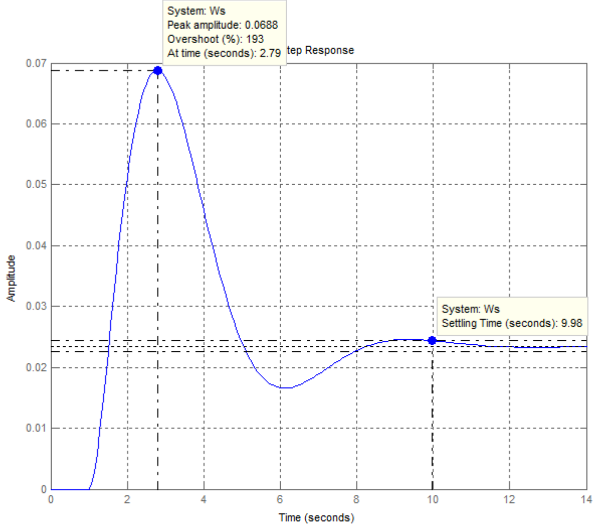
Slika 6: Amplitudna i fazna karakteristika otvorenog kola procesa opisanog prenosnom funkcijom $W_{P1}(s)$ za vrednost stepena prigušenja $\xi = 0,5$

Odziv sistema automatskog regulisanja za vrednosti stepena prigušenja $\xi = 0,5$ po referenci prikazan je na slici 4, a po poremećaju na slici 5. Amplitudna i fazna karakteristika otvorenog kola prikazana je na slici 6.

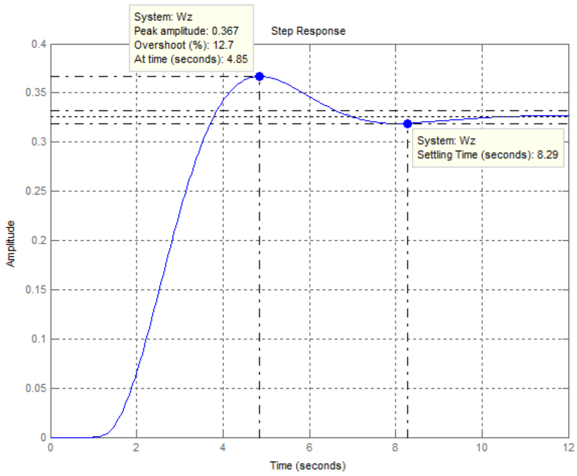
Drugi proces koji se razmatra opisan je prenosnom funkcijom $W_{P2}(s)$. Na slici 7 prikazana je parametarska ravan za ovaj proces, za stepen prigušenja $\xi = 0,5$ i granicu stabilnosti $\xi = 0,0$. Vrednosti proporcionalnog pojačanja K_p i odgovarajućeg diferencijalnog pojačanja K_d za vrednost stepena prigušenja $\xi = 0,5$ očitavaju se sa slike 7.



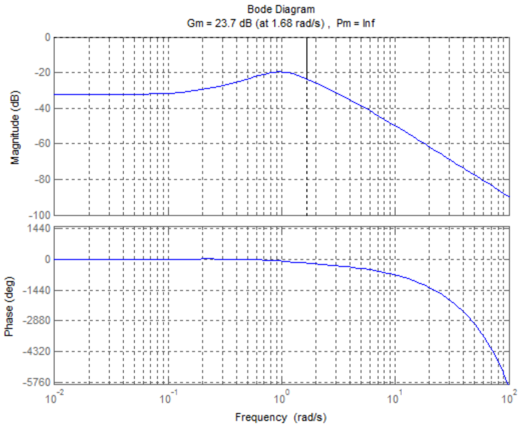
Slika 7: *Parametarska ravan za različite vrednosti stepena prigušenja zatvorenog kola za prenosnu funkciju* $W_{P2}(s)$



Slika 8: Odziv sistema regulisanja za vrednost stepena prigušenja $\xi = 0,5$ *za prenosnu funkciju* $W_{P2}(s)$



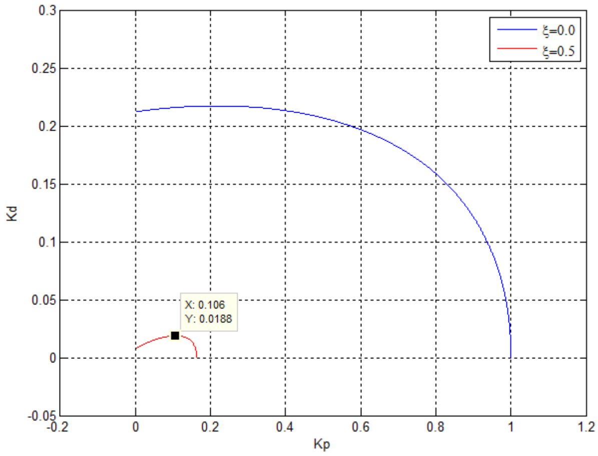
Slika 9: Odziv sistema automatskog regulisanja za vrednost stepena prigušenja $\xi = 0,5$ *pri potiskivanju opterećenja poremećaja za prenosnu funkciju* $W_{P2}(s)$



Slika 10: Amplitudna i fazna karakteristika otvorenog kola procesa opisanog prenosnom funkcijom $W_{P2}(s)$ *za vrednost stepena prigušenja* $\xi = 0,5$

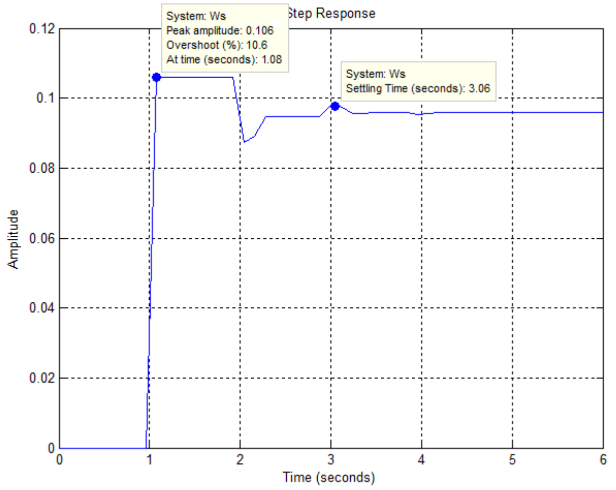
Odziv sistema automatskog regulisanja prenosne funkcije $W_{P2}(s)$ za vrednosti stepena prigušenja $\xi = 0,5$ po referenci prikazan je na slici 8, a po poremećaju na slici 9. Amplitudna i fazna karakteristika otvorenog kola prikazana je na slici 10.

Treći proces koji se razmatra opisan je prenosnom funkcijom $W_{P3}(s)$. Na slici 11 prikazana je parametarska ravan za proces opisan ovom prenosnom funkcijom, sa koje se očitavaju vrednosti proporcionalnog pojačanja K_p i odgovarajućeg diferencijalnog pojačanja K_d za stepen prigušenja $\xi = 0,5$.

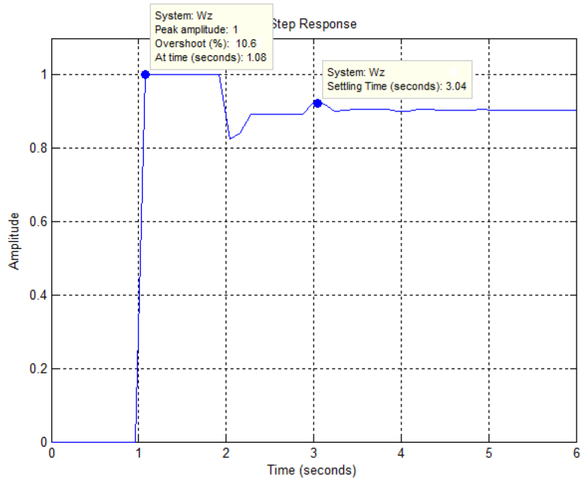


Slika 11: Parametarska ravan za različite vrednosti stepena prigušenja zatvorenog kola za prenosnu funkciju $W_{P3}(s)$

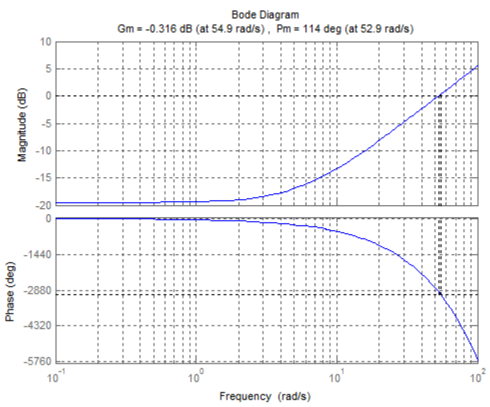
Odziv sistema automatskog regulisanja prenosne funkcije $W_{P3}(s)$ za vrednosti stepena prigušenja $\xi = 0,5$ po referenci prikazan je na slici 12, a po poremećaju na slici 13. Amplitudna i fazna karakteristika otvorenog kola prikazana je na slici 14.



Slika 12: Odziv sistema automatskog regulisanja za vrednost stepena prigušenja $\xi = 0,5$ za prenosnu funkciju $W_{P3}(s)$



Slika 13: Odziv sistema automatskog regulisanja za vrednost stepena prigušenja $\xi = 0,5$ pri potiskivanju opterećenja poremećaja za prenosnu funkciju $W_{P3}(s)$



Slika 14: Amplitudna i fazna karakteristika otvorenog kola procesa opisanog prenosnom funkcijom $W_{P3}(s)$ za vrednost stepena prigušenja $\xi = 0,5$

Vrednosti parametara regulatora sa performansama za prenosne funkcije $W_{P1}(s)$, $W_{P2}(s)$ i $W_{P3}(s)$, pri vrednosti stepena prigušenja $\xi = 0,5$, prikazane su u tabeli 1.

Tabela 1: Prikaz rezultata projektovanja PD regulatora sa performansama za prenosne funkcije $W_{P1}(s)$, $W_{P2}(s)$ i $W_{P3}(s)$, pri vrednosti stepena prigušenja $\xi = 0,5$

	$W_{P1}(s)$	$W_{P2}(s)$	$W_{P3}(s)$
Stepen prigušenja ξ	0,5	0,5	0,5
Proporcionalno pojačanje K_p	0,115	0,07208	0,106
Diferencijalno pojačanje K_d	0,3864	0,3286	0,0188
Preskok (referenca) [%]	48,3	193	10,6
Vreme smirenja (referenca) t_s [s]	37,8	9,98	3,06
Preskok (poremećaj) [%]	11,5	12,7	10,6
Vreme smirenja (poremećaj) t_s [s]	40,3	8,29	3,04
Pretek faze φ_m [°]	Inf	Inf	114
Pretek pojačanja g_m [dB]	15,7	23,7	-0,316

4. ZAKLJUČAK

Dolazi se do zaključka da PD regulator dobijen predloženom metodom daje dosta dobre rezultate i njegova prednost se ogleda u jednostavnosti izvođenja metode. Rezultati prikazani u ovom radu pokazuju da diferencijalno dejstvo minimizira efekte vremenskog kašnjenja i postiže stabilnost sistem sa velikim vremenskim kašnjenjem. Glavni nedostatak ovog tipa regulatora je nemogućnost eliminisanja statičke graške. Dalja istraživanja autora će se usmeriti ka projektovanju PID regulatora ovom metodom, čime će ovaj nedostatak biti otklonjen.

ZAHVALNOST

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Design of PD Controllers for Systems with High Time Delay

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PID controllers are the most used controllers in industry, with a share of 95% of all used controllers. Most of them are PI controllers, while PD controllers are less common. This paper proposes the design of PID controllers by D – decomposition method, as well as application of this type of controllers to the delay systems.

The results presented in this paper show that PD controller, obtained by the proposed method, gives good results, and that differential action minimizes effects of time delay and achieves stability of systems with high time delay.

Keywords: PD controller, D-decomposition, time delay systems, system stability, relative stability.

1. INTRODUCTION

PID controllers are the most used controllers in industry, with a share of 95% of all used controllers. Most of them are PI controllers [1]. The reason for the most frequently use of PI controllers in the industry is its relatively simple structure, which can be easily implemented in practice. Less used controllers are PD controllers. These controllers are also quite simple, but much less attention is devoted to them. Therefore, exactly this type of controller will be discussed in this paper.

The method that enables PID controller design for high order systems is *D* – decomposition. The *D* – decomposition method was developed by Neimark [2-3]. Significant expansion of *D* – decomposition method was given by Mitrovic, establishing a strong link between the processes values expressed through the corresponding degree of relative stability of the system [4]. The *D* – decomposition method was fully generalized in the algebraic method developed by Siljak [5-7]. For efficient interpretation of results obtained by the *D* – decomposition method it is necessary a suitable graphical interpretation which requires the corresponding software support. Modern control theory allows, for high order processes, to design controllers whose order will be equal to the order of the process [8-11]. Implementation of such controllers in industry is very complex and expensive. Because, there is a need for design of controllers of low order, which are most frequently present in industrial practice, for control of high order processes [12-17].

Time-delay systems are systems in which there is a time delay between the input or control and expression of the effects of these actions on the system. They are either the result of delays which are inherent in the system components or intentional introduction of delays for easy system control [18].

Processes with time delay require more attention when choosing a controller. In this paper, by applying the PD controller, it is examined the stability of the time-delay systems. For the PID controller design, it is used D – decomposition method. The basic assumption of this method is based on the requirement that, in the parameter plane, it is determined the set of all values of adjustable parameters for which the considered system, given by its characteristic equation, will be stable. [15]. By using this method, the parametric equations are obtained, on which the program was written in Matlab. With the assistance of this program it is performed the analysis of the proposed method of designing PD controller for time-delay systems.

2. MATHEMATICAL MODEL OF THE AUTOMATIC CONTROL SYSTEM WITH TIME DELAY

The transfer function of the PD controller is:

$$W_R = K_p + K_d \cdot s \tag{1}$$

The transfer function of the process is represented in the form:

$$W_P = \frac{N(s)}{M(s)} e^{-\tau s} = \frac{\sum_{k=0}^{m} b_k s^k}{\sum_{k=0}^{n} a_k s^k} e^{-\tau s}, m \le n \tag{2}$$

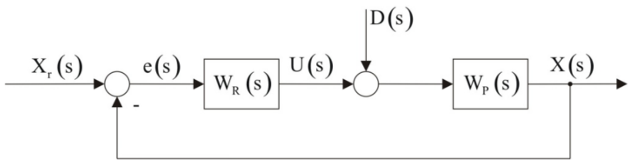

Figure 1: Automatic control system

The characteristic equation of the automatic control system from Figure 1 is determined by the equation:

$$f(s) = 1 + W_R(s) \cdot W_P(s) = 0 \tag{3}$$

$$f(s) = 1 + \left(K_p + K_d \cdot s\right) \cdot \frac{N(s)}{M(s)} \cdot e^{-\tau s} \tag{4}$$

$$f(s) = M(s) \cdot e^{\tau s} + \left(K_d \cdot s + K_p\right) \cdot N(s) = 0 \tag{5}$$

$$f_1(s) = M(s) \cdot e^{\tau s} = \sum_{k=0}^{n} a_k s^k \cdot e^{\tau s} \tag{6}$$

By connecting Equations (5) and (6), the final expression for the characteristic equation of the automatic control system in the complex domain is obtained:

$$f(s) = f_1(s) + \left(K_d \cdot s + K_p\right) \cdot N(s) = 0 \tag{7}$$

Taking into account Equation (7), it is necessary to express the complex number *s* in a suitable form and use it

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for establishing the relation between the damping degree ξ and the variable parameters of the controller, K_p and K_i, contained in the characteristic equation (7) for the automatic control system. This is how the area from the "s" plane below the straight line ξ = const. (Figure 2) is mapped in the area of the corresponding damping coefficient represented by the curve ξ = const., in the parameter plane of tuning parameters of the controller (K_p, K_d).

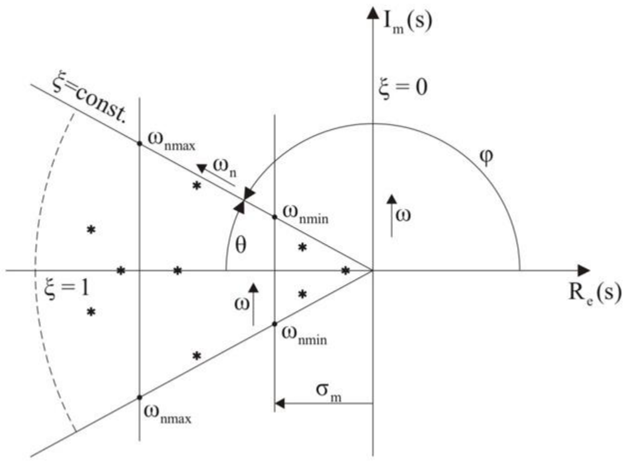

Figure 2: Area with the required settling time and relative stability [15]

Since:

$$s = -\omega_n \xi + j\omega_n \sqrt{1-\xi^2}, 0 \le \xi \le 1 \tag{8}$$

a pure delay term can be expressed in the form:

$$e^{\tau s} = e^{-\tau\xi\omega_n}\left(\cos\tau\omega_n\sqrt{1-\xi^2} + j\sin\tau\omega_n\sqrt{1-\xi^2}\right) \tag{9}$$

$$e^{\tau s} = e^{-p}\left(\cos q + j\sin q\right) \tag{10}$$

where

$$p = \tau\xi\omega_n, q = \tau\omega_n\sqrt{1-\xi^2} \tag{11}$$

If s is given by equation (8), then s^k is determined by the expression:

$$s^k = \omega_n^k\left(T_k(-\xi) + j\sqrt{1-\xi^2}U_k(-\xi)\right) \tag{12}$$

or:

$$s^k = \omega_n^k\left((-1)^k T_k(\xi) + j\sqrt{1-\xi^2}(-1)^{k+1}U_k(\xi)\right) \tag{13}$$

where T_k and U_k represent Chebyshev functions of the first and second kind for which the following recurrent equations hold:

$$T_{k+1}(\xi) = 2\xi T_k(\xi) - T_{k-1}(\xi) \tag{14}$$

$$U_{k+1}(\xi) = 2\xi U_k(\xi) - U_{k-1}(\xi) \tag{15}$$

$$T_0 = 1, T_1 = \xi, U_0 = 0, U_1 = 1 \tag{16}$$

By connecting (7) i (8) it is obtained:

$$f_1(\xi,\omega_n) + \left[K_d\left(-\xi\omega_n + j\omega_n\sqrt{1-\xi^2}\right) + K_p\right]\cdot N(\xi,\omega_n) = 0 \tag{17}$$

By connecting now equations (6), (10) and (13) we get:

$$f_1(\xi,\omega_n) + \left[K_d\left(-\xi\omega_n + j\omega_n\sqrt{1-\xi^2}\right) + K_p\right]\cdot N(\xi,\omega_n) = 0 \tag{18}$$

$$\begin{aligned} f_1(\xi,\omega_n) &= \left[\alpha(\xi,\omega_n)\cos q - \beta(\xi,\omega_n)\sin q\right]e^{-p} + \\ &+ j\left[\alpha(\xi,\omega_n)\sin q + \beta(\xi,\omega_n)\cos q\right]e^{-p} \end{aligned} \tag{19}$$

in which:

$$\alpha(\xi,\omega_n) = \sum_{k=0}^{n} a_k(-1)^k \omega_n^k T_k(\xi) \tag{20}$$

$$\beta(\xi,\omega_n) = \sqrt{1-\xi^2}\sum_{k=0}^{n} a_k(-1)^{k+1}\omega_n^k U_k(\xi) \tag{21}$$

The term $N(\xi,\omega_n)$ can be written in the following forms:

$$N(\xi,\omega_n) = \gamma(\xi,\omega_n) + j\delta(\xi,\omega_n) \tag{22}$$

$$N(\xi,\omega_n) = \sum_{k=0}^{m} b_k\omega_n^k\left((-1)^k T_k(\xi) + j\sqrt{1-\xi^2}(-1)^{k+1}U_k(\xi)\right) \tag{23}$$

Equalizing the expressions (22) and (23), and separating the real and imaginary part, it can be obtained:

$$\gamma(\xi,\omega_n) = \sum_{k=0}^{m} b_k\omega_n^k(-1)^k T_k(\xi) \tag{24}$$

$$\delta(\xi,\omega_n) = \sqrt{1-\xi^2}\sum_{k=0}^{m} b_k\omega_n^k(-1)^{k+1}U_k(\xi) \tag{25}$$

By connecting equation (17) with equations (18) - (25), and separating the real and imaginary part, we obtain the following system of equations:

$$\begin{aligned} &K_d\left[\xi\omega_n\gamma(\xi,\omega_n) + \sqrt{1-\xi^2}\omega_n\delta(\xi,\omega_n)\right] - \\ &- K_p\gamma(\xi,\omega_n) = (\alpha\cos q - \beta\sin q)e^{-p} \end{aligned} \tag{26}$$

$$\begin{aligned} &K_d\left[\xi\omega_n\delta(\xi,\omega_n) - \sqrt{1-\xi^2}\omega_n\gamma(\xi,\omega_n)\right] - \\ &- K_p\delta(\xi,\omega_n) = (\alpha\sin q + \beta\cos q)e^{-p} \end{aligned} \tag{27}$$

By solving the system of equations (26)-(27) at $\omega_n \ne 0, 0 \le \xi < 1$, the expressions for the parameters K_p and K_d of the PD controller are obtained. From this system of equations for $\omega_n = 0$ i $\omega_n = \infty$, the singular straight lines are defined, which are described by equations:

$$K_p(\xi, 0) = 0 \tag{28}$$

$$K_d(\xi, \infty) = 0 \tag{29}$$

3. APPLICATION OF PROPOSED METHOD FOR PID CONTROLLER DESIGN ON THE EXAMPLES FOR CONTROLLING OF PROCESSES WITH TIME DELAY

By using Matlab software package it is written a program to determine the parameters of the PD controller, based on a system of equations (26)-(27) and the singular straight lines (28)-(29).

This program automatically draws a depending curve K_d as a function of the proportional gain K_p in the parameter plane for the required damping degree of closed-loop of automatic control system.

To illustrate the proposed method of the PD controller design for processes with time-delay three examples are selected:

$$W_{P1}(s) = \frac{1}{(s+1)^3}e^{-15s} \tag{30}$$

$$W_{P2}(s) = \frac{1}{(s^2+s+1)(s+3)}e^{-s} \tag{31}$$

$$W_{P3}(s) = e^{-s} \tag{32}$$

An example of the transfer function of the process $W_{P1}(s)$ is characterized in terms of a relatively large time delay. In this process the ratio between the time delay and the dominant time constant is 15/3. In the process described by transfer function $W_{P3}(s)$, this ratio is even more extreme and theoretically tends to infinity.

In Figure 3, it is shown the parameter plane for the transfer function of the process $W_{P1}(s)$ with the boundaries of the absolute and relative stability for the required damping degree of the closed loop of automatic control system, which is $\xi = 0,5$. Stability limit is marked with $\xi = 0,0$. The values of proportional gain Kp and the corresponding differential gain Kd for the value of the damping degree $\xi = 0,5$ are read from the Figure 3.

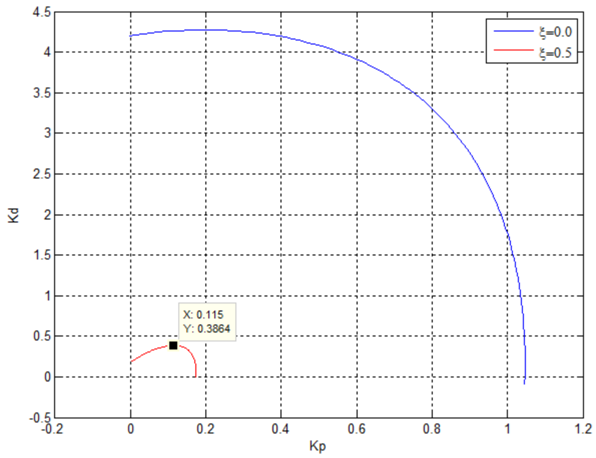

Figure 3: Parameter plane for different values of the damping coefficients of closed loop for transfer function $W_{P1}(s)$

Response of the automatic control system for the values of the damping degree $\xi = 0,5$ to the action of reference is shown in Figure 4, and to the action of disturbance in Figure 5. The amplitude and phase-frequent characteristic of the open loop is shown in Figure 6.

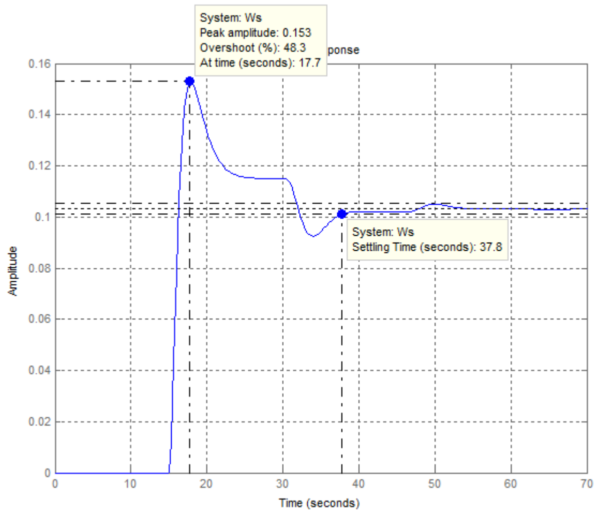

Figure 4: Response of the automatic control system, for the value of the damping degree $\xi = 0,5$ for transfer function $W_{P1}(s)$

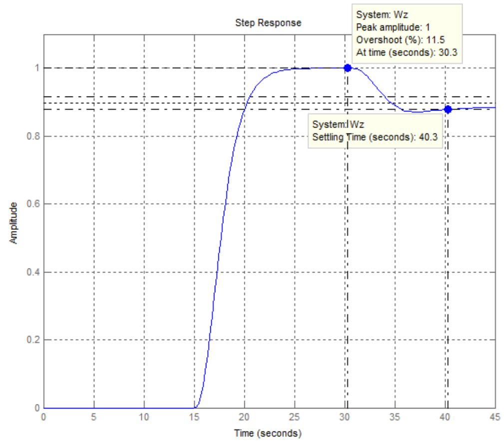

Figure 5: Response of the automatic control system for the value of the damping degree $\xi = 0,5$ with load disturbance rejection for transfer function $W_{P1}(s)$

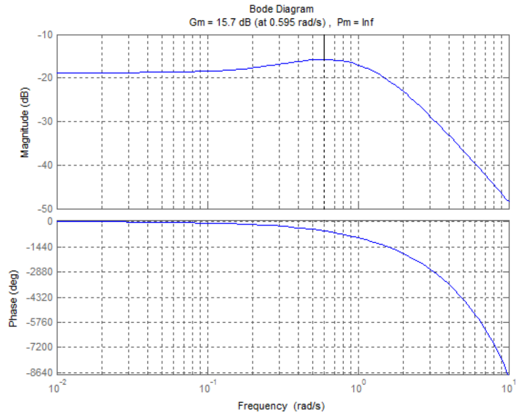

Figure 6: Amplitude and phase-frequent characteristic of the open loop of the process described by transfer function $W_{P1}(s)$ for the value of the damping degree $\xi = 0,5$

Second process, which is considered, is described by transfer function $W_{P2}(s)$. Figure 7 shows the parameter plane for this process, for damping degree $\xi = 0,5$ and stability limit $\xi = 0,0$. The values of proportional gain Kp and the corresponding differential gain Kd for the value of the damping degree $\xi = 0,5$ are read from the Figure 7.

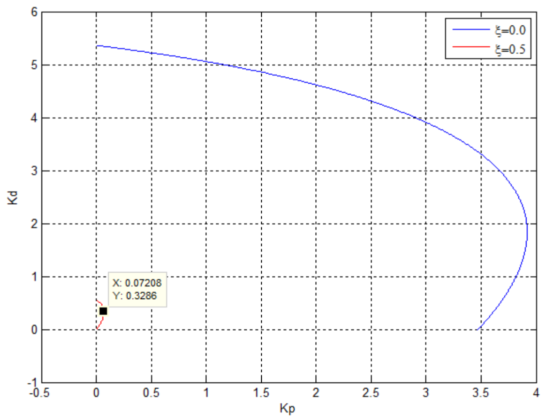

Figure 7: Parameter plane for different values of damping coefficient of the closed loop for transfer function $W_{P2}(s)$

Response of the automatic control system of the transfer function $W_{P2}(s)$ for the values of the damping degree $\xi = 0,5$ to the action of reference is shown in Figure 8, and to the action of disturbance in Figure 9. The amplitude and phase-frequent characteristic of the open loop is shown in Figure 10.

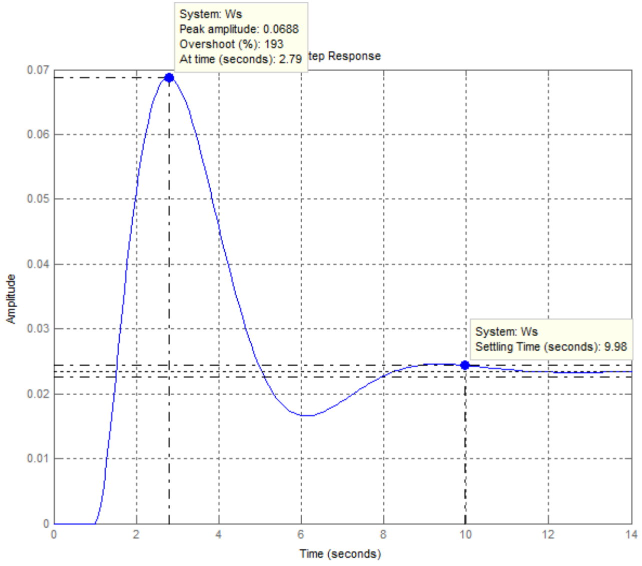

Figure 8: Response of the automatic control system for the value of the damping degree $\xi = 0,5$ for transfer function $W_{P2}(s)$

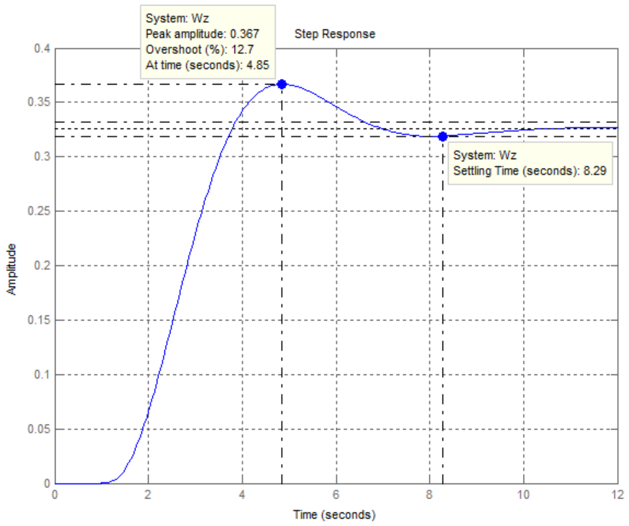

Figure 9: Response of the automatic control system for the value of the damping degree $\xi = 0,5$ with load disturbance rejection for transfer function $W_{P2}(s)$

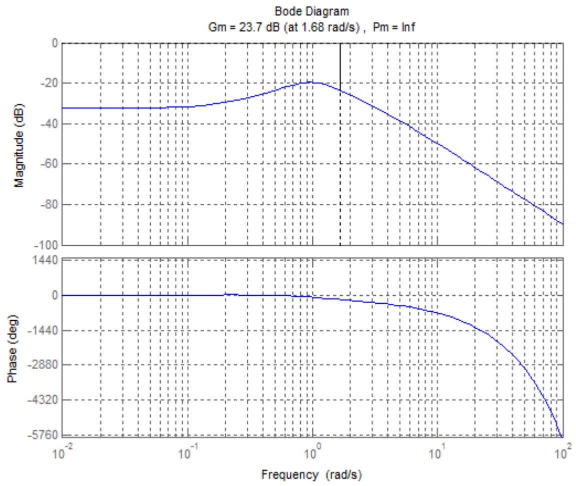

Figure 10: Amplitude and phase-frequent characteristic of the open loop of the process described by transfer function $W_{P2}(s)$ for the value of the damping degree $\xi = 0,5$

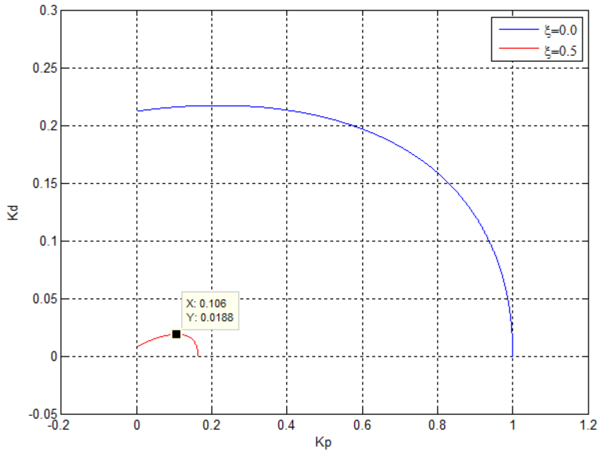

Figure 11: Parameter plane for different values of damping coefficient of the closed loop for transfer function $W_{P3}(s)$

Third process, which is considered, is described by transfer function $W_{P3}(s)$. Figure 11 shows the parameter plane for the process described by this transfer function, from which the values of proportional gain Kp and the corresponding differential gain Kd are read, for the value of the damping degree $\xi = 0,5$.

Response of the automatic control system of the transfer function $W_{P3}(s)$ for the values of the damping degree $\xi = 0,5$ to the action of reference is shown in Figure 12, and to the action of disturbance in Figure 13. The amplitude and phase-frequent characteristic of the open loop is shown in Figure 14.

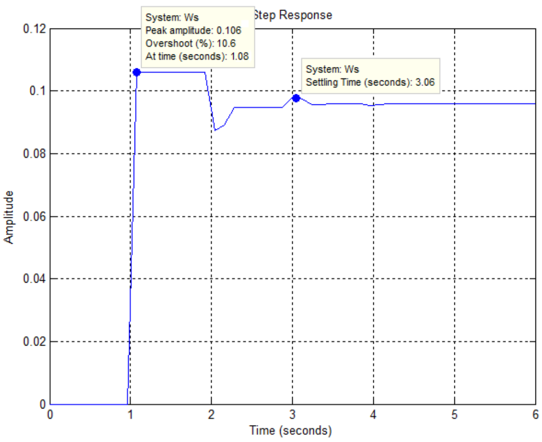

Figure 12: Response of the automatic control system for the value of the damping degree $\xi = 0{,}5$ for transfer function $W_{P3}(s)$

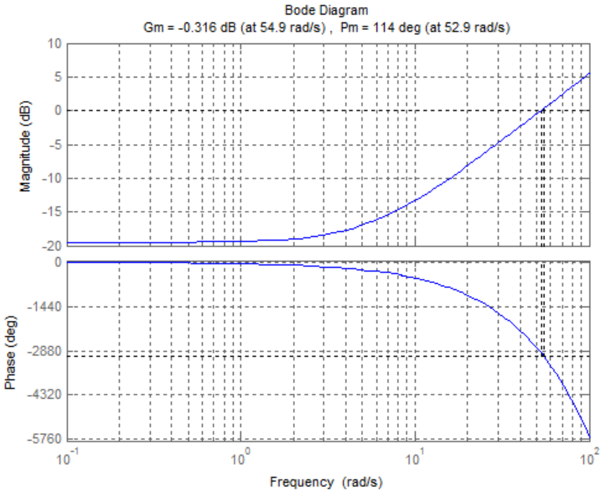

Figure 14: Amplitude and phase-frequent characteristic of the open loop of the process described by transfer function $W_{P3}(s)$ for the value of the damping degree $\xi = 0{,}5$

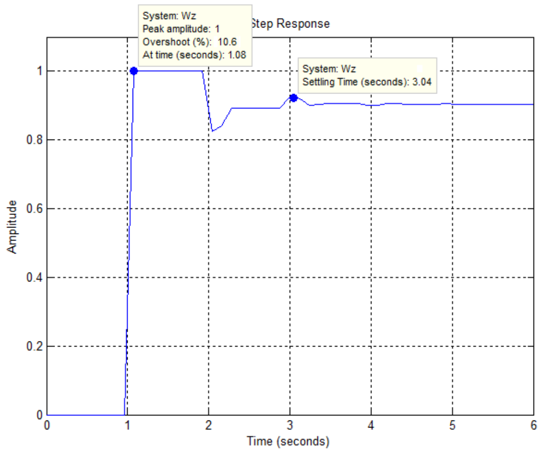

Figure 13: Response of the automatic control system for the value of the damping degree $\xi = 0{,}5$ with load disturbance rejection for transfer function $W_{P3}(s)$

The values of the controller parameters with performances of transfer functions $W_{P1}(s)$, $W_{P2}(s)$ i $W_{P3}(s)$, for values of the damping degree $\xi = 0{,}5$, are shown in Table 1.

Table 1: The values of the controller parameters with performances of transfer functions $W_{P1}(s)$, $W_{P2}(s)$ i $W_{P3}(s)$, for values of the damping degree $\xi = 0{,}5$, are shown in Table 1.

	$W_{P1}(s)$	$W_{P2}(s)$	$W_{P3}(s)$
Damping degree ξ	0,5	0,5	0,5
Proportional gain K_p	0,115	0,07208	0,106
Differential gain K_d	0,3864	0,3286	0,0188
Overshoot (reference) [%]	48,3	193	10,6
Settling time (reference) t_s [s]	37,8	9,98	3,06
Overshoot (disturbance) [%]	11,5	12,7	10,6
Settling time (disturbance) t_s [s]	40,3	8,29	3,04
Phase margin φ_m [°]	Inf	Inf	114
Gain margin g_m [dB]	15,7	23,7	-0,316

4. CONCLUSIONS

It has been concluded that the PD regulator obtained by the present method provides a lot of good results, and its advantage is the simplicity of the method. The results presented in this paper show that the differential action minimizes the effects of time delay and achieves stability of the system with large time delay. The main disadvantage of this type of controllers is inability to eliminate the static error.

Further authors' research will focus on PID controller design using this method, which this deficiency will be removed.

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