

Research of deflection - payload dependence of the auto crane articulated boom

Nebojša Zdravković
Teaching Assistant
University of Kragujevac
Faculty of Mechanical Engineering Kraljevo

Zoran Petković
Professor
University of Belgrade
Faculty of Mechanical Engineering

Milomir Gašić
Professor
University of Kragujevac
Faculty of Mechanical Engineering Kraljevo

Mile Savković
Associate Professor
University of Kragujevac
Faculty of Mechanical Engineering Kraljevo

One of requirements to be fulfilled by supporting structure is condition of deformation. Usually, it is set through displacement limits of certain points on deformed structure. The paper presents a method for determining the deflection of auto crane articulated boom due its payload on the example of three-segment articulated structure. The forces in joints were obtained in the segments' local coordinate systems. The equations of bending moments for each segment were defined, with respect to angles of segment inclination. Based on strain energy and Castigliano's second theorem, component displacements of the boom's tip were calculated in the global coordinate system XYZ. Finite element method (FEM) revealed that deviations of the analytical model are less than 5% for different boom shapes within work range.

Keywords: *articulated boom, payload, deflection, displacement, strain energy, FEM.*

1. INTRODUCTION

One in a series of requirements to be fulfilled by supporting structure, in addition to condition of material strength, elastic stability, connections safety, etc., is condition of deformation. This constraint restricts displacement of the characteristic points of the structure due to the effect of working loads.

Insufficient rigidity of structure, during operation, may cause excessive deformation occurrence and unwanted additional static and dynamic loads. This can cause unreliable operation and short service life of machines as a result. Thus, limiting the value of static deflection of the structure creates a prerequisite for achieving good exploitation properties.

For certain type of crane supporting structures, the allowed deflections are prescribed by standards. Thus, for example, allowed static displacement at mid-span of bridge crane main girder is $\frac{L}{600}$, where L - span of crane [1]. In the case of gantry crane, allowed static displacement at endpoint of overhang is $(1/400 \div 1/200)L_1$, where L_1 is overhang length, while at mid-span point it is $(1/1000 \div 1/600)L$ [2].

Calculating the deflection of the supporting structures of these types of cranes is easy thanks to their simplicity and unchangeable geometry. However, it is not always the case.

Auto-cranes articulated booms are typical representatives of carrying structures with variable geometry, in which the geometry of the structure changes depending on the working position [3].

Correspondence to: Nebojsa Zdravkovic, Teaching Assistant
Faculty of Mechanical Engineering Kraljevo
Dositejeva 19, 36000 Kraljevo, Serbia
E-mail: zdravkovic.n@mfkv.kg.ac.rs

The main loads, structural elements' self-weight and the action of the payload, remain constant in direction and intensity, while the geometry of the articulated boom varies according to the current horizontal reach and lifting height. This fact leads to variability in the load for all members of the mechanism and thus to the variable displacement of the characteristic points, depending on the boom's position.

Constraint imposed by static deformations may be dominant in the case of machine structures [4]. Analytical dependencies enable designer to use limited endpoint displacement ($f < f_{allowed}$) as a constraint function for optimization in order to reduce mass of the structure [4-8].

The paper presents a method for determining the deflection of auto crane articulated boom on the example of three-segment articulated boom driven by three hydraulic cylinders (Fig. 1).

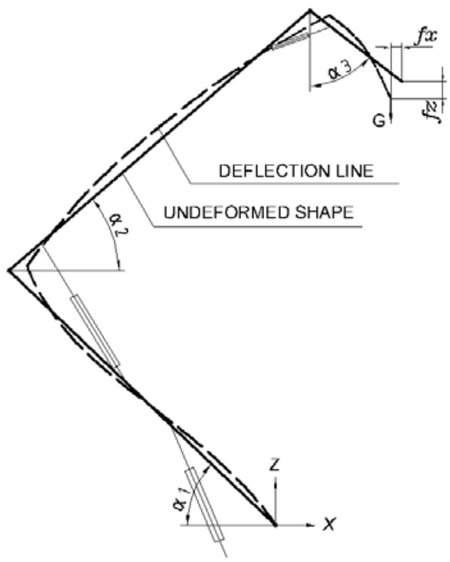

Figure 1. Three-segment articulated boom driven by hydro cylinders

2. ANALYTICAL MODEL FOR CALCULATION OF DEFLECTION

In order to carry out the procedure of obtaining deflection-payload dependence an analytical model of three-segment articulated structure is set in Fig. 2.

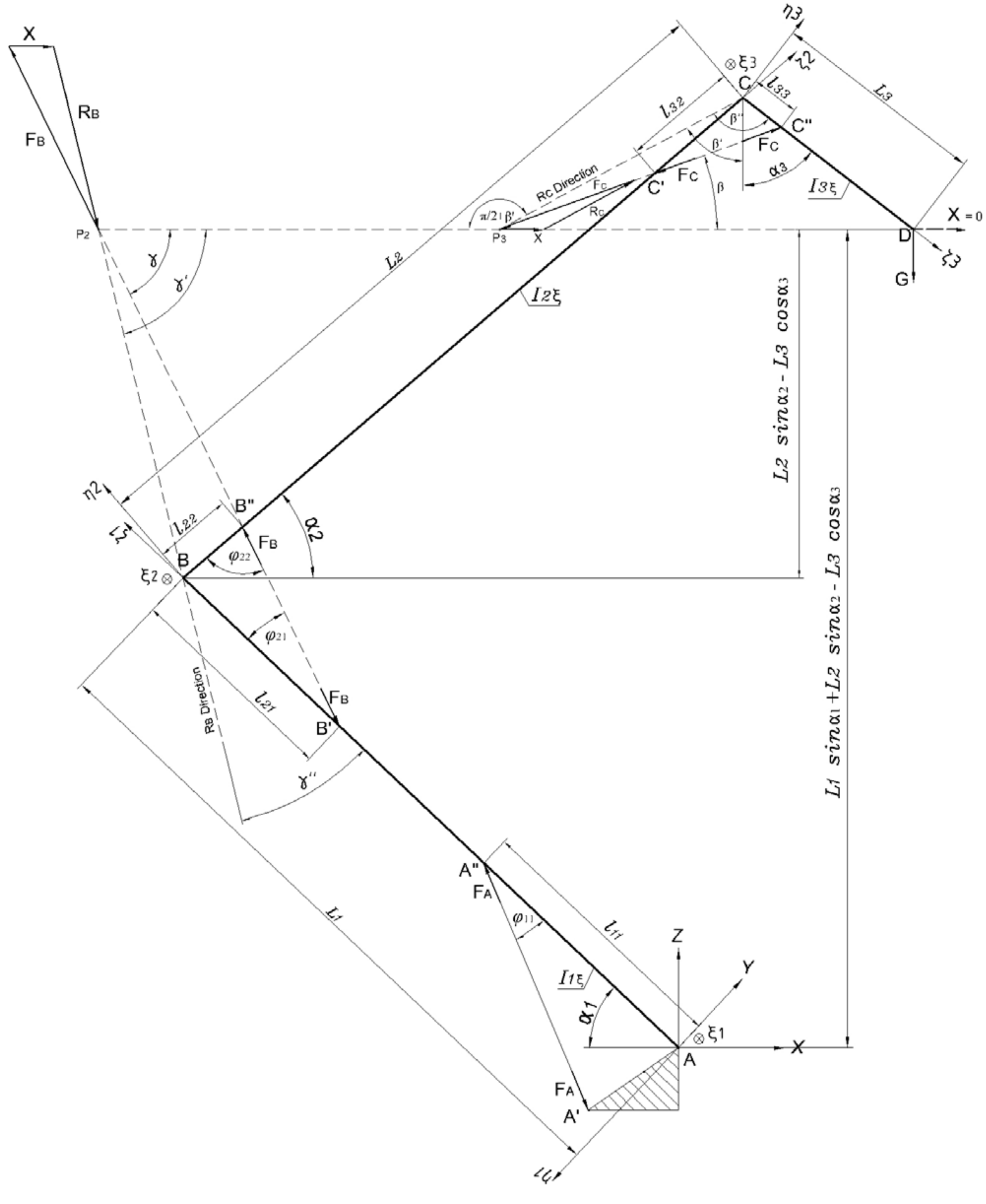

Figure 2. Analytical model of three-segment articulated structure used for determination of deflection-payload dependence using strain energy method

It is assumed that, principle of superposition applies for individual influences of payload *G* and virtual force *X=0*, which act on boom's tip. Also, when calculating strain energy, the influence of bending moments is being taken into account only, while other contributions are neglected.

In addition, the influence of the substructure's elasticity is ignored because its' stiffness is much greater than stiffness of the boom itself. Firstly, there was a static analysis of the structure and the loads were determined for each segment individually, as functions of inclination angles of segments α_1, α_2 and α_3, lengths and external load. The loads were obtained for the segments' local coordinate systems $\xi_i\eta_i\zeta_i$, $i=1,2,3$. It made possible to obtain the equations of bending moments for each segment, with respect to these angles. Also, based on strain energy and Castigliano's second theorem, component displacements of the boom's tip were calculated in the global coordinate system XYZ.

The payload *G* causes deflection of the structure wherein the boom's tip has displacements in X and Z directions, f_x and f_z (Fig. 1). In order to find f_x displacement due to payload influence, a virtual external force *X* is put at the boom's tip. The corresponding displacements are:

$$f_z = \frac{\partial A_d}{\partial G} = \frac{1}{E}\sum_{i=1}^{m}\frac{1}{I_i}\int_0^{l_i} M_i(s)\frac{\partial M_i(s)}{\partial G}ds\,. \tag{1}$$

$$f_x = \left.\frac{\partial A_d}{\partial X}\right|_{X=0} = \left.\frac{1}{E}\sum_{i=1}^{m}\frac{1}{I_i}\int_0^{l_i} M_i(s)\frac{\partial M_i(s)}{\partial X}ds\right|_{X=0}. \tag{2}$$

where: A_d - strain energy, E - modulus of elasticity, *m* - number of sections (subdivisions), $M_i(s)$ - bending moment of section *i*, *s* - floating section coordinate. The first action taken in order to define expressions for bending moments and corresponding partial derivatives for each section is to find analytical dependencies of joint reactions with the respect to inclination angles and for two separate load cases: case 1 - payload *G* acts only, case 2 - virtual force *X* acts only.

2.1 Determination of joint reactions due to external loads action

Joint reactions and forces in hydro cylinders when payload is acting were calculated already [9], so the task is now to resolve the case when virtual force *X* is acting only. To be more precise, it is necessary to derive the analytical expressions only for transverse components of joints' reactions which cause bending and deflection of the articulated structure. This components, according to Fig. 2, are projections of joint reactions onto the local axes η_i, $i=1,2,3$.

Firstly, segment 3 is analyzed. Out of moment equation set for point C, it is obtained:

$$F_{C\eta 3} = -F_C\sin\varphi_{33} = -\frac{X}{l_{33}}L_3\cos\alpha_3\,. \tag{3}$$

Applying second condition of static equilibrium for segment 3, it is:

$$R_{C\eta 3} = \frac{X}{l_{33}}\left(L_3 - l_{33}\right)\cos\alpha_3\,. \tag{4}$$

Out of moment equation for segment 2 set for point B, it is derived:

$$F_{B\eta 2} = F_B\sin\varphi_{22} = \frac{X}{l_{22}}\left(L_2\sin\alpha_2 - L_3\cos\alpha_3\right). \tag{5}$$

When calculating the transverse component of force in hydro cylinder F_C on segment 2, it is taken with opposite direction. Regarding (3) and applying sine theorem onto triangle CC'C" it is obtained:

$$F_{C\eta 2} = -F_C\sin\varphi_{32} = -X\frac{L_3}{l_{32}}\cos\alpha_3\,. \tag{6}$$

According to Fig. 2 it is written:

$$R_{C\eta 2} = R_C\cos\left[\pi + (\alpha_3 - \alpha_2) - \beta''\right]. \tag{7}$$

Having in mind relation

$$\beta'' = \beta' + \alpha_3\,. \tag{8}$$

(7) is transformed to the following form:

$$R_{C\eta 2} = R_C(\sin\beta'\sin\alpha_2 - \cos\beta'\cos\alpha_2)\,. \tag{9}$$

Expressions for sinβ' and cosβ' are derived from force graph of F_C, R_C and *X* by means of sine and cosine theorem:

$$\cos\beta' = \frac{F_C}{R_C}\sin\beta\,,\ \sin\beta' = \frac{F_C\cos\beta - X}{R_C}\,. \tag{10}$$

Using dependencies (10) in (9) it is obtained:

$$R_{C\eta 2} = -X\sin\alpha_2 - F_C\sin(\beta - \alpha_2)\,. \tag{11}$$

Considering following relation from triangle CC'C"

$$\beta = \alpha_3 + \varphi_{33} - \frac{\pi}{2}\,. \tag{12}$$

and applying some transformations, finally it is derived:

$$R_{C\eta 2} = \frac{X}{l_{32}}\left(L_3\cos\alpha_3 - l_{32}\sin\alpha_2\right). \tag{13}$$

Applying the condition of static equilibrium for segment 2, it is calculated:

$$R_{B\eta 2} = \frac{X}{l_{22}}\left(l_{22}\sin\alpha_2 - L_2\sin\alpha_2 + L_3\cos\alpha_3\right). \tag{14}$$

Similar procedure is carried out for segment 1. Out of moment equation set for point A, it is obtained:

$$F_{A\eta 1} = \frac{X}{l_{11}}\left(L_1\sin\alpha_1 + L_2\sin\alpha_2 - L_3\cos\alpha_3\right). \tag{15}$$

When calculating the transverse component of force in hydro cylinder F_B on segment 1, it is taken with opposite direction. Regarding (5) and applying sine theorem onto triangle BB'B" it is obtained:

$$F_{B\eta 1} = F_B \sin\varphi_{21} = \frac{X}{l_{21}}\left(L_2 \sin\alpha_2 - L_3 \cos\alpha_3\right). \quad (16)$$

According to Fig. 2 it is written:

$$R_{B\eta 1} = R_B \cos(\frac{3\pi}{2} - \gamma'') . \quad (17)$$

Having in mind relation

$$\gamma'' = \gamma' - \alpha_1 . \quad (18)$$

(17) is transformed to the following form:

$$R_{B\eta 1} = R_B (\sin\alpha_1 \cos\gamma' - \cos\alpha_1 \sin\gamma') . \quad (19)$$

Expressions for sinγ' and cosγ' are derived from force graph of F_B, R_B and X by means of sine and cosine theorem:

$$\cos\gamma' = \frac{F_B \cos\gamma - X}{R_B}, \ \sin\gamma' = \frac{F_B}{R_B}\sin\gamma . \quad (20)$$

Using dependencies (20) in (19) it is obtained:

$$R_{B\eta 1} = -X \sin\alpha_1 - F_B \sin(\gamma - \alpha_1) . \quad (21)$$

Considering following relation

$$\alpha_2 + \varphi_{22} + \gamma = \pi . \quad (22)$$

and applying some transformations, finally it is derived:

$$R_{B\eta 1} = \frac{X}{l_{21}}\left(-l_{21} \sin\alpha_1 - L_2 \sin\alpha_2 + L_3 \cos\alpha_3\right) . \quad (23)$$

Applying the condition of static equilibrium for segment 1, it is calculated:

$$R_{A\eta 1} = \frac{X}{l_{11}}\left(l_{11} \sin\alpha_1 - L_1 \sin\alpha_1 - L_2 \sin\alpha_2 + L_3 \cos\alpha_3\right) . \quad (24)$$

For clarity improving purpose, the obtained analytical expressions for transverse reactive forces upon all three segments are given in Table 1.

If multipliers of G and X are denoted as g_j and x_j ($j=1,...,11$) consequently, while using simplified denotation from Table 1 and applying the principle of superposition, total forces values can be noted as follows:

$$F_j = g_j G + x_j X \ , \ j = 1,...,11 \ . \quad (25)$$

Table 1. Transverse reactive forces upon segments

	Joint	Force	Force value in load case with *G* acting only [9]	Force value in load case with *X* acting only
segment I	A	$F_1 = R_{A\eta 1}$	$\frac{G}{l_{11}}\left(L_1 \cos\alpha_1 - L_2 \cos\alpha_1 - L_3 \sin\alpha_3 - l_{11} \cos\alpha_1\right)$	$\frac{X}{l_{11}}\left(l_{11} \sin\alpha_1 - L_1 \sin\alpha_1 - L_2 \sin\alpha_2 + L_3 \cos\alpha_3\right)$
	A"	$F_2 = F_{A\eta 1}$	$\frac{G}{l_{11}}\left(-L_1 \cos\alpha_1 + L_2 \cos\alpha_2 + L_3 \sin\alpha_3\right)$	$\frac{X}{l_{11}}\left(L_1 \sin\alpha_1 + L_2 \sin\alpha_2 - L_3 \cos\alpha_3\right)$
	B'	$F_3 = F_{B\eta 1}$	$\frac{G}{l_{21}}\left(L_2 \cos\alpha_2 + L_3 \sin\alpha_3\right)$	$\frac{X}{l_{21}}\left(L_2 \sin\alpha_2 - L_3 \cos\alpha_3\right)$
	B	$F_4 = R_{B\eta 1}$	$\frac{G}{l_{21}}\left(l_{21} \cos\alpha_1 - L_2 \cos\alpha_2 - L_3 \sin\alpha_3\right)$	$\frac{X}{l_{21}}\left(-l_{21} \sin\alpha_1 - L_2 \sin\alpha_2 + L_3 \cos\alpha_3\right)$
segment II	B	$F_5 = R_{B\eta 2}$	$\frac{G}{l_{22}}\left(l_{22} \cos\alpha_2 - L_2 \cos\alpha_2 - L_3 \sin\alpha_3\right)$	$\frac{X}{l_{22}}\left(l_{22} \sin\alpha_2 - L_2 \sin\alpha_2 + L_3 \cos\alpha_3\right)$
	B"	$F_6 = F_{B\eta 2}$	$\frac{G}{l_{22}}\left(L_2 \cos\alpha_2 + L_3 \sin\alpha_3\right)$	$\frac{X}{l_{22}}\left(L_2 \sin\alpha_2 - L_3 \cos\alpha_3\right)$
	C'	$F_7 = F_{C\eta 2}$	$\frac{G}{l_{32}} L_3 \sin\alpha_3$	$-\frac{X}{l_{32}} L_3 \cos\alpha_3$
	C	$F_8 = R_{C\eta 2}$	$\frac{G}{l_{32}}\left(-l_{32} \cos\alpha_2 - L_3 \sin\alpha_3\right)$	$\frac{X}{l_{32}}\left(L_3 \cos\alpha_3 - l_{32} \sin\alpha_2\right)$
segment III	C	$F_9 = R_{C\eta 3}$	$\frac{G}{l_{33}}\left(l_{33} - L_3\right)\sin\alpha_3$	$\frac{X}{l_{33}}\left(L_3 - l_{33}\right)\cos\alpha_3$
	C"	$F_{10} = F_{C\eta 3}$	$\frac{G}{l_{33}} L_3 \sin\alpha_3$	$-\frac{X}{l_{33}} L_3 \cos\alpha_3$
	D	$F_{11} = R_{D\eta 3}$	$-G \sin\alpha_3$	$X \cos\alpha_3$

2.2 Calculation of boom's tip displacements

Segments of articulated boom can be considered as independent separated girders. The whole structure is divided into 8 sections between joints and anchor points of hydro cylinders, as it is shown in Figure 3.

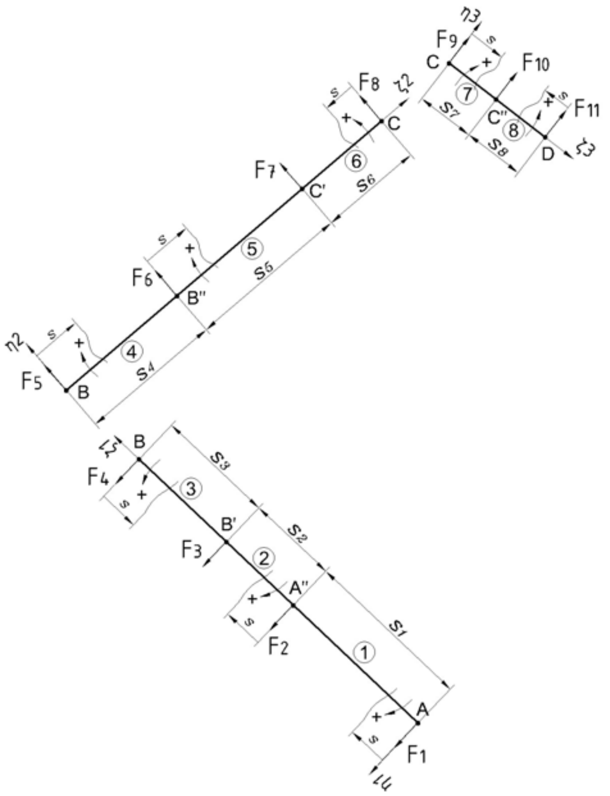

Figure 3. Articulated boom divided into sections

Regarding new simplified designations and adopted positive direction introduced in Figure 3, corresponding bending moments in relation to floating section coordinate *s* are given in Table 2.

Table 2. Bending moments by sections

Section	Section length	Moment of inertia	Bending moment
1	$s_1 = l_{11}$	I_1	$F_1 s$
2	$s_2 = L_1 - l_{11} - l_{21}$		$F_1\left(s + s_1\right) + F_2 s$
3	$s_3 = l_{21}$		$F_4 s$
4	$s_4 = l_{22}$	I_2	$F_5 s$
5	$s_5 = L_2 - l_{22} - l_{32}$		$F_5\left(s + s_4\right) + F_6 s$
6	$s_6 = l_{32}$		$F_8 s$
7	$s_7 = l_{33}$	I_3	$F_9 s$
8	$s_8 = L_3 - l_{33}$		$F_{11} s$

New forms of (1) and (2) are now:

$$f_z = \frac{\partial A_d}{\partial G} = \sum_{i=1}^{8} f_{zi} = \frac{1}{E}\sum_{i=1}^{8}\frac{1}{I_i}\int_0^{s_i} M_i(s)\frac{\partial M_i(s)}{\partial G}ds \,. \quad (26)$$

$$f_x = \left.\frac{\partial A_d}{\partial X}\right|_{X=0} = \sum_{i=1}^{8} f_{xi} = \frac{1}{E}\sum_{i=1}^{8}\frac{1}{I_i}\int_0^{s_i} M_i(s)\frac{\partial M_i(s)}{\partial X}ds\right|_{X=0} . \quad (27)$$

After integration, corresponding members in (26) and (27) are as follows:

$$f_{z1} = \frac{G g_1^2 s_1^3}{3EI_1} \,. \quad (28)$$

$$f_{z2} = \frac{G}{3EI_1(g_1+g_2)}\left\{\left[(g_1+g_2)s_2 + g_1 s_1\right]^3 - (g_1 s_1)^3\right\} . \quad (29)$$

$$f_{z3} = \frac{G g_4^2 s_3^3}{3EI_1} \,. \quad (30)$$

$$f_{z4} = \frac{G g_5^2 s_4^3}{3EI_2} \,. \quad (31)$$

$$f_{z5} = \frac{G}{3EI_2(g_5+g_6)}\left\{\left[(g_5+g_6)s_5 + g_5 s_4\right]^3 - (g_5 s_4)^3\right\} . \quad (32)$$

$$f_{z6} = \frac{G g_8^2 s_6^3}{3EI_2} \,. \quad (33)$$

$$f_{z7} = \frac{G g_9^2 s_7^3}{3EI_3} \,. \quad (34)$$

$$f_{z8} = \frac{G g_{11}^2 s_8^3}{3EI_3} \,. \quad (35)$$

$$f_{x1} = \frac{G g_1 x_1 s_1^3}{3EI_1} \,. \quad (36)$$

$$f_{x2} = \frac{G}{EI_1}\left[\begin{array}{l}\dfrac{(g_1+g_2)(x_1+x_2)}{3}s_2^3 + g_1 x_1 s_1^2 s_2 \\ +\dfrac{x_1(g_1+g_2)+g_1(x_1+x_2)}{2}s_1 s_2^2\end{array}\right] . \quad (37)$$

$$f_{x3} = \frac{G g_4 x_4 s_3^3}{3EI_1} \,. \quad (38)$$

$$f_{x4} = \frac{G g_5 x_5 s_4^3}{3EI_2} \,. \quad (39)$$

$$f_{x5} = \frac{G}{EI_2}\left[\begin{array}{l}\dfrac{(g_5+g_6)(x_5+x_6)}{3}s_5^3 + g_5 x_5 s_4^2 s_5 \\ +\dfrac{x_5(g_5+g_6)+g_5(x_5+x_6)}{2}s_4 s_5^2\end{array}\right] . \quad (40)$$

$$f_{x6} = \frac{G g_8 x_8 s_6^3}{3EI_2} \,. \quad (41)$$

$$f_{x7} = \frac{G g_9 x_9 s_7^3}{3EI_3} \,. \quad (42)$$

$$f_{x8} = \frac{G g_{11} x_{11} s_8^3}{3EI_3} \,. \quad (43)$$

3. THE RESULTS OBTAINED FROM ANALYTICAL MODEL AND FEM

Numerical data taken for verification of analytical calculation of boom's tip displacement are:

L_1=760[cm], L_2=820[cm], L_3=240[cm], l_{11}=300[cm], l_{21}=336[cm], l_{22}=150[cm], l_{32}=130[cm], l_{32}=54[cm], I_1=9969[cm], I_2=5240[cm], I_3=2306[cm], G=2[kN], E=21000[kN/cm^2].

Numerical calculation was done for five different operation positions of the three-segment articulated boom, Figure 4.

Figure 4. Articulated boom operation shapes within numerical example

Software packages MatLab and SAP200 were used for numerical calculations of boom's tip displacements. Numerical results obtained from analytical model and finite element method are given in Table 3.

Table 3. Numerical results and relative error

Boom's shape $\alpha_1-\alpha_2-\alpha_3(°)$	Model (cm)		FEM (cm)		Relative Error (%)	
	f_z	f_x	f_z	f_x	δ_z	δ_x
15-15-30	-4.259	-0.387	-4.335	-0.396	1.75	2.27
30-30-45	-4.357	1.040	-4.417	1.051	1.36	1.05
45-45-60	-3.893	2.268	-3.928	2.305	0.89	1.61
60-60-75	-2.993	2.966	-2.937	2.867	1.9	3.45
75-75-90	-1.896	2.949	-1.980	3.095	4.24	4.71

4. CONCLUSION

Analytical form of displacement of articulated boom's tip enables designer to have the ultimate control upon geometric parameters through design process of the hinged structure.

Comparative analysis of results revealed that relative deviations are less than 5%, which makes introduced method appropriate for determination of articulated boom's deflection.

Contribution of axial and transverse forces to total amount of strain energy can be neglected.

The obtained analytical dependence of boom endpoint displacement as a function of geometric parameters and payload could be the basis for design and optimization of cross-sections of boom segments and/or their lengths.

ACKNOWLEDGMENT

This paper is a contribution to the research project TR 35038 funded by Serbian Ministry of Science and Technological Development.

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