MODEL OF MULTICRITERIA OPTIMIZATION USING COMPLEX CRITERIA FUNCTIONS

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Abstract: *Specific problem in multicriterial optimization is rating of large number of alternatives by using complex multicriteria functions which are composed of subcriteril functions. These papers suggest model which is based at use of average values of pure flow Φ subcriteria functions derived by PROMETHEE III method.*

Key words: *multicriteria optimization, complex multicriteria functions*

1. INTRODUCTION

Family of methods with the full name „Preference Ranking Organization Method for Enrichment Evaluations" which are better known by abbrevation PROMETHEE for multicriterial optimization problem solution has been developed in a variants I, II, III и IV by several authors leadnig with BRANS (BRANS/1982/, BRANS & MARECHAL/1984, 1992, 1994/, BRANS & VINCKE/1985/, MARECHAL/1986, 1988/).

The main features of these methods compared to the corresponding criteria of decision maker are six generelized criteria. Method PROMETHEE I ensures partial order, method PROMETHEE II gives full order, method PROMETHEE III gives interval order of compared alternatives and PROMETHEE IV considers a continuous series of alternatives .

The adoption of decisions on multicriterial optimization problem solving using PROMETHEE methods consists of sequence of delicate phases:

- definition of multicriteria base i.e. system of criteria for alternatives sorting,
- selection of generalized criteria for displaying of preference in relation to the adequate criteria,
- determination of criteria relative weight,
- sorting of alternatives by one of PROMETHEE I-IV, methods,
- decision making
- Specific problems that occurs in multicriterial optimization are criteria for alternatives sorting which consist of subcriterial functions where level of function decomposistion can raise up to (*r*-th) level [3].

Below was shown a model for resolving the aforementioned problems and which is based on transformation of intermediate values of pure stream F subcriterial functions (which are calculated in interval order of compared alternatives of PROMETHEE III method) at level of criterial function of higher order until creation of unique first order criteria.

2. MODEL SOLUTION OF ALTERNATIVES RANKING PROBLEM USING COMPLEX CRITERIAL FUNCTION

Formal writing of problem is given at Table 1, where are:

m – number of alternatives

n – number of r-th level criterial functions

s – number of $(r-1)$-th level criterial functions

l - number of *2-nd* level criterial functions

k - number of *1-st* level criterial functions

Criteria functions are characterized by K_j and proper index for observed ranking level. So are the basic criteria presented by criterial function of *1-st* level and characterized by K_j^1. Subcriterial functions i.e. 5-th level are characterized by K_j^5.

Each criteria has its own relative weight which expresses by weight coefficient characterized by W_j^s and by demand for function (criteria) minimization or maximization. It is not necessarily for subcriteria fiunctions of certain level to have the same demand for minimization or maximization.

The problem is solved by using average values of pure flow Φ on the following way:

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- real values of criteria functions are used only at last r-th level,
- at other ranking levels, as values of subcriteria functions are introduced transformed values of pure flow Φ from *k*-th level at *(k-1)*-th ranking level with the process repeating until *1*-st (basic) level,
- transformation of average values of pure flow are calculated using:

$$e_i = \frac{\overline{\Phi}(a_i) - min\overline{\Phi}}{R} \qquad (1.1)$$

where:

$$R = max\overline{\Phi} - min\overline{\Phi} \qquad (1.2)$$

and represents i.e. difference between maximum minimum value of pure flow.

Algorithm of proposed method for solving of multicriteria optimization with complex criteria functions which in themselves contain subcriteria functions is shown at figure 1.

3. EXAMPLE

For example is taken choice of products (alternatives **A_1 - A_8**) based on three criteria at first level [4]:

- time for product developement - t
- product development costs - T
- product quality – Q

Each of the above criteria is composed of multitude subcriteria functions. Decomposition is derived only to second level. Function t is composed of subcriteria functions marked as t1, t2, t3, t4 and t5, function T is composed of 10 subcriteria functions (T_1, T_2, ... ,T_{10}) and function Q is composed of 24 subcriteria functions (q_1, q_2, ... , q_{24}).

Basic data for algorithm execution are shown in table 2.

Table 2. Algorithm data

Number of alternatives	m=8
Number of *1-st* level criterial functions	n=3
Number of *2-nd* level criterial functions	$K_t^2 = 5$ $K_T^2 = 10$ $K_Q^2 = 24$

Multicriteria analysis is executed in two phases based on experimental data shown in table 3.

I phase – multicriteria analysis with criteria functions of II level using PROMETHEE III method:

a) multicriteria analysis based on criteria t_1, t_2, t_3, t_4 и t_5;

b) multicriteria analysis based on criteria T_1, T_2, ... ,T_{10}

c) multicriteria analysis based on criteria q_1, q_2, ... ,q_{24}.

Values of pure flows that are calculated based on this analisys are transformed using formula (1.1) и (1.2) and they are shown in table 4.

Table 4. Transformed values of pure florws after phase I

	C_{ij}		
	Time	**Costs**	**Quality**
A_1	0,00000	0,00000	0,00000
A_2	0,49822	0,31443	0,21040
A_3	0,37571	0,40197	0,20665
A_4	0,71401	0,75213	0,56167
A_5	0,07791	0,51209	0,35125
A_6	0,77862	0,55234	0,76741
A_7	0,73510	0,64564	0,66410
A_8	1,00000	1,00000	1,00000

II phase – multicriteria analysis of I level criteria functions using PROMETHEE I, II и III methods. Entry data are transformed values of pure flows calculated at previous phase. Results of this phase are shown in table 5. The best alternative is A8.

For all calculations that are performed at I and II phase of proposed methodology is used software MODIPROM в.1.0 [4].

Table 1. Formalized inscription of multicriteria analisys problem

CRITERIA LEVEL					ALTERNATIVES						Relative weight coefficients							
1	2	...	r-1	r	**A_1**	**A_2**	**...**	**A_i**	**...**	**A_m**	**r**	max/ min	**...**	**2**	max / min	**1**	max / min
K_1^1	K_1^2	...	K_1^{r-1}	K_1^r	C_{11}	C_{21}	...	C_{i1}	...	C_{m1}	W_1^r	max/min	...	W_1^2	max/min	W_1^1	max/min
				K_2^r	C_{12}	C_{22}	...	C_{i2}	...	C_{m2}	W_2^r	max/min					
			K_2^{r-1}	K_3^r	C_{13}	C_{23}	...	C_{i3}	...	C_{m3}	W_3^r	max/min	...				
				K_4^r	C_{14}	C_{24}	...	C_{i4}	...	C_{m4}	W_4^r	max/min					
	K_2^2	...	K_3^{r-1}	K_5^r	.	.		.		.	W_5^r	max/min	...	W_2^2	max /min		
	K_3^2	...	K_4^{r-1}	K_6^r	.	.		.		.	W_6^r	max/min	...	W_3^2	max/min		
			K_5^{r-1}	K_7^r	.	.		.		.	W_7^r	max/min	...				
				K_8^r	.	.		.		.	W_8^r	max/min					
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K_j^1			K_j^{r-1}	K_j^r	C_{1j}	C_{2j}	...	C_{ij}	...	C_{mj}	W_j^r	max/min				W_j^1	max/min
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	K_l^2				.	.		.		.				:	:		
			K_s^{r-1}	K_n^r	C_{1n}	C_{2n}	...	C_{in}	...	C_{mn}	W_n^r	max/min					

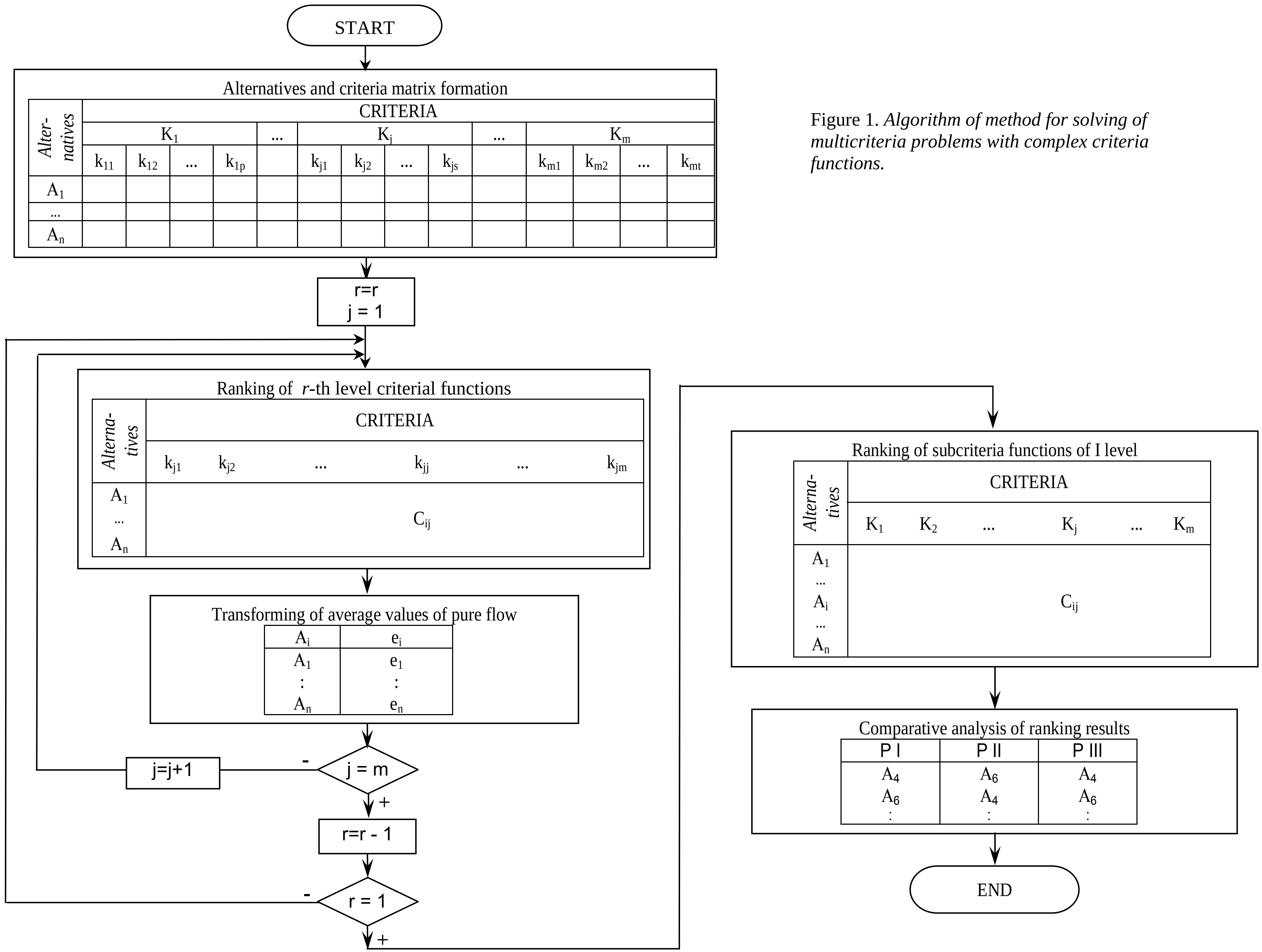

Figure 1. *Algorithm of method for solving of multicriteria problems with complex criteria functions.*

Table 3. Entry data for multicriteria analysis

Criteria		ALTERNATIVES								W_j	max/ min
		A_1	**A_2**	**A_3**	**A_4**	**A_5**	**A_6**	**A_7**	**A_8**		
t	t_1	4	4	4	4	4	4	4	4	1	min
	t_1	6	7	6	7	7	7	7	7	1	min
	t_1	43	28	27	18.5	39	28	27.5	18	1	min
	t_1	27	20	27	20	24	14	16	18	1	min
	t_1	16	14	15	14	32	21	21	13	1	min
T	T_1	520	520	520	520	520	520	520	520	1	min
	T_2	1200	1400	1200	2100	1400	2100	2100	2100	1	min
	T_3	7000	4200	4100	2600	5440	5110	5110	2470	1	min
	T_4	1600	1400	1300	1100	1400	1400	1300	1000	1	min
	T_5	2700	2000	2700	2000	2400	1400	1600	1800	1	min
	T_6	2560	2240	2400	2240	2240	2240	2240	2080	1	min
	T_7	1445	1859	852	680	767	529	625	359	1	min
	T_8	3375	2760	3059	1797	2119	1883	1323	636	1	min
	T_9	3175	3060	1920	1480	1600	2030	1710	575	1	min
	T_{10}	415	348	375	110	115	215	78	65	1	min
Q	q_1	0.82	0.86	0.84	0.90	0.87	0.95	0.92	0.98	0.20	max
	q_2	0.81	0.85	0.85	0.91	0.89	0.94	0.93	0.98	0.10	max
	q_3	0.80	0.83	0.95	0.96	0.90	0.92	0.96	0.97	0.05	max
	q_4	0.80	0.82	0.90	0.93	0.89	0.93	0.94	0.97	0.03	max
	q_5	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.04	max
	q_6	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.04	max
	q_7	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.04	max
	q_8	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.03	max
	q_9	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.03	max
	q_{10}	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.04	max
	q_{11}	0.80	0.84	0.82	0.89	0.87	0.93	0.90	0.98	0.04	max
	q_{12}	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.04	max
	q_{13}	0.83	0.87	0.85	0.92	0.89	0.96	0.93	0.99	0.05	max
	q_{14}	0.82	0.86	0.84	0.90	0.87	0.95	0.93	0.99	0.05	max
	q_{15}	0.97	0.97	0.97	0.97	0.97	0.97	0.97	0.99	0.04	max
	q_{16}	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.98	0.03	max
	q_{17}	0.83	0.87	0.85	0.90	0.87	0.95	0.93	0.99	0.01	max
	q_{18}	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.02	max
	q_{19}	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.01	max
	q_{20}	0.89	0.92	0.90	0.94	0.90	0.96	0.94	0.98	0.02	max
	q_{21}	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.02	max
	q_{22}	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.03	max
	q_{23}	0.82	0.86	0.84	0.90	0.87	0.95	0.92	0.98	0.03	max
	q_{24}	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.01	max

Table 5. Comparative analysis of ranking results

Rank	PROMETHEE I	PROMETHEE II	PROMETHEE III
1	A8	A8	A8
2	A6	A6	A4
3	A4	A7	A6
4	A7	A4	A7
5	A2	A2	A2
6	A3	A5	A3
7	A5	A3	A5
8	A1	A1	A1

4. CONCLUSION

Complex conditions for bussines that are present today, require multicriteria approach at process of solving bussines problems for objective comparation of alternatives which are given in different units, with different factor of importancy and with opposite extremization demands.

Thera are many methods for multicriteria analysis problem solving like: ELECTRE I-IV, PROMETHEE I-IV, AHP, VIKOR, etc. and softwer[5].

Implementation of these methods at bussines decision making is neccessary to defende chosen alternative with strong arguments.

Specific problem occurs when criteria functions on the basis of which alternatives ranking should be done consists of subcriteria functions. Problem becomes mor complex with increasing number of levels of subcriteria functions. Algorithm that was developed permits to resolve this problem using classical methods of multicriteria optimizationоптимизације on a relatively easy way so that through several phases of iteration get an unique solution.

The proposed procedure can have practical application at solving of different problems of multicriteria ranking of alternative solution particularly to choose an investment alternative where criteria can be given at quantitative and qualitative form.

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