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Multicriterion optimization of the box section of the main girder of the bridge crane

The paper considers the problem of optimization of the box section of the main girder of the bridge crane for the case of placing the rail above the web plate. The method of Lagrange multipliers was used as the methodology for determination of optimum dependencies of geometrical parameters of the box section. The criteria of strength, local stability of plates, lateral stability of the girder, dynamic stiffness, deflection and cost effectiveness were simultaneously applied as the constraint functions. The results were obtained in explicit form, which is very favourable for discussion of solutions. Verification of the obtained results of geometrical parameters was carried out through numerical examples.

Keywords: *box section ,bridge crane, optimization, Lagrange multipliers**,** strength**,** stability.*

1. INTRODUCTION

The main task in the process of designing the carrying structure of the bridge crane is determination of optimum dimensions of the main girder box section. The main girder is the most responsible part of the bridge crane and therefore it is necessary, during optimization, to affect the increase in its carrying capacity with simultaneous reduction of its mass. The mass of the main girder has the largest share in the total mass of the bridge crane, so it is very important to perform its optimization in order to reduce the total costs of manufacturing the whole carrying structure.

That is the reason why the selection of the optimum shape and geometrical parameters, which influence the reduction of mass and costs of manufacturing, is the subject of research of many authors regardless of whether they deal specifically with cranes or carrying structures in general [1-9]. Most authors set permissible stress or two constraint functions: permissible stress and permissible deflection as the constraint function.

2. MATEMATHICAL FORMULATION OF THE OPTIMIZATION PROBLEM

The task of optimization is to define geometrical parameters of the cross section of the girder as well as their mutual relations, which result in its minimum area. The area of the cross section primarily depends on: height and width of the girder, thickness of plates and their mutual relations.

The optimization problem defined in this way can be given the following general mathematical formulation:

$$\text{minimize } f(X), \tag{1}$$

$$\text{subject to: } g_j(\boldsymbol{X}) \le 0,\ j=1,...,m, \tag{2}$$

where: $f(\boldsymbol{X})$ the objective function; $g(\boldsymbol{X}) \le 0$ the constraint function, m number of constraints.

$\boldsymbol{X} = \{x_1,...,x_D\}^T$ represents the design vector made of D design variables.

In this paper optimization for the criterion of strength (3), lateral stability (4), local stability of plates (5), deflection (6) and dynamic stiffness (7) was performed:

$$g_1 = \sigma_{r1} - \sigma_{k1} \le 0. \tag{3}$$

$$g_2 = \sigma_{r2} - \sigma_{k2} \le 0. \tag{4}$$

$$g_3 = \sigma_{r3} - \sigma_{k3} \le 0. \tag{5}$$

$$g_{41} = f_v - f_{v,dop} \le 0,\ g_{42} = f_h - f_{h,dop} \le 0. \tag{6}$$

$$g_5 = T - T_d \le 0. \tag{7}$$

where: σ_{r1}, σ_{k1} - the calculation and permissible bending stresses of the girder, σ_{r2}, σ_{k2} - the calculation and permissible stresses in lateral buckling of the girder, σ_{r3}, σ_{k3} - the calculation and permissible stresses local stability of plates, f_v, f_h - deflection in vertical and horizontal plane, $f_{v,dop}, f_{h,dop}$ - permissible deflection in vertical and horizontal plane, T - period of oscillation, T_d - permissible period of oscillation.

By using the Lagrange function:

$$\Phi = A + \lambda_1 \cdot g_1 + \lambda_2 \cdot g_2 + \lambda_3 \cdot g_3 + \lambda_4 \cdot g_4 + \lambda_5 \cdot g_5. \tag{8}$$

along with the elimination of parameter λ, where condition $\lambda_i \ne 0,\ i = 1 \div 5$ is fulfilled, the equations for determination of optimum parameters of girder cross section are obtained:

$$\frac{\partial A}{\partial b} \cdot \frac{\partial g_1}{\partial h} = \frac{\partial A}{\partial h} \cdot \frac{\partial g_1}{\partial b} \ \wedge \ g_1 = 0. \tag{9}$$

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$$\frac{\partial A}{\partial b}\cdot\frac{\partial g_2}{\partial h}=\frac{\partial A}{\partial h}\cdot\frac{\partial g_2}{\partial b}\quad\wedge\quad g_2=0\,. \tag{10}$$

$$\frac{\partial A}{\partial b}\cdot\frac{\partial g_3}{\partial h}=\frac{\partial A}{\partial h}\cdot\frac{\partial g_3}{\partial b}\quad\wedge\quad g_3=0\,. \tag{11}$$

$$\frac{\partial A}{\partial b}\cdot\frac{\partial g_4}{\partial h}=\frac{\partial A}{\partial h}\cdot\frac{\partial g_4}{\partial b}\quad\wedge\quad g_4=0\,. \tag{12}$$

$$\frac{\partial A}{\partial b}\cdot\frac{\partial g_5}{\partial h}=\frac{\partial A}{\partial h}\cdot\frac{\partial g_5}{\partial b}\quad\wedge\quad g_5=0\,. \tag{13}$$

3. OBJECTIVE AND CONSTRAINT FUNCTIONS

3.1 Objective function

The objective function is represented by the area of the cross section of the box girder. The paper treats two optimization parameters *(h, b).* The wall thicknesses t_1 and t_2 (Fig. 1) are not treated as optimization parameters for the purpose of simplification of the procedure. Their values were adopted in accordance with the recommendations [8-12].

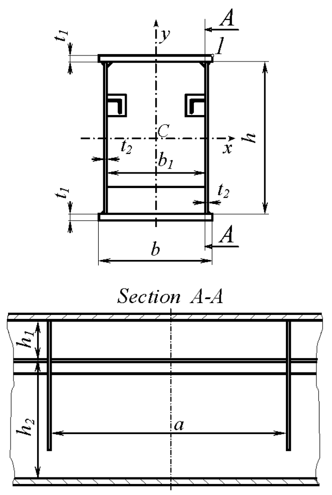

Figure 1. The box section of the main girder of the bridge crane

The area of the cross section, i.e. the objective function, is:

$$A(h,b)=f(h,b)=2\cdot(e\cdot b\cdot h+h^2)/s\,. \tag{14}$$

where: $e=t_1/t_2$, $e=t_1/t_2$, $s=h/t_2$, $k=h/b$ - the ratio between geometrical parameters of the plates (Fig. 1) [13].

3.2 Constraint functions

3.2.1 The criterion of strength

Maximum bending stress occurs at point 1 (Fig. 1). Constraint function according to this criterion is:

$$g_1=g_1(h,b)=\frac{M_{cv}+c\cdot A}{\alpha_x\cdot h\cdot A}+\frac{M_{ch}+k_a\cdot c\cdot A}{\alpha_y\cdot b\cdot A}-\sigma_{k1}\le 0\,. \tag{15}$$

where: M_{cv},M_{ch} bending moments in vertical and horizontal plane; σ_{k1} - critical stress; c - weight per unit volume coefficient ; α_x,α_y - the dimensionless coefficient of the resistance moment of inertia for corresponding axes [7-12]; k_a - coefficient of dynamic load in horizontal plane.

Partial derivatives (15) have following form:

$$\frac{\partial g_1}{\partial b}=-\left[\frac{M_{cv}}{\alpha_x\cdot h}\cdot\frac{1}{A^2}\cdot\frac{\partial A}{\partial b}+\frac{M_{ch}}{\alpha_y\cdot b}\cdot\frac{1}{A^2}\cdot\frac{\partial A}{\partial b}+\frac{M_{ch}}{\alpha_y}\cdot\frac{1}{A}\cdot\frac{1}{b^2}+\frac{k_a\cdot c}{\alpha_y}\cdot\frac{1}{b^2}\right];$$

$$\frac{\partial g_1}{\partial h}=-\left[\frac{M_{cv}}{\alpha_x\cdot h}\cdot\frac{1}{A^2}\cdot\frac{\partial A}{\partial h}+\frac{M_{cv}}{\alpha_x}\cdot\frac{1}{A}\cdot\frac{1}{h^2}+\frac{c}{\alpha_x}\cdot\frac{1}{h^2}+\frac{M_{ch}}{\alpha_y\cdot b}\cdot\frac{1}{A^2}\cdot\frac{\partial A}{\partial h}\right];$$

$$\frac{\partial A}{\partial b}=2\cdot e\cdot h/s;\quad \frac{\partial A}{\partial h}=2\cdot(e\cdot b+2\cdot h)/s; \tag{16}$$

By using the partial derivatives (16) and relation (9), with appropriate transformations, the optimum value of parameter k according to this criterion, is obtained:

$$k_1=\sqrt{\frac{e\cdot\alpha_y}{\alpha_x}\cdot\frac{M'_{cv}}{M'_{ch}}}\,. \tag{17}$$

By using the obtained dependences from the constraint function according to the criterion of strength, the objective function can be written in the following form:

$$A_1(h)\ge\frac{M_{cv}/\alpha_x+M_{ch}/\alpha_y\cdot k_1}{\sigma_{k1}\cdot h-c/\alpha_x-k_a\cdot c/\alpha_y\cdot k_1}\,. \tag{18}$$

3.2.2 The criterion of lateral stability

Testing of the box girder stability against lateral buckling was carried out in compliance with the Serbian standards of the group [14,15]. In accordance with these standards, the compression zone of the box girder is observed as an independent bar which is controlled against buckling due to the equivalent force arising from the bending moment of the girder.

This criterion is fulfilled if the condition of lateral stability is satisfied:

$$g_2=\sigma_{r2}-\sigma_{k2}=\frac{D}{A_p}\cdot\frac{1}{\chi}+0,9\cdot\frac{M_{ch1}}{W_y}-\sigma_{k2}\le 0\,. \tag{19}$$

where: χ - the buckling coefficient.

The buckling coefficient χ has values [14,15]: $\chi=1$, if the relative slenderness of the bar is $\bar{\lambda}\le 0,2$, i.e.:

$$\chi=\frac{2}{\beta+\sqrt{\beta^2-4\cdot\bar{\lambda}}}\,. \tag{20}$$

if the relative slenderness of the bar is $\bar{\lambda}>0,2$.

For the box cross section of the main girder of the crane, the coefficient β is:

$$\beta=1+0,489\cdot(\bar{\lambda}-0,2)+\bar{\lambda}^2\,. \tag{21}$$

Derived expressions are substituted in (19) and corresponding members of this relation get transformed into form:

$$\frac{D}{A_p}\cdot\frac{1}{\chi}=\frac{2\cdot\gamma_x}{\beta_x^{\,2}\cdot h\cdot A}\cdot\left(M_{cv}+c\cdot A\right)\cdot f(b)/\left(\beta_y\cdot b\right)^2 \tag{22}$$

Function (19) for σ_{r2} can be written in the following manner:

$$\sigma_{r2} = \sigma_{r21} + \sigma_{r22} . \quad (23)$$

The values of particular stresses in expression (23) are as follows:

$$\sigma_{r21} = K_1 \cdot \left(M_{cv} \cdot f_1 + c \cdot f_2 \right),$$

$$\sigma_{r22} = K_2 \cdot \left(M_{ch} \cdot f_3 + k_a \cdot c \cdot f_4 \right),$$

where: $K_1 = \dfrac{2 \cdot \gamma_x}{\beta_x^2 \cdot \beta_y^2}$; $f_1 = \dfrac{f(b)}{h \cdot A \cdot b^2}$; $f_2 = \dfrac{f(b)}{h \cdot b^2}$; $K_2 = 0,9/\alpha_y$; $f_3 = 1/(b \cdot A)$; $f_4 = 1/b$.

In order to apply the method of Lagrange multipliers, for the criterion of lateral stability, it is necessary to find the corresponding partial derivatives (10) with using the relation (24):

$$\frac{\partial g_3}{\partial b} = \frac{\partial \sigma_{r21}}{\partial b} + \frac{\partial \sigma_{r22}}{\partial b}, \quad \frac{\partial g_3}{\partial h} = \frac{\partial \sigma_{r21}}{\partial h} + \frac{\partial \sigma_{r22}}{\partial h} . \quad (24)$$

With substitution of derived expressions in eq. (24) with appropriate transformation, it is obtained:

$$\begin{aligned} \frac{0,9}{4} \cdot \frac{\beta_x^2}{\gamma_x} \cdot \frac{M_{ch}}{M_{cv}} &= \frac{1}{h^2} \cdot (e \cdot a \cdot m^2 + e \cdot g \cdot m \cdot \beta_y \cdot b) + \\ &+ \frac{1}{h^2} \cdot (e \cdot d \cdot \beta_y^2 \cdot b^2 - 2 \cdot a \cdot m^2 \cdot k - g \cdot m \cdot \beta_y \cdot h) \end{aligned}, \quad (25)$$

From this equation, the optimum value of parameter *k* according to the criterion of lateral stability, is obtained in relation to parameter *e*.

$$\frac{\left(k_2 + 3e\right)}{\left(2e + k_2\right)} + 0,442 \frac{1}{k_2} - 10,08 \frac{1}{k_2^2} - 0,442 = 0 \cdot \quad (26)$$

By using the obtained dependences from the constraint function according to the criterion of lateral stability, the objective function can be written in the following form:

$$A_2(h) \geq \frac{K_1 \cdot M_{cv} \cdot k_2^2 \cdot f(h) + K_2 \cdot M_{ch} \cdot k_2 \cdot h^2}{\sigma_{k_2} \cdot h^3 - K_1 \cdot c \cdot k_2^2 \cdot f(h) - K_2 \cdot k_a \cdot c \cdot k_2 \cdot h^2}, . (27)$$

3.2.3 Local stability of plates

Testing of the local stability of flange and vertical plate was carried out in accordance with the standards group [16]. It is necessary to check the stability of the flange plate with the width b_1 and the thickness t_1 (Fig. 1), the stability of the web plate above the longitudinal stiffener (length *a*, height h_1 and thickness t_2 – Fig. 1), as well as the stability of the web plate under the longitudinal stiffener (length *a*, height h_2 and thickness t_2 – Fig. 1).

The criterion of local stability of flange plate is fulfilled if the following condition is satisfied:

$$g_3 = \frac{M_{cv} + c \cdot A}{\alpha_x \cdot h \cdot A} + f \cdot \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b \cdot A} - \sigma_{k3} \leq 0 \cdot \quad (29)$$

where: σ_{k3} - critical stress.

A reduction factor according to Euro code recommendations is to be defined:

$$\kappa_x = c_e \cdot \left(\frac{1}{\lambda_x} - \frac{0,22}{\lambda_x^2} \right) \leq 1 \text{ for } \lambda_x > 0,673 , \quad (30)$$

$$\kappa_x = 1 \text{ for } \lambda_x \leq 0,673 .$$

For average values, coefficient $\psi_e = \sigma_2 / \sigma_1$ can be also written by eq. [17]:

$$\psi_p \approx 0,83 - 0,06 \cdot k , \quad (31)$$

so, the values of other involved parameters are defined by expressions:

$$K\sigma = \frac{8,2}{\psi_e + 1,05}; \; c_p \approx 1,15 + 0,0072 \cdot k ;$$

$$\lambda_{xp} \approx \frac{Ko}{\sqrt{K\sigma_p}} \cdot \frac{s \cdot f}{e \cdot k}; \; Ko = \frac{1}{\pi} \cdot \sqrt{\frac{12 \cdot (1 - \nu^2) \cdot f_y}{E}} ; \quad (32)$$

$$K\sigma_p = \frac{8,2}{1,88 - 0,06 \cdot k}; \; \sigma_e = \frac{\pi^2 \cdot E}{12 \cdot (1 - \nu^2)} \cdot \left(\frac{e \cdot k}{s \cdot f} \right)^2 .$$

Fig. 2 shows that for average and expected parameters values, the value of coefficient is $\kappa_x = 1$, wherein classification class is 2 and material is S235JRG2 .

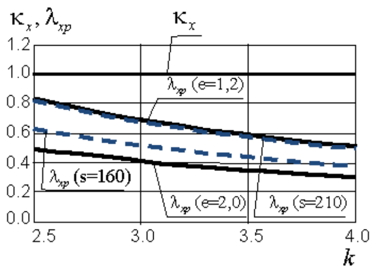

Figure 2. Change of the coefficients λ_{xp} and κ_x as the function of the parameter *k*

We can get the optimum value of the parameter *k* according to the criterion of local stability of the flange plate by determining corresponding partial derivatives (11) and with including relations (14) and (29) and some transformations:

$$k_3 = \sqrt{\frac{e \cdot \alpha_y}{f \cdot \alpha_x} \cdot \frac{c_1}{k_a}} . \quad (33)$$

By using the obtained dependences from the constraint function according to this criterion, the objective function can be written in the following form:

$$A_3(h) \geq \frac{M_{cv}/\alpha_x + f \cdot M_{ch}/\alpha_y \cdot k_3}{\sigma_{k3} \cdot h - c/\alpha_x - f \cdot k_a \cdot c/\alpha_y \cdot k_3} . \quad (34)$$

Testing of the local stability of the web plate in area 1 and area 2, whose dimensions are given in Fig. 1, was carried out. The case when, in addition to vertical stiffeners at midspan at section *I*, a row of horizontal stiffeners is also placed at the distance of $\left(0,25 \div 0,33\right) \cdot h$ was considered, according to crane manufacturers. The areas 1 and 2 are analyzed.

Area 1: The criterion is fulfilled if the following condition is satisfied:

$$\left(\frac{|\sigma_{Sd1,x}|}{f_{b,Rd1,x}}\right)^{e_{1x}} + \left(\frac{|\sigma_{Sd1,y}|}{f_{b,Rd1,y}}\right)^{e_{1y}} - (\kappa_{1x}\cdot\kappa_{1y})^{6}\cdot\left(\frac{|\sigma_{Sd1,x}\cdot\sigma_{Sd1,y}|}{f_{b,Rd1,x}\cdot f_{b,Rd1,y}}\right) \le 1, \quad (35)$$

where: $|\sigma_{Sd1,x}|$, $|\sigma_{Sd1,y}|$ - the highest value of the stresses in the corresponding directions; $f_{b,Rd1,x}$, $f_{b,Rd1,y}$ - the values of critical stresses; $e_{1x} = 1+\kappa_{1x}^4$, $e_{1y} = 1+\kappa_{1y}^4$ corresponding coefficients; κ_{1x}, κ_{1y} - corresponding reduction factors for area 1.

Analogous to the previous, for average values, coefficient ψ_p can be approximately defined as follows:

$$\psi_{1p} \approx 0,54 + 0,015\cdot k\,. \quad (36)$$

so, the values of other included parameters are defined by expresssions:

$$K\sigma_{1p} = \frac{8,2}{1,59+0,015\cdot k}\,;\; \lambda_{1xp} \approx \frac{Ko}{\sqrt{K\sigma_{1p}}}\cdot\frac{s}{4}\,; \quad (37)$$

Fig. 3 shows that for average and expected parameters values, the value of coefficient is $\kappa_{1x} = 1$, wherein classification class is 2 and material is S235JRG2 .

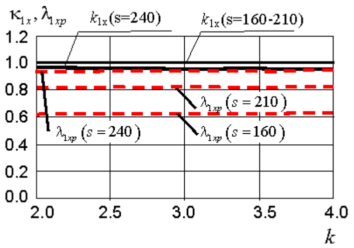

Figure 3. Change of the coefficients $\lambda_{1,xp}$ and κ_{1x} as the function of the parameter k

Further on, for loads in y direction, according to Euro code recommendations, a reduction factor is [16]:

$$\kappa_{1y} = 1,13\cdot\left(\frac{1}{\lambda_{1y}} - \frac{0,22}{\lambda_{1y}^{\;2}}\right) \le 1 \text{ for } \lambda_{1y} > 0,831, \quad (38)$$

$$\kappa_{1y} = 1 \text{ for } \lambda_{1y} \le 0,831.$$

The non-dimensional slenderness coefficient is:

$$\lambda_{1y} = \frac{Ko}{\sqrt{K\sigma_{1y}\cdot a/c_{1r}}}\cdot\frac{s}{4}\,. \quad (39)$$

where: $K\sigma_{1y}$ - buckling coefficient [16]; c_{1r} - the width over which the transverse load is distributed (corresponds to l_{1r}). Fig. 4 shows that for average and expected parameters values, the value of coefficient is $\kappa_{1y} = 1$, wherein classification class is 2 and material is S235JRG2 .

On the basis of the obtained values the relation (35) can be written:

$$\sqrt{|\sigma_{Sd1,x}|^2 + |\sigma_{Sd1,y}|^2 - |\sigma_{Sd1,x}\cdot\sigma_{Sd1,y}|} \le f_y/\gamma_m\,. \quad (40)$$

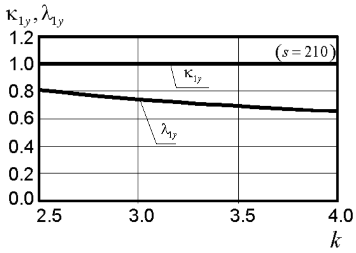

Figure 4. Change of the coefficients $\lambda_{1,y}$ and κ_{1y} as the function of the parameter k

If it is adopted that $|\sigma_{Sd1,x}|$ is critical stress, equation (40) is correct if it is fulfilled that $f_l(L) \ge f_d(L)$, which is true for real parameters values, Fig. 5 [9,16].

Similar procedure is applied for area 2 and by using the same assumptions it can be written:

$$\left(|\sigma_{Sd2,x}|/f_{b,Rd2,x}\right)^{e_{2x}} + \left(|\sigma_{Sd2,y}|/f_{b,Rd2,y}\right)^{e_{2y}} \le 1\,. \quad (41)$$

To do analysis, the ratio between the maximum stress in this area and the stress that occurred.

Equation (41) becomes:

$$(\psi_2/\kappa_{2x})^{1+\kappa_{2x}^4} + \left(|\sigma_{Sd2,y}|\cdot\gamma_m/(\kappa_{2y}\cdot f_y)\right)^{1+\kappa_{2y}^4} \le 1\,. \quad (42)$$

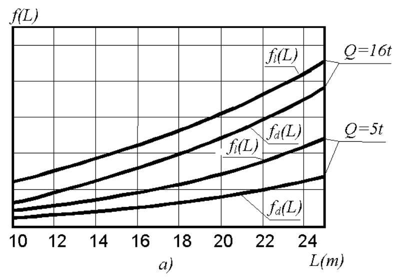

Figure 5. Testing of the stability of the flange plate for variable parameters

Fig. 6 shows that for average parameters values and with variation of loads and internal distance, this relation varies, where it was taken $s = 160$.

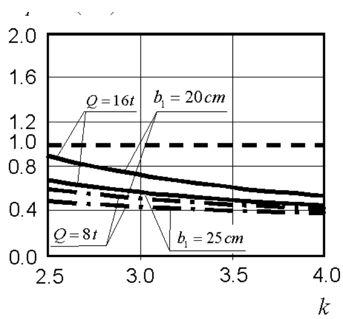

Figure 6. Change of the value of the function (42) depending on the carrying capacity Q and the width b_1

Based on all mentioned above, it is possible to adopt such parameters that make the local stability criterion fulfilled. The area 1 is critical one for the investigation of vertical plates local stability. The highest stress is analogous to flange plate stress. Since factors κ_{1x} i κ_{1y}

take value 1, it appears that the same curve or constraint can be taken into account for this criterion (g_3), as well as the optimal value for *k*.

According to Serbian standard [15], for flange plate local stability, we have:

$$\kappa_x = \frac{0,6}{\sqrt{\bar{\lambda}_p^{\,2} - 0,13}} \le 1 \,;\; c_\sigma = 1,25 - 0,25 \cdot \psi_e \,, \qquad (43)$$

$$c_\sigma \le 1,25 \text{ for } K\sigma_p \cdot \alpha_e^{\,2} > 2 \,.$$

In section *I* vertical stiffeners are placed at distance 2*h*, so it can be written:

$$K\sigma = \frac{8,4}{\psi_e + 1,1} \,;\; c_p \approx 1,042 + 0,015 \cdot k \,;$$

$$K\sigma_p \approx \frac{8,4}{1,93 - 0,06 \cdot k} > 4 \,; \qquad (44)$$

Fig. 7 shows that for average and expected parameters values, the value of coefficient is $\kappa_x = 1$, wherein classification class is 2 and material is S235JRG2. Incresing the slenderness *s* leads to lower values.

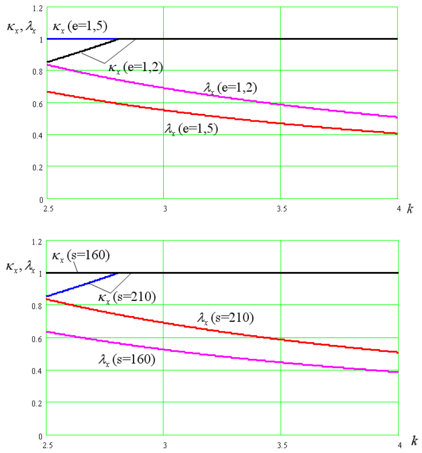

Figure 7. Change of the coefficients λ_x and κ_x as the function of the parameter k

The same relations (33) and (34) from previous cases are valid for determining the optimum value of parameter *k* as the objective function. In order to define optimum values of parameters for vertical plates stability in area 1, the following expressions are used:

$$\frac{|\sigma_1|}{\sigma_{kr1}} + \frac{\sigma_M}{\sigma_{Mkr1}} \le 0,9 \,;\; \sigma_y = \frac{\gamma \cdot F_1 \cdot h}{15,91 \cdot \gamma_y \cdot A} \,; \qquad (45)$$

$$g_{32,1} = \sigma_{zv1} + f \cdot \sigma_{zh1} + 3 \cdot \sigma_y - 0,6 \cdot \sigma_{kr1} \,, \qquad (46)$$

Appliance of the method of Lagrange multipliers leads to equation:

$$\frac{M_{cv} + c \cdot A}{\alpha_x \cdot h^2} \cdot e - \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b^2} \cdot f = \frac{3 \cdot \gamma \cdot F_1 \cdot e}{15,91 \cdot \gamma_y} \,. \qquad (47)$$

By solving the equation system (47) and using condition $g_{32,1} = 0$, the optimum ratio k (k_{p1}) is obtained. Cross section area function for area 1 is:

$$A_{p1}(h) \ge \frac{M_{cv}/\alpha_x + M_{ch} \cdot f/\alpha_y \cdot k_{p1} + \dfrac{3 \cdot \gamma \cdot F_1 \cdot h^2}{15,91 \cdot \gamma_y}}{0,6 \cdot \sigma_{kr1} \cdot h - c/\alpha_x - k_a \cdot c \cdot f/\alpha_y \cdot k_{p1}} \,. \qquad (48)$$

For area 2, in a similar way, starting from expression

$$\frac{|\sigma_3|}{\sigma_{kr2}} + \frac{0,4 \cdot \sigma_M}{\sigma_{Mkr2}} \le 1 \,; \qquad (49)$$

$$g_{32,2} = 3 \cdot \sigma_{zv1} + 6 \cdot f \cdot \sigma_{zh1} + 18,3 \cdot \sigma_y - 4 \cdot \sigma_{kr2} \,., \qquad (50)$$

and by appliance of the method of Lagrange multipliers it is derived:

$$3 \cdot \frac{M_{cv} + c \cdot A}{\alpha_x \cdot h^2} \cdot e - 6 \cdot \frac{M_{ch} + k_a \cdot c \cdot A}{\alpha_y \cdot b^2} \cdot f = \frac{18, \cdot \gamma \cdot F_1 \cdot e}{15,91 \cdot \gamma_y} \,, \qquad (51)$$

By solving the equation system (51) and using condition $g_{32,2} = 0$, the optimum ratio k (k_{p2}) is obtained.

Cross section area function for area 2 is:

$$A_{p2}(h) \ge \frac{3 \cdot M_{cv}/\alpha_x + 6 \cdot M_{ch} \cdot f/\alpha_y \cdot k_{p2} + \dfrac{18,3 \cdot \gamma \cdot F_1 \cdot h^2}{15,91 \cdot \gamma_y}}{4 \cdot \sigma_{kr1} \cdot h - 3 \cdot c/\alpha_x - 6 \cdot k_a \cdot c \cdot f/\alpha_y \cdot k_{p2}} \,, \qquad (52)$$

3.2.4 Deflection criterion

In order to satisfy this criterion, it is necessary that the deflections in the corresponding plane have the values smaller than the permissible ones. The maximum values of deflection must be within the following limits: the deflection in the vertical plane

$$f_v = \frac{F_1 \cdot L^3}{48 \cdot E \cdot I_x} \cdot \left[1 + w \cdot (1 - 6 \cdot p^2)\right] \le f_{v,dop} = K_v \cdot L \,. \qquad (53)$$

the deflection in the horizontal plane

$$f_h = \frac{k_a \cdot F_{1h} \cdot L^3}{48 \cdot E \cdot I_y} \cdot \left[1 + w \cdot (1 - 6 \cdot p^2)\right] \le f_{h,dop} = K_h \cdot L \,. \qquad (54)$$

where: $w = F_2 / F_1 \le 1$; $p = d/L$; d - the distance between the wheels of the trolley.

The constraint functions in this case have the following form:

$$g_4 = \frac{g_{42}}{g_{41}} = \frac{k_a \cdot F_{1h} \cdot I_x}{F_1 \cdot I_y} - \frac{K_h}{K_v} \le 0 \,. \qquad (55)$$

In order to apply the method of Lagrange multipliers, for the criterion of girder deflection, it is necessary to find the corresponding partial derivatives, in accordance with the expression (12):

$$\frac{\partial g_4}{\partial b} = -2 \cdot \frac{k_a \cdot F_{1h} \cdot \beta_x^{\,2} \cdot h^2}{F_1 \cdot \beta_y^{\,2} \cdot b^3} \,,\; \frac{\partial g_4}{\partial h} = \frac{2 \cdot k_a \cdot F_{1h} \cdot \beta_x^{\,2} \cdot h}{F_1 \cdot \beta_y^{\,2} \cdot b^2} \,. \qquad (56)$$

By replacing the expression (56) in (12), dividing by the member $\partial A / \partial h$, with the corresponding transformations, it is obtained that $e \cdot b = -h$.

By using the obtained dependences from the constraint function according to the criterion of deflection, the objective function can be written in the following form:

$$A_f = A(h) \geq \frac{F_{1h} \cdot L^2 \cdot \left[1 + w \cdot (1 - 6 \cdot p^2)\right]}{48 \cdot E \cdot \beta_x^{\ 2} \cdot K_f \cdot h^2}. \qquad (57)$$

3.2.5 Dynamic stiffness

In order to determine the optimum ratio of optimization parameters according to the criterion of dynamic stiffness, it is necessary to analyze oscillation of the main girder in the vertical plane. The analysis procedure was performed in compliance with the [18].

The mass m_1 is determined according to the expression [18]:

$$m_1 = 0,5 \cdot (Q + m_0) + 0,486 \cdot m_m. \qquad (58)$$

where: $0,486 \cdot m_m$ the reduced continual mass of the girder at midspan for the assumed function of displacement of the elastic line of the adopted discrete dynamic model for the simple girder.

The time of damping of oscillation is determined from the expression [18]:

$$T = 3 \cdot \tau / \gamma_d \leq T_d. \qquad (59)$$

where: τ the period of oscillation *(s)*, γ_d the logarithmic decrement which shows the rate of damping of oscillation; T_d the permissible time of damping of oscillation (s), which depends on the purpose of the crane. If the denotation:

$$C_d = \frac{1,0}{48 \cdot E \cdot \beta_x^{\ 2}}. \qquad (60)$$

the constraint function for the criterion of dynamic stiffness is:

$$g_5(h,b) = \frac{6 \cdot \pi}{\gamma_d} \cdot \sqrt{\frac{C_d \cdot M \cdot L^3}{h^2 \cdot A} + \frac{C_d \cdot r \cdot L^4}{h^2}} - T_d \leq 0. \qquad (61)$$

In order to apply the method of Lagrange multipliers for the criterion of dynamic stiffness, it is necessary to find the corresponding partial derivatives, in accordance with the expression (13):

$$\frac{\partial g_5}{\partial h} = \frac{6 \cdot \pi}{\gamma_d} \cdot \frac{1}{\sqrt{\delta_{11} \cdot m_1}} \cdot \frac{C_d \cdot M \cdot L^3}{h^2} \cdot \left(-\frac{1}{A^2}\right) \cdot \frac{\partial A}{\partial h} -$$

$$- \frac{6 \cdot \pi}{\gamma_d} \cdot \frac{1}{\sqrt{\delta_{11} \cdot m_1}} \cdot \frac{C_d \cdot M \cdot L^3}{h^2} \cdot \left[\frac{2 \cdot C_d \cdot M \cdot L^3}{h^3 \cdot A} + \frac{2 \cdot C_d \cdot r \cdot L^4}{h^3}\right];$$

$$\frac{\partial g_5}{\partial b} = \frac{6 \cdot \pi}{\gamma_d} \cdot \frac{1}{\sqrt{\delta_{11} \cdot m_1}} \cdot \frac{C_d \cdot M \cdot L^3}{h^2} \cdot \left(-\frac{1}{A^2}\right) \cdot \frac{\partial A}{\partial b};. \qquad (62)$$

By replacing the expression (63) in (13), after rearrangement, it is obtained that:

$$\frac{e + k}{k} - \frac{(1 - 2 \cdot t_2 \cdot e)}{2 \cdot [G(m) - 1]} = 0, \qquad (63)$$

where: $G(m) = \dfrac{0,486 \cdot m_m}{0,5 \cdot (m_Q + m_k)}$.

By using the obtained dependences from the constraint function according to the criterion of dynamic stiffness, the objective function can be written in the following form:

$$A_5 = A(h) \geq \frac{C_d \cdot M \cdot L^3}{\left(T_d \cdot \gamma_d / 6\pi\right)^2 \cdot h^2 - C_d \cdot r \cdot L^4}. \qquad (64)$$

4. NUMERICAL REPRESENTATION OF THE RESULTS OBTAINED

Using the expressions (17), (26) and (33) the optimum value of the parameter k is obtained, according to the considered criteria of strength, lateral and local stability of plates. Optimum values of the parameter *k* as a function of the member *e* are presented in Table 1.

Table 2 shows charasteristic results according to relations (47) and (51). The optimum values for k are greater when span increases, but are smaller when increasing payload.

Table 3 shows characteristic results for dynamic stiffness criterion.

Table 1

e	1,2	1,3	1,4	1,5
k_1	3,90	4,00	4,10	4,25
k_2	3,35	3,33	3,31	3,30
k_3	4,20	4,30	4,40	4,55

Table 2

Q	$L = 18m$		$L = 12m$	
	k_{p1}	k_{p2}	k_{p1}	k_{p2}
5	2,73	1,04	2,39	0,81
8	1,99	0,62	1,55	0,47
10	1,58	0,48	1,17	0,36
16	0,85	0,28	0,62	0,21

Table 3

k=2.5					
e	1.20	1.40	1.60	1.80	2.00
G(m)	0.53	0.43	0.33	0.24	0.17
k=3.5					
e	1.20	1.40	1.60	1.80	2.00
G(m)	0.48	0.35	0.25	0.14	0.05
k=4.5					
e	1.20	1.40	1.60	1.80	2.00
G(m)	0.44	0.31	0.19	0.07	-0.04

Expressions (18), (27), (34), (48), (52), (57) and (64) are objective functions derived from constraint functions according to considered criteria and along with objective function (14) can be presented graphically. Expressions (18), (27), (34), (48), (52), (57) and (64) are presented by dashed line, while expression (14) for optimum values of *k* for considered criteria - by continuous line (Fig. 8 and Fig. 9). It can be noticed that the position of the intersection point changes according to considered criteria taken from Euro code (Fig. 8), i.e., national standard (Fig. 9), where adopted values are $L=18\ m$ and $Q=8\ t$.

In order to perform a comparative analysis of optimization results, it is necessary to define the initial parameters of cranes. To start the analysis, the middle range values can be adopted, so for S235JRG2 it is $s=210$, while for S355JR it is $s=170$. Other parameters values, at this phase, are: $e=1,33$; $f=0,85$; $\psi=1,15$; $k_a=0,1$; $e_k=2,3m$; $G_k=15kN$.

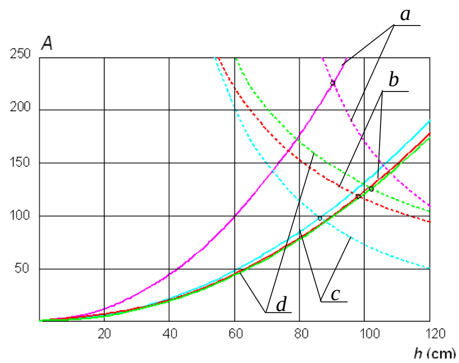

Figure 8. Optimum values of the girder height and the cross section area according to the criterion of: a) dynamic stiffness, b) strength, c) deflection, d) local stability according to Euro code

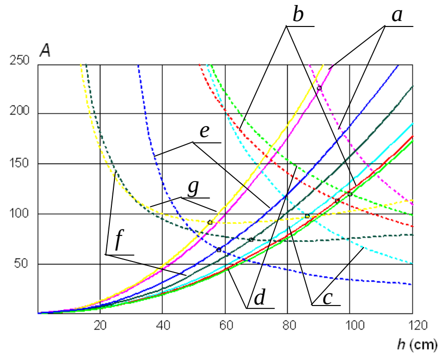

Figure 9. Optimum values of the girder height and the cross section area according to the criterion of: a) dynamic stiffness, b) strength, c) deflection, d) local stability of flange plate according to JUS, e) lateral stability, f) local stability of vertical plate in area 1, g) local stability of vertical plate in area 2

Analysis was carried out for classification class 2, which is most frequent in real life, according to Serbian standard for cranes [14]. For this class of cranes, the following values are appropriate: $\gamma=1,05,\ \alpha=1,20,\ K=0,08,\ m_o=1,20$.

Analysis was carried out for steel S235JRG2. In order to do the analysis, it is necessary to consider the recomendations cited in standards, as well as those from crane manufacturers [19,20]. Serbian crane industry recommends that minimum value for b_1 is $b_1>30\,cm$, while foreign crane manufacturers give $b_1>20\,cm$, wherefrom we get the following expressions:

$$k \le f \cdot h/30, \qquad (65)$$

$$k \le f \cdot h/20. \qquad (66)$$

Stability condition of upper flange plate, along with appropriate transformations, is defined as follows:

$$k \ge \frac{s \cdot f}{65 \cdot e} \cdot \sqrt{\frac{23.5}{R_{ei}}}. \qquad (67)$$

If the expression (14) is made equal with (18), (27), (34), (48), (52), (57) and (64), the dependences of the parameter k according to the considered criteria are obtained. But, it must be underlined that dependencies for k_1, k_3, k_{p1}, k_{p2} are presented for this criterion for both, Euro code and Serbian standard.

The foolowing figures show optimum geometric parameters' values that are obtained for characteristic payloads and spans of double-girder bridge cranes.

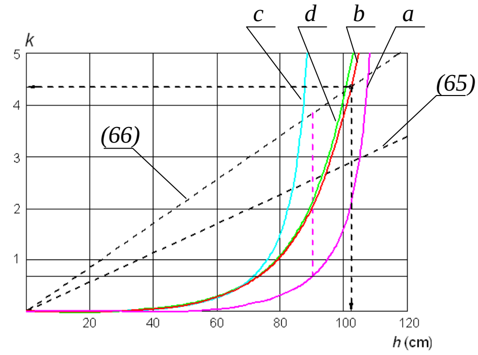

Figure 10. Multicriteria determination of the optimum value of the parameter k according to criterion of: a) dynamic stiffness, b) strength, c) deflection, d) local stability according to Euro code

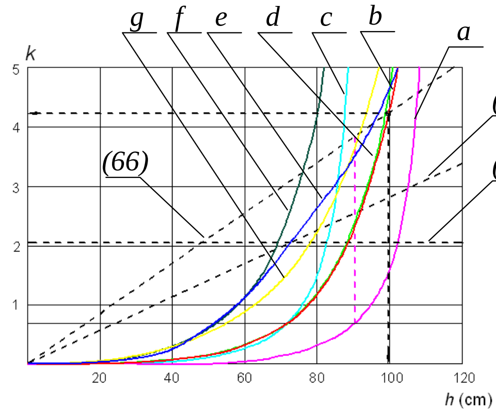

Figure 11. Multicriteria determination of the optimum value of the parameter k according to criterion of: a) dynamic stiffness, b) strength, c) deflection, d) local stability of flange plate according to JUS, e) lateral stability, f) local stability of vertical plate in area 1, g) local stability of vertical plate in area 2

It can be noticed that the curves defined by Euro code and national standard are very close. Also, the influence of manufacturing technology on optimum result can be evaluated (intersection points of dashed and continuous lines).

5. CONCLUSION

The paper defined optimum dimensions of the box section of the main girder of the bridge crane in an analytical form, by using the method of Lagrange multipliers, according to criterion of strength, lateral stability, local stability of plates, deflection and dynamic stiffness.

It was shown that the proper selection of girder height and plate thickness can considerably influence the reduction in the cross sectional area at the same time satisfying all constraint functions. It was also shown that it is possible to carry out successful optimization by the Lagrange procedure with the simultaneous use of multiple constraint functions. The results were obtained in explicit form, which is very favourable for discussion of solutions as well as for consideration of influences of individual geometrical parameters and their ratios. Comparison of the obtained results with certain solutions of bridge cranes shows that the obtained cross sectional areas are smaller, which verifies the optimization results. The comparative analysis of the optimization results represents the basis for recommendations that are important for designers during the construction of the main girder of the bridge crane.

In addition, the usage of the method of Lagrange multipliers is justified because the optimization results are obtained in analytical form, which allows getting conclusions about influences of particular parameters and further researches toward mass reduction.

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