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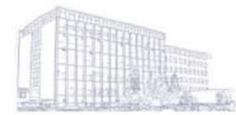

Comparative Analysis and Optimization of Different Cross-sections of Crane Hook Subject to Stresses According to Winkler-Bach Theory

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***Abstract*— The paper presents comparative analysis and optimization of the geometric parameters of different cross-sections of crane hook at the critical places. The reduction of the cross-sectional area of crane hook is primary goal of this research. The selected sections are trapezoidal, elliptic, parabolic and their special cases. Criterion of maximum permissible stress in the characteristic points of cross-section is set as constraint function. Some geometric limits are taken, too. Stresses are observed according to Winkler-Bach theory. Optimization procedure is performed by using the method of Lagrange multiplier and GRG2 algorithm. The obtained results of optimization for different types of cross-sections are compared with each other and with the geometric values of standard hook solution.**

***Keywords*— Crane hook, Optimization, Stresses, Winkler-Bach theory**

I. INTRODUCTION

Crane hooks are devices for grabbing and lifting loads of heavy duty by mean of devices such as crane hoists. Crane hooks are highly responsible components that are typically used for handling material in industries. It is basically a hoisting fixture designed to engage a ring or link of a lifting chain or the pin of a shackle or cable socket and must follow the health and safety guidelines.

The lifting of material generally occurs on construction sites, in factories and other industrial situations. Correct lifting can move large objects efficiently and reduce manual handling operations. Improper design of crane hook lead to disastrous accidents.

Optimization is a procedure through which the best possible values of decision variables are obtained under the given set of constraint functions and in accordance to objective function. The most common optimization procedure applies to a design that will minimize the total mass or any other specific objective.

There are a large number of papers and publications who dealing with the problems of optimization and analysis of stresses and deformations of crane hooks.

There are many methods for analysis and optimization for this type of structure. In the paper [1], optimization process was performed using by software for design of curved beam, for circular, rectangular, triangular, trapezoidal, T and I – sections. In [2], for weight optimization of lifting hook, GA algorithm was used in MATLAB software and analysis was done in ANSYS software. Authors in [3] studied Taguchi method that can be used for optimization of crane hook. The optimum combination of input parameters for minimum Von-Mises stresses are determined. Similarly to previous, in the paper [4], the Von-Mises stresses from FEM method are compared with Taguchi L9 orthogonal array for specific results.

Authors in the paper [5] discussed on stress analysis of crane hook and validation by photoelasticity. These results were compared with results from FEM analysis. Authors in [6] studied the stress pattern of crane hook in its loaded condition, and solid model of crane hook is prepared with ABAQUS software. The stress distribution pattern is verified for its correctness on an acrylic model of crane hook using by Caustic method, and the shape of the hook is modified to increase its working life and reduce the failure rates.

As can be seen, most of the authors applied FEM analysis for these type of structures. Most authors treat the problems of optimization and analysis using by FEM, in different software packages ([2]-[14]). In [7], designing of the hook is done through analytical method with different area of cross-sections (trapezoidal, triangular, rectangular and circular) and are analyzed for stresses and deformations through ANSYS software. Similarly to previous, for same cross-sections, in [8] using by CAE software, and the results obtained were compared with theoretical analysis. Stress and deformation analysis in ANSYS software was performed in the paper [9] for different cross-sections such as trapezoidal, rectangular and circular. Similar analysis was performed in [10], whereby the results from FEM analysis were compared with the analytical results. Modal and fatigue analysis was taken in consideration, too. Fatigue analysis is very important for these types of structures. In [11], structural and fatigue analysis using by ANSYS Workbench and ANSYS Ncode Designlife was performed for trapezoidal cross-section. The trapezoidal cross-section is most prevalent in relation to other cross-sections. In the paper [12], analysis of crane hook with different types of material was carried out for crane hook with trapezoidal cross-section.

In addition to the mentioned cross-sections, other cross-sections are applied, as shown in papers [13] and [14].

Thus, the aim of the work is to analyse and optimize the cross-sections of a crane hook using Winkler-Bach theory. Having in mind these results, the aim of this paper is to define the optimum values of geometric parameters of for crane hook, too.

II. Mathematical Formulation of the Optimization Problem

The optimization of crane hook cross-section is based on stresses, according to Winkler-Bach theory. The total deformations of the fibers in curved beams are proportional to the distances of the fibers from the neutral surfaces. The strains of the fibers are not proportional to these distances because the fibers are not of equal length, where as in straight beam the fibers are of equal length and fibers are of equal length and hence the strains in a straight beam, as well as total deformations are proportional to the distances of fibers from neutral axis. But for bending stresses that do not exceed the elastic strength of the material the stress on any fiber in the beam is proportional to the strain of the fiber, and hence the elastic stresses in the fibers of curved beam are not proportional to the distances from the neutral surface. For the same reason, neutral axis in a curved beam does not pass through the centroid of section.

The optimization problem is defined in following way:

$$\min f(X) \tag{1}$$

subject to:

$$g_l(X) \le 0 \tag{2}$$

and

$$X_j \ge 0 \tag{3}$$

where:

$f(X)$ - the objective (target) function

$g_l(X)$ - the constraint function

X - the design vector made of two design variables

j - number of design variables

The objective and constraint functions are presented in the next chapters.

Design variables are the values that should be defined during the optimization procedure.

Optimization procedure will be performed using by the method of Lagrange multiplier and by GRG2 algorithm.

The Lagrange function is defined in the following way:

$$\Phi = f(X) + \lambda \cdot g_l \tag{4}$$

where:

Φ - Lagrange function

λ - Lagrange multiplier

By using the Lagrange function, along with the elimination of parameter λ, system of equations for determination of optimum parameters of hook cross-section are obtained:

$$\frac{\partial f(X)}{\partial x_1} \cdot \frac{\partial g_l}{\partial x_2} = \frac{\partial f(X)}{\partial x_2} \cdot \frac{\partial g_l}{\partial x_1} \wedge g_l = 0 \tag{5}$$

GRG method is based upon elimination of variables by using constraints equality. The idea is to convert the constraints problem into one without constraints by direct substitution. Ms Excel Solver uses GRG2 algorithm for optimization of nonlinear problems.

Fig. 1 shows critical place (I-I) for cross-section stress check of crane hook.

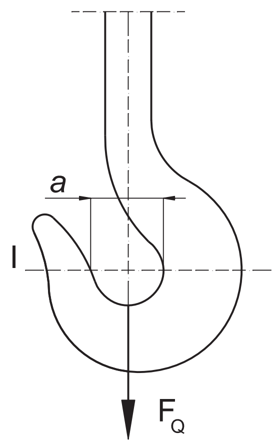

Fig. 1 Crane hook

The objective function is the cross-sectional area of crane hook (Fig. 2, Fig. 3 and Fig. 4):

$$f(X) = A(X) = A(x_1, x_2) \tag{6}$$

The vector of the given parameters is:

$$\vec{x} = (Q,\ a,\ \sigma_d) \tag{7}$$

where:

Q - the carrying capacity of crane hook

a - diameter of inner fiber of hook (inner geometric parameter of hook)

σ_d - critical stress

The constraint function, subject to stresses, in this case has the following form:

$$g_l = \sigma_1 = \frac{F_Q}{A} + \frac{M_{\max}}{S_x} \cdot \frac{h_1}{R_1} \le \sigma_d \tag{8}$$

or

$$g_l = |\sigma_2| = \frac{F_Q}{A} - \frac{M_{\max}}{S_x} \cdot \frac{h_2}{R_2} \le \sigma_d \tag{9}$$

where:

$$h_1 = r - R_1 \tag{10}$$

$$h_2 = R_2 - r \tag{11}$$

$$h = h_1 + h_2 \tag{12}$$

$$R_1 = \frac{a}{2} \tag{13}$$

$$R_2 = \frac{a}{2} + h \tag{14}$$

$$R_c = R_1 + e_1 \tag{15}$$

$$y_o = R_c - r \tag{16}$$

$$r = \frac{A}{\int_A \frac{dA}{\rho}} \tag{17}$$

$$F_Q = Q \cdot g \tag{18}$$

$$M_{\max} = F_Q \cdot R_c \tag{19}$$

$$S_x = A \cdot y_o \tag{20}$$

where:

R_1 - radius of inner fiber

R_2 - radius of outer fiber

R_c - radius of centroidal axis

r - radius of neutral axis

y_o - distance between centroidal axis and neutral axis

F_Q - axial force

$M_{\max}$ - maximum bending moment

S_x - static moment of area

A. Objective function and geometrical parameters for trapezoidal cross-section

The objective function is represented by the area of trapezoidal cross-section of crane hook (Fig. 2).

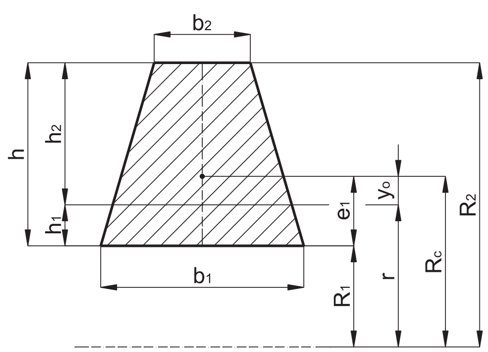

Fig. 2 Trapezoidal cross-section

Geometric relations (Fig. 2) are defined in following way:

$$e_o = \frac{b_2}{b_1} \tag{21}$$

$$A = b_1 \cdot \frac{1 + e_o}{2} \cdot h \tag{22}$$

$$e_1 = \frac{h}{3} \cdot \frac{1 + 2 \cdot e_o}{1 + e_o} \tag{23}$$

$$\int_A \frac{dA}{\rho} = b_1 \cdot \left[\left(e_o + \frac{a + 2 \cdot h}{2} \cdot \frac{1 - e_o}{h} \right) \cdot \ln \frac{a + 2 \cdot h}{a} - (1 - e_o) \right] \tag{24}$$

$$r = \frac{(1 + e_o) \cdot h}{2 \cdot \left[\left(e_o + \frac{a + 2 \cdot h}{2} \cdot \frac{1 - e_o}{h} \right) \cdot \ln \frac{a + 2 \cdot h}{a} - (1 - e_o) \right]} \tag{25}$$

For $e_o = 0$, trapezoidal cross-section becomes triangular cross-section:

$$A = \frac{b_1 \cdot h}{2} \tag{26}$$

$$e_1 = \frac{h}{3} \tag{27}$$

$$r = \frac{h}{2 \cdot \left(\frac{a + 2 \cdot h}{2 \cdot h} \cdot \ln \frac{a + 2 \cdot h}{a} - 1 \right)} \tag{28}$$

For $e_o = 1$, trapezoidal cross-section becomes rectangular cross-section:

$$A = b_1 \cdot h \tag{29}$$

$$e_1 = \frac{h}{2} \tag{30}$$

$$r = \frac{h}{\ln \frac{a + 2 \cdot h}{a}} \tag{31}$$

For $b_1 = h$, rectangular cross-section becomes square cross-section:

$$A = h^2 \tag{32}$$

B. Objective function and geometrical parameters for elliptic cross-section

The objective function is represented by the area of elliptic cross-section of crane hook (Fig. 3).

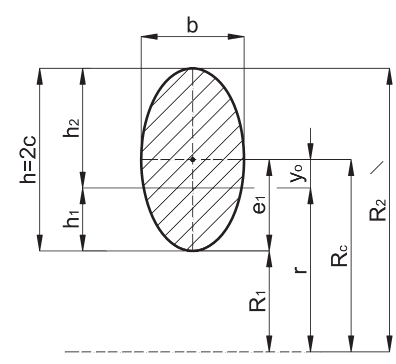

Fig. 3 Elliptic cross-section

Geometric relations (Fig. 3) are defined in following way:

$$h = 2 \cdot c \tag{33}$$

$$A = \frac{b \cdot c \cdot \pi}{2} \tag{34}$$

TABLE I THE VALUES OF OPTIMUM PARAMETERS FOR TRAPEZOIDAL CROSS-SECTION AND SAVINGS

e_o	Method of Lagrange multiplier				GRG2 method				As (cm^2)
	b_1 (cm)	h (cm)	A (cm^2)	Saving	b_1 (cm)	h (cm)	A (cm^2)	Saving	
0,0	6,081	30,21	91,847	16,43%	6,082	30,205	91,847	16,43%	109,9
0,1	6,056	27,862	92,808	15,55%	6,054	27,874	92,808	15,55%	109,9
0,2	6,006	26,352	94,97	13,59%	6,006	26,352	94,97	13,59%	109,9
0,3	5,948	25,278	97,732	11,07%	5,948	25,279	97,732	11,07%	109,9
0,4	5,887	24,46	100,803	8,28%	5,888	24,457	100,803	8,28%	109,9
0,5	5,826	23,806	104,028	5,34%	5,828	23,798	104,028	5,34%	109,9
0,6	5,766	23,264	107,312	2,35%	5,769	23,255	107,319	2,35%	109,9
0,7	5,708	22,802	110,623	-0,66%	5,709	22,797	110,623	-0,66%	109,9
0,8	5,65	22,401	113,907	-3,65%	5,651	22,397	113,907	-3,65%	109,9
0,9	5,593	22,047	117,151	-6,60%	5,595	22,039	117,151	-6,60%	109,9
1,0	5,538	21,729	120,34	-9,50%	5,54	21,722	120,34	-9,50%	109,9
1,25	5,404	21,058	128,058	-16,52%	5,407	21,049	128,031	-16,50%	109,9
1,5	5,277	20,511	135,283	-23,10%	5,277	20,509	135,283	-23,10%	109,9
2,0	5,036	19,656	148,472	-35,10%	5,037	19,65	148,472	-35,10%	109,9

$$e_1 = c \tag{35}$$

$$\int_A \frac{dA}{\rho} = \frac{b \cdot \pi}{c} \cdot \left(R_c - \sqrt{R_c^{\,2} - c^2} \right) \tag{36}$$

$$r = \frac{c^2}{2 \cdot \left(R_c - \sqrt{R_c^{\,2} - c^2} \right)} \tag{37}$$

For $b = 2 \cdot c$, elliptic cross-section becomes circular cross-section:

$$A = c^2 \cdot \pi \tag{38}$$

C. Objective function and geometrical parameters for parabolic cross-section

The objective function is represented by the area of parabolic cross-section of crane hook (Fig. 4).

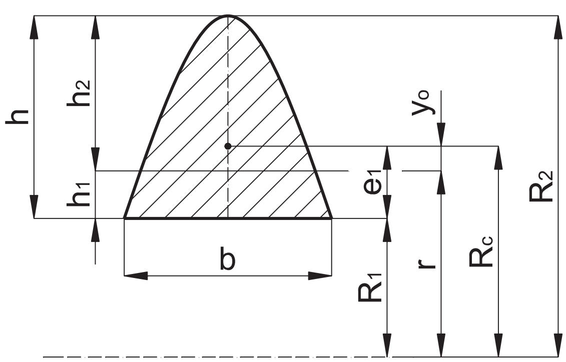

Fig. 4 Parabolic cross-section

Relation for parabolic function is:

$$y(x) = \frac{a}{2} + h - \frac{4 \cdot h}{b^2} \cdot x^2 \tag{39}$$

Geometric relations (Fig. 4) are defined in following way:

$$A = \frac{2 \cdot b \cdot h}{3} \tag{40}$$

$$e_1 = \frac{2 \cdot h}{5} \tag{41}$$

$$\int_A \frac{dA}{\rho} = \frac{b}{\sqrt{h}} \cdot \left(\sqrt{R_2} \cdot \ln \frac{2 \cdot \left(\sqrt{R_2} + \sqrt{h} \right)^2}{a} - 2 \cdot \sqrt{h} \right) \tag{42}$$

$$r = \frac{2 \cdot h \cdot \sqrt{h}}{3 \cdot \left(\sqrt{R_2} \cdot \ln \frac{2 \cdot \left(\sqrt{R_2} + \sqrt{h} \right)^2}{a} - 2 \cdot \sqrt{h} \right)} \tag{43}$$

III. NUMERICAL REPRESENTATION OF THE OBTAINED RESULTS

The optimization is done by the method of Lagrange multiplier in MathCad software package, using system of equations (5) and by GRG2 algorithm, using Solver Tool in Analysis module from Ms Excel software package.

A. Obtained results for trapezoidal cross-section

Variable parameters are height h and base width b_1 of crane hook cross-section (Fig. 2). Relation e_o is constant parameter, in first phase of optimization process.

Geometric parameter a was not taken into consideration, it was taken as the input parameter, according to the standard. This parameter can also be subject of optimization.

Input parameters is: $F_Q=100\ kN$, $a=12,5\ cm$ and $\sigma_d=8\ kN/cm^2$. The values of relation e_o is shown in Table I.

The cross-sectional area of the standard hook profile is: $A_s=109,9\ cm^2$.

Table I shows the results of the optimization (optimal values) for taken values of parameter e_o,

where:

A_s - value of the area of standard hook, [15]

A_o - optimal value of the area of hook (Table I, Table II)

The results obtained by both methods are almost identical. It is noticeable that the height h of the cross-section is a quite large.

It can be seen that the smallest area is for triangle cross-section, when $e_o=0$, and the largest for rectangular cross-section, when $e_o=1$, which means that the optimal cross-section in this case is triangular cross-section.

For optimization problem where parameter e_o is variable, this value is approximate to zero, which should have been expected. In this case, the following results were obtained: $A_o=91,849\ cm^2$, $h=29,884\ cm$, $b_1=6,14\ cm$, $e_o=0,00116$.

Also, it can be seen that the relation $e_o>1$ began unfavorable, because it comes to increase in cross-sectional area, so as to continue in this analysis will not be considered.

For square cross-section, when $h=b_1$, the following is obtained: $h=b_1=11,37\ cm$, $A_o=129,27\ cm^2$,
which is an unfavorable case, too, because the area of cross-section is a quite large.

It can be seen that in all cases is a quite large height h of cross-section (in comparison to standard value). For this reason, additional geometrical constraint has been introduced, so the results of optimization (using by GRG2 algorithm) are as follows, shown in Table II:

TABLE II OPTIMUM PARAMETERS AND SAVINGS FOR TRAPEZOIDAL CROSS-SECTION WITH GEOMETRICAL CONSTRAINT AND SAVINGS

e_o	**GRG2 method**			
	b_1 (cm)	h (cm)	A_o (cm^2)	*Saving*
0	14,451	14	101,16	7,95%
0,1	13,022	14	100,27	8,76%
0,2	12,072	14	101,40	7,73%
0,3	11,357	14	103,51	5,81%
0,4	10,829	14	106,13	3,43%
0,5	10,382	14	109,01	0,81%
0,6	10,003	14	112,03	-1,94%
0,7	9,674	14	115,12	-4,75%
0,8	9,382	14	118,21	-7,56%
0,9	9,12	14	121,29	-10,36%
1,0	8,881	14	124,34	-13,14%

Additional geometric constraint is the cross-section height $h_s=14\ cm$, according to standard [15].

For optimization problem where parameter e_o is variable, the following results are obtained: $A_o=100,23\ cm^2$, $h=14\ cm$, $b_1=13,24\ cm$, $e_o=0,08418$.

B. Obtained results for elliptic cross-section

Variable parameters are c and b of elliptic cross-section (Fig. 3).

The results obtained by both methods are identical: $A_o=168,75\ cm^2$, $b=11,459\ cm$, $c=9,375\ cm$.

A special case is circular cross-section, when $b = 2 \cdot c$, and the following is obtained: $A_o=170,61\ cm^2$, $c=7,369\ cm$.

C. Obtained results for parabolic cross-section

Variable parameters are b and h of parabolic cross-section (Fig. 4).

The results obtained by both methods are almost identical, as in the first case: $A_o=101,312\ cm^2$, $b=5,856\ cm$, $h=25,952\ cm$, for method of Lagrange multiplier and $A_o=101,312\ cm^2$, $b=5,856\ cm$, $h=25,949\ cm$ for GRG2 method.

It can be seen that the optimal value of area is smaller than value defined by standard, but it value is similar to the values of trapezoidal and triangular cross-section area, which are a a quite large.

By introducing additional geometrical constraint function, as in the first case, the following is obtained: $A=107,912\ cm^2$, $b=11,562\ cm$, $h=14\ cm$.

IV. CONCLUSIONS

The paper presented optimum dimensions of various types of cross-sections of crane hook (trapezoidal, triangular, rectangular, square, elliptic, circular and parabolic cross-sections), subject to stresses according to Winkler-Bach theory of cross-section on their critical place of structure, in the characteristic points, using the method of Lagrange multiplier and by GRG2 algorithm. The objective function was minimum cross-sectional area, whereby given constraint conditions were satisfied. Based on the optimization theory and the procedure for calculation of cross-section of crane hook, this paper combines the optimum design philosophy and the design of crane hook. The cross-sectional area is optimized using by system of equations in MathCad software package and making full use of the optimization functions in Ms Excel software package.

The optimization task – minimization of the cross-sectional area was successfully realized, which is seen in the comparison of the results obtained with standard values.

Justification of application of these methods resulted in significant savings in material, about 9 % .

Based on obtained results for trapezoidal cross-section, it can be seen that with increase in the value of ratio e_o the value of area increase, too (Table II), while for the lower values of ratio e_o it gets smaller value of area than the one according to standard value, while the most optimal value for area is about 9% lower than the value of standard one. Rectangular cross-section gives a slightly larger area, while this value is a quite large for square cross-section.

For triangular cross-section, under given conditions and for the observed example, the cross-sectional area is not with the lowest value, so that trapezoidal cross-section is the most optimal for concrete value obtained for optimal value of ratio e_o.

It can be concluded from obtained results that the circle and elliptic cross-sections are not suitable for shapes for these structure, because too large cross-sectional area is obtained.

For parabolic cross-section, by introducing geometrical constraint function, the value of area is a slightly smaller than the one defined by standard. This cross section gave a slightly larger value for area in comparison to trapezoidal cross-section, but it is again much more favorable in relation to the square, elliptic or circular cross-sections.

The general conclusion is that the most optimal shapes for this type of structures are triangular and trapezoidal cross-sections. Parabolic cross-section gives a slightly larger value for area, but obtained results are satisfactory. Geometrical constraint is particularly important in this analysis. A slightly larger values for areas are for the rectangular and square cross-section in comparison to parabolic cross-section. The most unfavorable shapes are circular and elliptical cross-sections.

For further studies and research should include other parameters of the hook geometry, which are important for reducing its mass. It is necessary to observe and analyze the conditions on the characteristic segments of crane hook. In addition to stress analysis, deformation as well as fatigue should be covered. Also, all other potential shapes of these structures, as well as materials, should be analyzed.

In addition to the analytical solutions, the results can be compared with those obtained using by some of FEM software packages.

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