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Analytical Form for Total Static Deflection of the Articulated Boom of the Mobile Elevating Work Platform

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This paper deals with a method for obtaining an analytical form for total deflection of the articulated boom of the mobile elevating work platform (MEWP). Analysis was carried out on the example of a three-segment articulated structure where the influences of self-weight and payload were considered. The force reactions in joints were defined in relation to inclination angles, loading and geometrical parameters for each segment local coordinate system. The whole structure was divided into sections with the corresponding geometrical parameters and equations of bending moments. Based on strain energy and Castigliano's second theorem, component displacements of the boom tip were calculated in the global coordinate system XYZ. The Finite element method (FEM) was used for comparison and model validation. The deviations of the analytical model are less than 5% for different boom shapes within the work range.

Keywords: **deflection, articulated boom, mobile elevating work platform, self-weight, payload, strain energy, FEM.**

0. INTRODUCTION

Steel supporting structures of cranes, subjected to various types of loads such as self-weight and payload, must fulfill a series of requirements. Besides the condition of material strength, elastic stability, connections strength, etc., there is a condition of deformation or deflection, which often happens to be the most restricting one. This requirement restricts displacements of the characteristic points of the structure, caused by loads acting during operation. Insufficient stiffness of construction, while operating, may cause excessive deformation occurrence and unwanted additional static and dynamic loads. As a result, an unreliable operation and short service life of machines and equipment could take place. Therefore, limiting the value of static deflection of the structure makes a prerequisite for achieving good exploitation properties.

Allowed deflections are prescribed by standards for certain type of crane supporting structures. Thus, for example, allowed static deflection at the mid-span point of the bridge crane main girder is $\frac{L}{600}$, where L - the span of the crane [1]. In the case of gantry crane, the allowed static deflection at the endpoint of overhang is $\left(\frac{1}{400}\div\frac{1}{200}\right)L_1$, where L_1 is the overhang length, while at the mid-span point it is $\left(\frac{1}{1000}\div\frac{1}{600}\right)L$ [2].

Calculating the deflection of the supporting structures of these types of cranes is easy thanks to their simplicity and unchangeable geometry. However, it is not always the case.

MEWP with articulated booms (Fig. 1) are typical representatives of steel carrying structures with variable geometry, which changes depending on the working position [3]. The main loads, structural elements self-weight and the weight of the payload remain constant in direction and intensity, but the geometry of the articulated boom varies according to the current horizontal reach and the lifting height within its workspace, Fig. 2. This fact leads to variability in load for all members of the mechanism and thus the variable displacement of the characteristic points, depending on the boom's position.

Fig. 1. *MEWP with articulated booms*

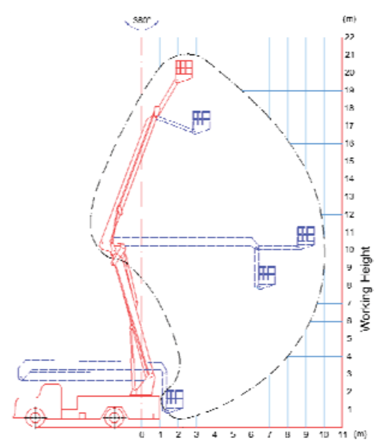

Fig. 2. *Workspace diagram of MEWP*

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The constraint imposed by static deformations may be dominant in the case of machine structures [4]. Analytical dependencies enable the designer to use limited endpoint displacement ($f < f_{allowed}$) as a constraint function for design optimization in order to reduce the mass of the structure [4-8]. Self-weight influence can significantly participate in overall deflection of the carrying structure.

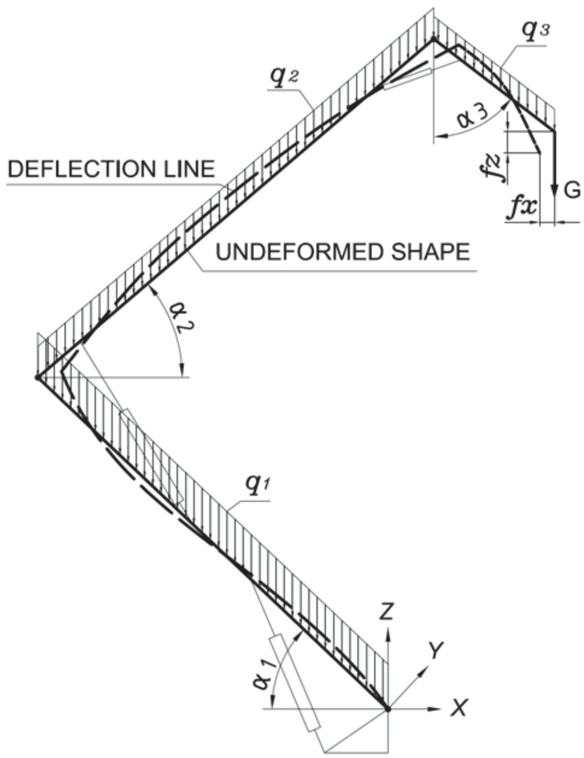

Fig. 3. *Deflection of the articulated boom and displacement of the boom tip under payload and self-weight*

The paper presents a method for determining the overall static deflection of the MEWP articulated boom, taking into account the influence of payload and segments self-weight as well. Analysis is carried out on the model of three-segment articulated mechanism driven by three hydraulic cylinders (Fig. 3).

1. APPROXIMATE ANALYTICAL SOLUTION

The model of three-segment MEWP articulated boom, which was the subject of analysis, is shown in Fig.4.

The structure consists of three segments and three hydro cylinders connected by joints. There is a global coordinate system *XYZ*, while each segment has its own local coordinate system $\xi_i\eta_i\zeta_i$, $i=1,2,3$ respectively.

The orientation of all local systems is such that the axial axis is ζ_i , while the bending of each segment occurs about ξ_i-axis, which is parallel to the global *Y*-axis.

Joint connections between segments are designated as *A*, *B* and *C*, while payload is taken to act in *D*, being the tip of the boom and endpoint of the third segment as well.

Each hydro cylinder connects a segment with a lower one, causing its rotation about the connection joint, while extracting or retracting.

In such a manner, all work points from MEWP workspace are reached (Fig. 2). So, the boom tip position depends on segments lengths and the inclination angles α_1, α_2 and α_3.

1.1 Strain energy method and basic assumptions

The payload *G* and self-weigth of segments cause the structure to deflect in the vertical *XZ* plane, where the boom tip has displacements in both directions *X* and *Z*, f_x and f_z (Fig. 3).

In order to find the f_x displacement, a virtual external force *X* is introduced at the boom tip. Based on strain energy and Castigliano's second theorem, the corresponding displacements of the boom's tip in the *Z* and *X* directions are:

$$f_z = \frac{\partial A_d}{\partial G} = \frac{1}{E}\sum_{i=1}^{m}\frac{1}{I_i}\int_0^{l_i} M_i(s)\frac{\partial M_i(s)}{\partial G}ds \qquad (1)$$

$$f_x = \left.\frac{\partial A_d}{\partial X}\right|_{X=0} = \left.\frac{1}{E}\sum_{i=1}^{m}\frac{1}{I_i}\int_0^{l_i} M_i(s)\frac{\partial M_i(s)}{\partial X}ds\right|_{X=0} \qquad (2)$$

where: A_d - the strain energy, *E* - the modulus of elasticity, *m* - the number of sections (subdivisions), $M_i(s)$ - the bending moment of section *i*, *s* - the floating section coordinate.

The first action taken in order to define expressions for bending moments and corresponding partial derivatives for each section is to find analytical dependencies of joint reactions and forces in hydrocylinders with the respect to inclination angles.

To make analysis easier, the process of joint reactions determination is carried out for three independent load cases: case 1 - self-weight acts only, case 2 - payload *G* acts only and case 3 - virtual force *X* acts only. It is assumed that the principle of superposition is applied to individual influences of self-weight, payload *G* and virtual force *X*, so total joint reactions and forces in hydro cylinders are obtained by superposition of these influences.

The segments of the articulated boom are considered as beams with a constant cross-section along their lengths. Therefore, their weigths are taken into account as uniformly distributed loads q_1, q_2 and q_3 (Figures 3 and 4).

Also, when calculating strain energy, the influence of bending moments is taken into account only, while other contributions are neglected.

In addition, the influence of the substructure elasticity (vehicle chassis) is ignored because its stiffness is much greater than the stiffness of the boom itself.

1.2 Joint reactions and hydro cylinders forces in relation to the work position of the boom

Firstly, there was a static analysis of the structure and the joint reactions and hydro cylinders forces were determined for each segment individually, as the functions of inclination angles of segments α_1, α_2 and α_3, lengths and external load.

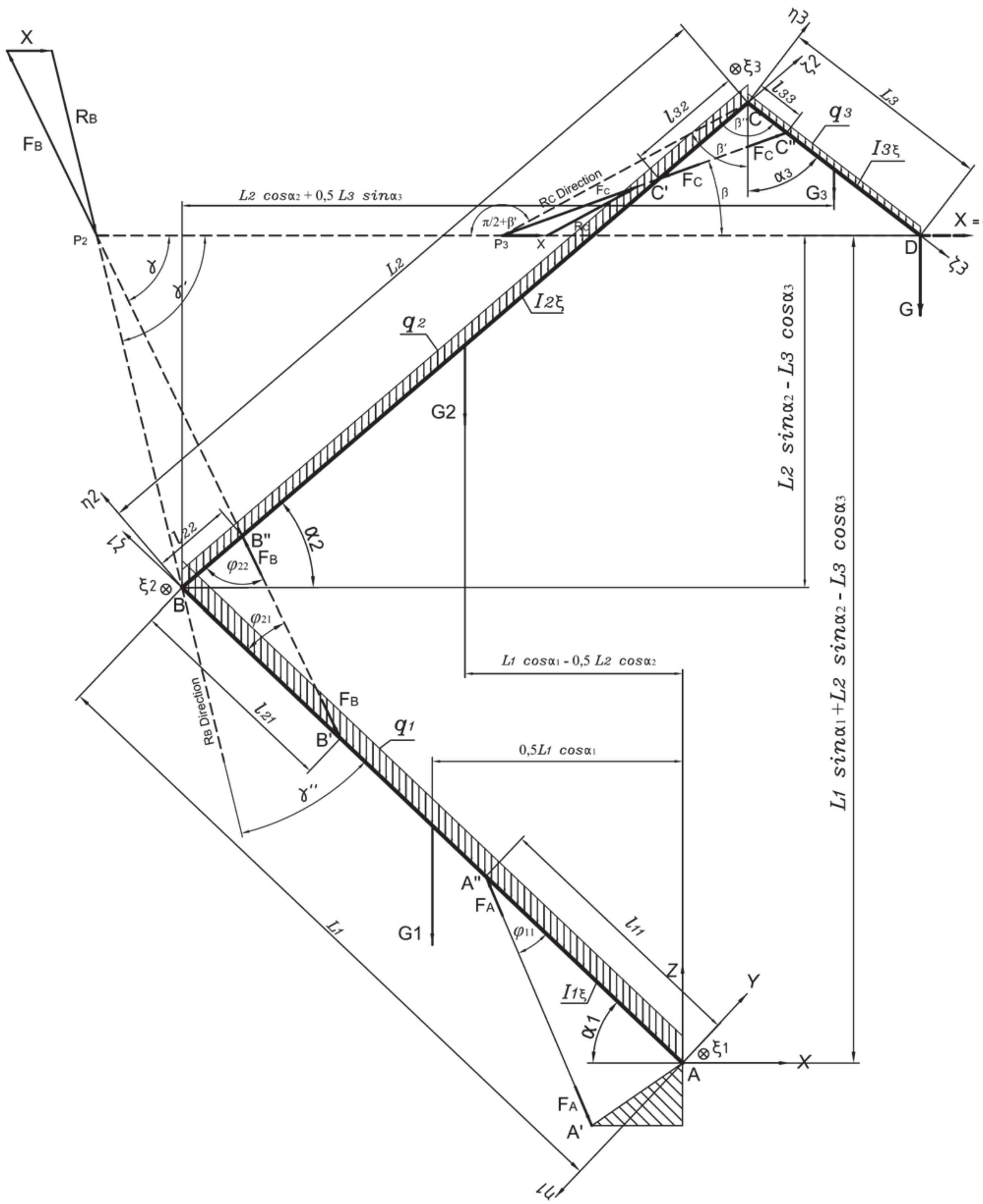

Fig. 4. *Model of the MEWP three-segment articulated boom used for determination of total deflection*

The reaction forces for each segment were obtained in the corresponding local coordinate system $\xi_i\eta_i\zeta_i$, $i=1,2,3$. It made possible to obtain the equations of bending moments for each segment, with respect to these angles.

Joint reactions and forces in hydro cylinders when payload and self-weight are acting have already been calculated [9], so the task is now to resolve the case when virtual force X is acting only. To be more precise, it is necessary to derive the analytical expressions only for transverse components of joints reactions which cause bending and deflection of the articulated structure. These components, according to Fig. 4, are projections of joint reactions and hydro cylinders forces onto the local axes η_i, $i=1,2,3$.

Firstly, segment *3* is analyzed. Out of a moment equation set for point *C*, it is obtained:

$$F_{C\eta 3} = -F_C \sin\varphi_{33} = -\frac{X}{l_{33}} L_3 \cos\alpha_3 \quad (3)$$

Applying the second condition of static equilibrium for segment *3*, it is:

$$R_{C\eta 3} = \frac{X}{l_{33}}\left(L_3 - l_{33}\right)\cos\alpha_3 \quad (4)$$

Out of a moment equation for segment *2* set for point *B*, it is derived:

$$F_{B\eta 2} = F_B \sin\varphi_{22} = \frac{X}{l_{22}}\left(L_2 \sin\alpha_2 - L_3 \cos\alpha_3\right) \quad (5)$$

When calculating the transverse component of force in hydro cylinder F_C on segment *2*, it is taken with the opposite direction. Regarding (3) and applying a sine theorem onto triangle *CC'C"* it is obtained:

$$F_{C\eta 2} = -F_C \sin\varphi_{32} = -X\,\frac{L_3}{l_{32}}\cos\alpha_3 \quad (6)$$

According to Fig. 2 it is written:

$$R_{C\eta 2} = R_C \cos\left[\pi + (\alpha_3 - \alpha_2) - \beta''\right] \quad (7)$$

Having in mind the relation

$$\beta'' = \beta' + \alpha_3 \quad (8)$$

(7) is transformed into the following form:

$$R_{C\eta 2} = R_C(\sin\beta'\sin\alpha_2 - \cos\beta'\cos\alpha_2) \quad (9)$$

Expressions for sinβ' and cosβ' are derived from the force graph of F_C, R_C and X by means of sine and cosine theorem:

$$\cos\beta' = \frac{F_C}{R_C}\sin\beta\,,\ \sin\beta' = \frac{F_C\cos\beta - X}{R_C} \quad (10)$$

Using dependencies (10) in (9) it is obtained:

$$R_{C\eta 2} = -X\sin\alpha_2 - F_C\sin(\beta - \alpha_2) \quad (11)$$

Considering the following relation from triangle *CC'C"*

$$\beta = \alpha_3 + \varphi_{33} - \frac{\pi}{2} \quad (12)$$

and applying some transformations, finally it is derived:

$$R_{C\eta 2} = \frac{X}{l_{32}}\left(L_3\cos\alpha_3 - l_{32}\sin\alpha_2\right) \quad (13)$$

Applying the condition of static equilibrium for segment *2*, it is calculated:

$$R_{B\eta 2} = \frac{X}{l_{22}}\left(l_{22}\sin\alpha_2 - L_2\sin\alpha_2 + L_3\cos\alpha_3\right) \quad (14)$$

A similar procedure is carried out for segment *1*. Out a of moment equation set for point *A*, it is obtained:

$$F_{A\eta 1} = \frac{X}{l_{11}}\left(L_1\sin\alpha_1 + L_2\sin\alpha_2 - L_3\cos\alpha_3\right) \quad (15)$$

When calculating the transverse component of force in hydro cylinder F_B on segment *1*, it is taken with the opposite direction. Regarding (5) and applying sine theorem onto triangle *BB'B"* it is obtained:

$$F_{B\eta 1} = F_B\sin\varphi_{21} = \frac{X}{l_{21}}\left(L_2\sin\alpha_2 - L_3\cos\alpha_3\right) \quad (16)$$

According to Fig. 2 it is written:

$$R_{B\eta 1} = R_B\cos\left(\frac{3\pi}{2} - \gamma''\right) \quad (17)$$

Having in mind the relation

$$\gamma'' = \gamma' - \alpha_1 \quad (18)$$

(17) is transformed into the following form:

$$R_{B\eta 1} = R_B(\sin\alpha_1\cos\gamma' - \cos\alpha_1\sin\gamma') \quad (19)$$

Expressions for sinγ' and cosγ' are derived from force graph of F_B, R_B and X by means of sine and cosine theorem:

$$\cos\gamma' = \frac{F_B\cos\gamma - X}{R_B}\,,\ \sin\gamma' = \frac{F_B}{R_B}\sin\gamma \quad (20)$$

Using dependencies (20) in (19) it is obtained:

$$R_{B\eta 1} = -X\sin\alpha_1 - F_B\sin(\gamma - \alpha_1) \quad (21)$$

Considering the following relation

$$\alpha_2 + \varphi_{22} + \gamma = \pi \quad (22)$$

and applying some transformations, finally it is derived:

$$R_{B\eta 1} = \frac{X}{l_{21}}\left(-l_{21}\sin\alpha_1 - L_2\sin\alpha_2 + L_3\cos\alpha_3\right) \quad (23)$$

Applying the condition of static equilibrium for segment *1*, it is calculated:

$$R_{A\eta 1} = \frac{X}{l_{11}}\begin{pmatrix} l_{11}\sin\alpha_1 - L_1\sin\alpha_1 - \\ -L_2\sin\alpha_2 + L_3\cos\alpha_3 \end{pmatrix} \quad (24)$$

For clarification, the obtained analytical expressions for transverse components of reactive forces upon all three segments are given in Table 1. If the self-weight members in force reactions are denoted as Q_j and multipliers of G and X are denoted as g_j and x_j ($j=1,...,11$), total forces values can be noted as follows:

$$F_j = Q_j(G_1, G_2, G_3) + g_j G + x_j X\ ,\ j = 1,...,11 \quad (25)$$

where: $G_1=q_1L_1$, $G_2=q_2L_2$ and $G_3=q_3L_3$ are segments self-weights which are taken to act in the mid-points.

Table 1. *Transverse components of force reactions*

	Joint	Force	Load case with self-weight acting only [9]	Load case with G acting only [9]	Load case with X acting only (this paper)
segment I	A	$F_1 = R_{A\eta 1}$	$\frac{L_1}{l_{11}}(\frac{G_1}{2}+G_2+G_3)\cos\alpha_1 - \frac{L_2}{l_{11}}(\frac{G_2}{2}+G_3)\cos\alpha_2 - \frac{L_3}{l_{11}}\frac{G_3}{2}\sin\alpha_3 - (G_1+G_2+G_3)\cos\alpha_1$	$\frac{G}{l_{11}}\left(L_1\cos\alpha_1 - L_2\cos\alpha_2 - L_3\sin\alpha_3 - l_{11}\cos\alpha_1\right)$	$\frac{X}{l_{11}}\left(l_{11}\sin\alpha_1 - L_1\sin\alpha_1 - L_2\sin\alpha_2 + L_3\cos\alpha_3\right)$
	A”	$F_2 = F_{A\eta 1}$	$-\frac{L_1}{l_{11}}(\frac{G_1}{2}+G_2+G_3)\cos\alpha_1 + \frac{L_2}{l_{11}}(\frac{G_2}{2}+G_3)\cos\alpha_2 + \frac{L_3}{l_{11}}\sin\alpha_3\frac{G_3}{2}$	$\frac{G}{l_{11}}\left(-L_1\cos\alpha_1 + L_2\cos\alpha_2 + L_3\sin\alpha_3\right)$	$\frac{X}{l_{11}}\left(L_1\sin\alpha_1 + L_2\sin\alpha_2 - L_3\cos\alpha_3\right)$
	B’	$F_3 = F_{B\eta 1}$	$\frac{L_2}{l_{21}}(\frac{G_2}{2}+G_3)\cos\alpha_2 + \frac{L_3}{l_{21}}\frac{G_3}{2}\sin\alpha_3$	$\frac{G}{l_{21}}\left(L_2\cos\alpha_2 + L_3\sin\alpha_3\right)$	$\frac{X}{l_{21}}\left(L_2\sin\alpha_2 - L_3\cos\alpha_3\right)$
	B	$F_4 = R_{B\eta 1}$	$(G_3+G_2)\cos\alpha_1 - \frac{L_2}{l_{21}}(G_3+\frac{G_2}{2})\cos\alpha_2 - \frac{L_3}{l_{21}}\frac{G_3}{2}\sin\alpha_3$	$\frac{G}{l_{21}}\left(l_{21}\cos\alpha_1 - L_2\cos\alpha_2 - L_3\sin\alpha_3\right)$	$\frac{X}{l_{21}}\left(-l_{21}\sin\alpha_1 - L_2\sin\alpha_2 + L_3\cos\alpha_3\right)$
segment II	B	$F_5 = R_{B\eta 2}$	$(G_2+G_3)\cos\alpha_2 - \frac{L_2}{l_{22}}\cos\alpha_2(G_3+\frac{G_2}{2}) - \frac{L_3}{l_{22}}\sin\alpha_3\frac{G_3}{2}$	$\frac{G}{l_{22}}\left(l_{22}\cos\alpha_2 - L_2\cos\alpha_2 - L_3\sin\alpha_3\right)$	$\frac{X}{l_{22}}\left(l_{22}\sin\alpha_2 - L_2\sin\alpha_2 + L_3\cos\alpha_3\right)$
	B”	$F_6 = F_{B\eta 2}$	$\frac{L_2}{l_{22}}(\frac{G_2}{2}+G_3)\cos\alpha_2 + \frac{L_3}{l_{22}}\frac{G_3}{2}\sin\alpha_3$	$\frac{G}{l_{22}}\left(L_2\cos\alpha_2 + L_3\sin\alpha_3\right)$	$\frac{X}{l_{22}}\left(L_2\sin\alpha_2 - L_3\cos\alpha_3\right)$
	C’	$F_7 = F_{C\eta 2}$	$\frac{l_3}{l_{32}}\frac{G_3}{2}\sin\alpha_3$	$\frac{G}{l_{32}}L_3\sin\alpha_3$	$-\frac{X}{l_{32}}L_3\cos\alpha_3$
	C	$F_8 = R_{C\eta 2}$	$-G_3\cos\alpha_2 - \frac{L_3}{l_{32}}\frac{G_3}{2}\sin\alpha_3$	$\frac{G}{l_{32}}\left(-l_{32}\cos\alpha_2 - L_3\sin\alpha_3\right)$	$\frac{X}{l_{32}}\left(L_3\cos\alpha_3 - l_{32}\sin\alpha_2\right)$
segment III	C	$F_9 = R_{C\eta 3}$	$(1-\frac{L_3}{2l_{33}})G_3\sin\alpha_3$	$\frac{G}{l_{33}}\left(l_{33} - L_3\right)\sin\alpha_3$	$\frac{X}{l_{33}}\left(L_3 - l_{33}\right)\cos\alpha_3$
	C”	$F_{10} = F_{C\eta 3}$	$\frac{L_3}{l_{33}}0{,}5G_3\sin\alpha_3$	$\frac{G}{l_{33}}L_3\sin\alpha_3$	$-\frac{X}{l_{33}}L_3\cos\alpha_3$
	D	$F_{11} = R_{D\eta 3}$	0	$-G\sin\alpha_3$	$X\cos\alpha_3$

1.3 Derivation of equations for boom tip displacements

At this point of analysis, the whole structure can be disassembled and the segments can be considered as independent beams. In order to make a further procedure easer, the articulated boom is divided into 8 sections between joints and support points of hydro cylinders, as it is shown in Figure 5.

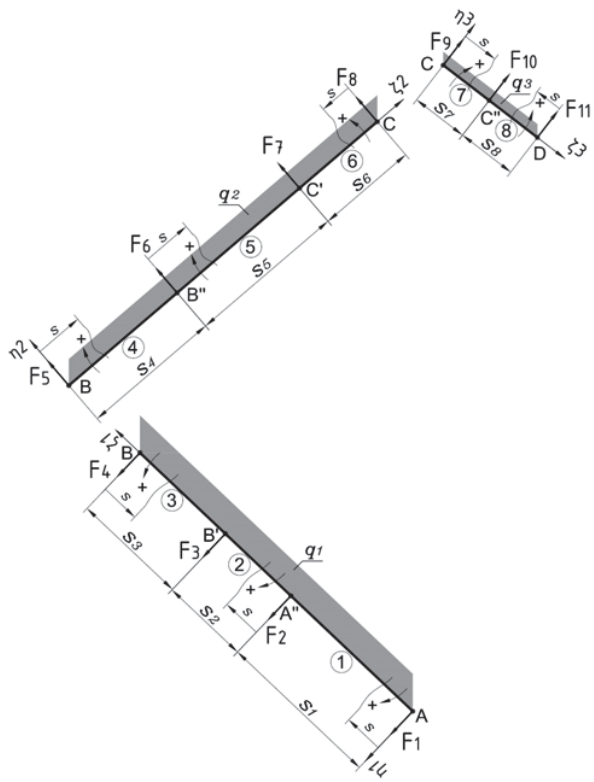

Fig. 5. *Articulated boom divided into sections*

Considering new simplified designations from equation (25) and the adopted positive direction introduced in Figure 5, the corresponding expressions for bending moments in relation to floating sections coordinate *s* are given in Table 2.

New forms of expressions (1) and (2) are now:

$$f_z = \frac{\partial A_d}{\partial G} = \sum_{i=1}^{8} f_{zi} = \frac{1}{E}\sum_{i=1}^{8}\frac{1}{I_i}\int_0^{s_i} M_i(s)\frac{\partial M_i(s)}{\partial G}ds \qquad (26)$$

$$f_x = \left.\frac{\partial A_d}{\partial X}\right|_{X=0} = \sum_{i=1}^{8} f_{xi} = \frac{1}{E}\sum_{i=1}^{8}\frac{1}{I_i}\int_0^{s_i} M_i(s)\frac{\partial M_i(s)}{\partial X}ds\Bigg|_{X=0} \qquad (27)$$

As constitutive members of functions that are to be integrated in expressions (26) and (27), there are partial derivatives of bending moments upon payload *G* and virtual horizontal force *X*. Having in mind the condensed equation for force reactions (25), the expressions for these partial derivatives are also given in Table 3.

Table 2. *Expressions for bending moments by sections*

Section	Section length	Moment of inertia	Bending moment
1	$s_1 = l_{11}$	I_1	$F_1 s + \frac{q_1 s^2}{2}\cos\alpha_1$
2	$s_2 = L_1 - l_{11} - l_{21}$		$F_1(s+s_1) + F_2 s + \frac{q_1(s_1+s)^2}{2}\cos\alpha_1$
3	$s_3 = l_{21}$		$F_4 s + \frac{q_1 s^2}{2}\cos\alpha_1$
4	$s_4 = l_{22}$	I_2	$F_5 s - \frac{q_2 s^2}{2}\cos\alpha_2$
5	$s_5 = L_2 - l_{22} - l_{32}$		$F_5(s+s_4) + F_6 s - \frac{q_2(s_4+s)^2}{2}\cos\alpha_2$
6	$s_6 = l_{32}$		$F_8 s - \frac{q_2 s^2}{2}\cos\alpha_2$
7	$s_7 = l_{33}$	I_3	$F_9 s - \frac{q_3 s^2}{2}\sin\alpha_3$
8	$s_8 = L_3 - l_{33}$		$F_{11} s - \frac{q_3 s^2}{2}\sin\alpha_3$

Table 3. *Expressions for partial derivatives*

Section *i*	$\frac{\partial M_i(s)}{\partial G}$	$\frac{\partial M_i(s)}{\partial X}$
1	$g_1 s$	$x_1 s$
2	$g_1(s+s_1)+g_2 s$	$x_1(s+s_1)+x_2 s$
3	$g_4 s$	$x_4 s$
4	$g_5 s$	$x_5 s$
5	$g_5(s+s_4)+g_6 s$	$x_5(s+s_4)+x_6 s$
6	$g_8 s$	$x_8 s$
7	$g_9 s$	$x_9 s$
8	$g_{11} s$	$x_{11} s$

After integration, corresponding members in (26) and (27) are as follows:

$$f_{z1} = \frac{g_1}{24EI_1}\left(8F_1 s_1^3 + 3q_1\cos\alpha_1 s_1^4\right)\Bigg|_{X=0} \qquad (28)$$

$$f_{z2}=\frac{1}{24EI_1}\left\{\begin{array}{l}4(F_1g_2+F_2g_1)(3s_1s_2^2+2s_2^3)+\\8F_1g_1\left[\left(s_1+s_2\right)^3-s_1^3\right]+8F_2g_2s_2^3+\\3q_1g_1\cos\alpha_1\left[\left(s_1+s_2\right)^4-s_1^4\right]+\\q_1g_2\cos\alpha_1(6s_1^2s_2^2+8s_1s_2^3+3s_2^4)\end{array}\right\}\Bigg|_{X=0} \quad (29)$$

$$f_{z3}=\frac{g_4}{24EI_1}\left(8F_4s_3^3+3q_1\cos\alpha_1s_3^4\right)\Big|_{X=0} \quad (30)$$

$$f_{z4}=\frac{g_5}{24EI_2}\left(8F_5s_4^3-3q_2\cos\alpha_2s_4^4\right)\Big|_{X=0} \quad (31)$$

$$f_{z5}=\frac{1}{24EI_2}\left\{\begin{array}{l}4(F_5g_6+F_6g_5)(3s_4s_5^2+2s_5^3)+\\8F_5g_5\left[\left(s_4+s_5\right)^3-s_4^3\right]+8F_6g_6s_5^3-\\3q_2g_5\cos\alpha_2\left[\left(s_4+s_5\right)^4-s_4^4\right]-\\q_2g_6\cos\alpha_2(6s_4^2s_5^2+8s_4s_5^3+3s_5^4)\end{array}\right\}\Bigg|_{X=0} \quad (32)$$

$$f_{z6}=\frac{g_8}{24EI_2}\left(8F_8s_6^3-3q_2\cos\alpha_2s_6^4\right)\Big|_{X=0} \quad (33)$$

$$f_{z7}=\frac{g_9}{24EI_3}\left(8F_9s_7^3-3q_3\sin\alpha_3s_7^4\right)\Big|_{X=0} \quad (34)$$

$$f_{z8}=\frac{g_{11}}{24EI_3}\left(8F_{11}s_8^3-3q_3\sin\alpha_3s_8^4\right)\Big|_{X=0} \quad (35)$$

$$f_{x1}=\frac{x_1}{24EI_1}\left(8F_1s_1^3+3q_1\cos\alpha_1s_1^4\right)\Big|_{X=0} \quad (36)$$

$$f_{x2}=\frac{1}{24EI_1}\left\{\begin{array}{l}4(F_1x_2+F_2x_1)(3s_1s_2^2+2s_2^3)+\\8F_1x_1\left[\left(s_1+s_2\right)^3-s_1^3\right]+8F_2x_2s_2^3+\\3q_1x_1\cos\alpha_1\left[\left(s_1+s_2\right)^4-s_1^4\right]+\\q_1x_2\cos\alpha_1(6s_1^2s_2^2+8s_1s_2^3+3s_2^4)\end{array}\right\}\Bigg|_{X=0} \quad (37)$$

$$f_{x3}=\frac{x_4}{24EI_1}\left(8F_4s_3^3+3q_1\cos\alpha_1s_3^4\right)\Big|_{X=0} \quad (38)$$

$$f_{x4}=\frac{x_5}{24EI_2}\left(8F_5s_4^3-3q_2\cos\alpha_2s_4^4\right)\Big|_{X=0} \quad (39)$$

$$f_{x5}=\frac{1}{24EI_2}\left\{\begin{array}{l}4(F_5x_6+F_6x_5)(3s_4s_5^2+2s_5^3)+\\8F_5x_5\left[\left(s_4+s_5\right)^3-s_4^3\right]+8F_6x_6s_5^3-\\3q_2x_5\cos\alpha_2\left[\left(s_4+s_5\right)^4-s_4^4\right]-\\q_2x_6\cos\alpha_2(6s_4^2s_5^2+8s_4s_5^3+3s_5^4)\end{array}\right\}\Bigg|_{X=0} \quad (40)$$

$$f_{x6}=\frac{x_8}{24EI_2}\left(8F_8s_6^3-3q_2\cos\alpha_2s_6^4\right)\Big|_{X=0} \quad (41)$$

$$f_{x7}=\frac{x_9}{24EI_3}\left(8F_9s_7^3-3q_3\sin\alpha_3s_7^4\right)\Big|_{X=0} \quad (42)$$

$$f_{x8}=\frac{x_{11}}{24EI_3}\left(8F_{11}s_8^3-3q_3\sin\alpha_3s_8^4\right)\Big|_{X=0} \quad (43)$$

2. COMPARISON OF RESULTS OBTAINED FROM THE ANALYTICAL MODEL AND FINITE ELEMENT ANALYSIS

For the purpose of evaluation of the analytical model for boom tip displacement, a numerical example with the following parameters is calculated:
L_1=760[cm], L_2=820[cm], L_3=240[cm], l_{11}=300[cm], l_{21}=336[cm], l_{22}=150[cm], l_{32}=130[cm], l_{32}=54[cm], I_1=9969[cm^4], I_2=5240[cm^4], I_3=2306[cm^4], G=2[kN], E=21000[kN/cm^2], q_1=661.942·10^{-5}[kN/cm], q_2=460.588·10^{-5}[kN/cm], q_3=349.752·10^{-5}[kN/cm],

Calculation is done for five different operation positions of the three-segment articulated boom, Figure 6.

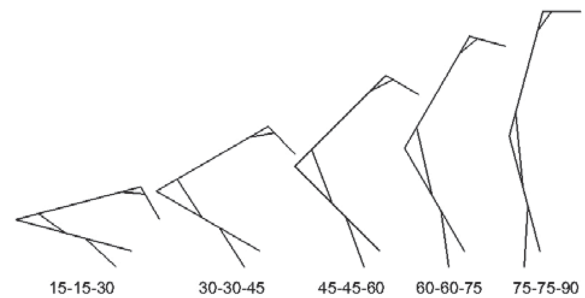

Fig. 6. *Articulated boom shapes represented by combination of inclination angles α_1 -α_2 - α_3 with calculated tip point displacements*

The software package MatLab was used for numerical calculations, while the FEM model was built in software SAP2000. The model for FEM analysis consists of beam finite elements with linear elastic behaviour. The numerical results obtained from analytical model and the finite element method are given in Table 4.

Table 4. *Numerical results from the analytical model and the FEM model with relative deviations*

Boom's shape $\alpha_1-\alpha_2-\alpha_3$(°) (Fig. 6)	f_z (cm)		δ_z	f_x (cm)		δ_x
	Model	FEM	(%)	Model	FEM	(%)
15-15-30	-7.1014	-7.2429	1.95	-0.7629	-0.7876	3.14
30-30-45	-7.0037	-7.1005	1.36	1.3760	1.3750	0.07
45-45-60	-6.0154	-6.0728	0.95	3.1441	3.1739	0.94
60-60-75	-4.4013	-4.3353	1.52	4.0676	3.9547	2.85
75-75-90	-2.5940	-2.7160	4.49	3.8992	4.0998	4.89

3. CONCLUSION

The introduced approach and the applied strain energy method result in good compliance of the gained analytical model and finite element analysis. It has been shown that the relative deviations are less than 5%, which makes the introduced method appropriate for determination of articulated boom deflection.

Contribution of axial and transverse forces to the total amount of the strain energy can be neglected.

Analytical form for displacement of articulated boom tip enables the designer to have control upon geometric parameters through the design process of such structure.

The obtained analytical dependence of boom tip displacement as a function of geometric parameters, self-weight and payload could be the basis for design and optimization of cross-sections of boom segments and/or their lengths. On the other hand, total static deflection under self-weight and payload is a needed parameter for dynamic analysis of such structure.

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4. REFERENCES

[1] Eurocode 3: *Design of steel structures - Part 6: Crane supporting structures*, EN 1993-6

[2] Petković Z., Ostrić D.: *Metal structures in mechanical engineering I* (in Serbian), Faculty of Mechanical Engineering Belgrade,1995

[3] Bošnjak, S., Zrnić, N., Dragović, B. (2009): "Dynamic Response of Mobile Elevating Work Platform under Wind Excitation", *Strojniški vestnik - Journal of Mechanical Engineering*, Vol. 55, No. 2 (2009), pp. 104-113. ISSN 0039-2480.

[4] Farkas Jozsef.: *Optimum Design of Metal Structures*, Akademiai Kiado, Budapest, 1984.

[5] Gasic M, Savkovic M, Bulatovic R, Petrovic R.: "Optimization of a pentagonal cross section of the truck crane boom using Lagrange's multipliers and differential evolution algorithm", *Meccanica* (2011) 46:845–853. doi:10.1007/s1 1012-010-9343-7

[6] Gasic M, Savkovic M, Bulatovic R: "Optimization of trapezoidal cross section of the truck crane boom by Lagrange's multipliers and by differential evolution algorithm (de)", *Strojniški vestnik – Journal of Mechanical Engineering*, 57(2011)4, 304-312, doi: 10.5545/sv-jme.2008.029

[7] Pinca BC, Tirian OG, Socalici VA, Ardeleadn DE: "Dimensional optimization for the strength structure of a traveling crane", *WSEAS Transactions on Applied and theoretical Mechanics 4 (4)*, pp. 147-156

[8] Selmic R, Cvetkovic R, Mijailovic R: *Optimization of crosssection in structures*, monograph. The Faculty of Transport and Traffic Engineering, Belgrade

[9] Zdravković N., Gašić M., Savković M., Petrović D.: "Research of the force values dependences in hydro cylinders of the mobile elevating work platform articulated boom on the work position and load weight", *Proceedings of the 7th international conference research and development of mechanical elements and systems IRMES2011*, pp. 271-278, 27th&28th April 2011, Zlatibor, Serbia, ISBN 978-86-6055-012-7