# Application of the multi-criteria decision making in the selection of materials of composite shaft

IOP Conf. Series: Materials Science and Engineering **659** (2019) 012024 doi:10.1088/1757-899X/659/1/012024

# Application of the multi-criteria decision making in the selection of materials of composite shaft

**Z Djordjevic[1], S Jovanovic[1], S Kostic[2], M Blagojevic[1] and D Nikolic[1]**

[1]University of Kragujevac, Faculty of Engineering, Sestre Janjić 6, 34000 Kragujevac, Serbia
[2]Technical College of Applied Studies in Kragujevac, Kosovska 8, 34000, Kragujevac, Serbia

E-mail: dviks@kg.ac.rs

**Abstract.** Thanks to the outstanding combination of construction and special features such as high resistance and stiffness, increased wear resistance to fatigue, higher values of critical speeds, lower mass, long lifetime, and so on, today a growing number of shafts, especially in the automotive industry, are made of composite materials. The optimal values of these parameters can be obtained by selecting the appropriate material.
The selection of materials in the process of product design requires efficient decision-making. Incorporating hybrid materials, in addition to providing the potential for improved performance, can make the decision-making process in the selection of material difficult.
In this sense, the use of multi-criteria decision making (MCDM) was used as an adequate tool and support for making optimum decisions. This method has the potential to improve practically all areas of decision making in engineering from design to product.

## 1. Introduction

The shafts belong to the group of elements for rotational motion. They carry on their machinery machine parts (gears, belts, sprockets, couplings, working elements, etc.), making with them a functional system, providing the conditions for rotation with the required speed and load transmission.
The shafts are exposed to complex stress: torsion (due to transmit a torque from one rotating part to the other), bending (due to the force in the gears of the gear teeth, etc.) and axial stress (due to the existence of axial forces). Also, the shafts are loaded with their own weight as well as the weight of the parts on it, the inertial forces due to the unbalance of the mass, the forces of sudden shock and overload, etc. However, the stresses due to these loads are relatively low intensity, and most often during the calculation of the shafts, they are neglected.

For the construction of shafts most commonly used general structural steels, tempering steels or cementing steels, depending on the required strength, durability, cost of production, etc.
In recent years, due to its good characteristics, composite materials increasingly replace steel in shaft production. These advantages are reflected in the high accuracy of production, without the need for subsequent mechanical and heat treating, the possibility of combining different materials of the matrices and fiber, thus achieving the improvement of mechanical properties, energy savings, etc.

Despite the advantages of composite in comparison with conventional metal shafts, their application is limited for now. The main reason for this is still the high cost of raw materials and the production process, which is mostly individual. It is a challenge for constructors to reach a

compromise between the tendency to obtain the minimum mass of the shaft with the maximum mechanical characteristics and the cost and difficulties in the organization of production.

The most commonly used shafts are carbon or glass fibers in combination with epoxy or polyester resins or hybrid constructions obtained by combining these fibers. Also, the laminates obtained by the combination of a metal matrix-composite are suitable for high resistance, load and specific stiffness.

When choosing metal materials for shaft production, more criteria must be considered (required load capacity, stiffness, stability, ...). When it comes to composite materials, the problem becomes even more complicated. Namely, it is known that the composite characteristics may vary depending on the type of material applied, the amount, the angle of orientation of the fibers, etc.

In order to include as many criteria as possible when deciding on material selection, the multi-criteria decision making process (MCDM) is recommended. Such a decision-making process can be encountered in a large number of papers.

In the paper [1], it is emphasized how much the proper choice of materials plays a major role in the product development process. The selection was carried out using two methods of multi-criteria decision-making (VIKOR and ELECTRA). The application of these methods has been demonstrated through concrete examples of analysis.

The problem of material selection is very complex, as explained in [2]. By using hybrid materials, such as composites, which seek to improve some of the characteristics, the problem becomes even more complicated. That's why more and more approach to multi-criteria decision-making in the selection of materials, using different software.

Paper [3] proposes a multi-criteria decision making approach (MCDM) when selecting materials for engineering needs. The paper emphasizes how this method is actually very simple to implement, and includes numerous qualitative and quantitative properties of the material in selection. The results obtained showed good agreement with other methods of analysis.

A multi-criteria decision making approach (MCDM) in the selection of materials has also been proposed in [4]. According to the authors, the number of selection criteria should be limited to seven plus/minus two. The paper emphasizes the advantages of the VIKOR method compared to other MCDM decision-making methods.

Paper [5] deals with analysis of multi-criteria decision-making (MCDM) in material selection, taking into account the ecological aspect. The model is based on engineering analysis of the entire life cycle of the product - from design, product development and product recycling opportunities after the end of the service life.

## 2. Multi-criteria decision making in material selection

Multi-criteria decision making is a relatively new discipline that, through its development, should provide support to decision-makers who, in most cases, face very numerous and often contrary influencing factors. Through the review of the historical development of this method [6], we discover the fact that the term Multi-Criteria Decision-Making (MCDM) was introduced for the first time in 1972 in the management sciences in the United States. The European "version" of this method is an analysis of multicriterial decision making (MCDA - Multi-Criteria Decision-Analysis). These methods aim to achieve an optimal and compromise solution within a transparent process and to maximize the quality of decisions that involve meeting multiple criteria.

In the usual MCDM approach, the final set of alternatives (options, potential solutions) $A_i \in A$. Any alternative solution can be evaluated from the aspect of multiple criteria (attributes), $C_j \in C$. Alternatives ($A_i$) and criteria ($C_j$) and their interrelations can be presented in the form of a matrix table or the so-called decision matrix (Table 1).

**Table 1.** Correlation between alternatives and criteria – the decision matrix.

| | | **Criteria** | | | | | |
|---|---|---|---|---|---|---|---|
| | | $C_1$ ($W_1$) | $C_2$ ($W_2$) | … | $C_j$ ($W_j$) | … | $C_n$ ($W_n$) |
| **Alternative (scenario)** | $A_1$ | $x_{11}$ | $x_{12}$ | … | $x_{1j}$ | … | $x_{1n}$ |
| | $A_2$ | $x_{21}$ | $x_{22}$ | … | $x_{2j}$ | … | $x_{2n}$ |
| | ⋮ | ⋮ | ⋮ | … | ⋮ | … | ⋮ |
| | $A_i$ | $x_{i1}$ | $x_{i2}$ | … | $x_{ij}$ | … | $x_{in}$ |
| | ⋮ | ⋮ | ⋮ | … | ⋮ | … | ⋮ |
| | $A_m$ | $x_{m1}$ | $x_{m2}$ | … | $x_{mj}$ | … | $x_{mn}$ |
| | max/min | max | max | … | min | … | min |

Each of the criteria ($C_j$) can be the maximization (max) or minimization (min) type according to the nature of the size it characterizes. In Table 1, the size $x_{ij}$ represents the value of the *i*-th alternative in relation to the j-th criterion, and $W_j$ is the weight coefficient of the *j*-th criterion. Each of the criteria has no equal importance for decision-makers, so their significance is a weight coefficient - $W_j$. These coefficients have their absolute (ratios) and relative values (weight coefficients). In this segment of the MCDM procedure, in determining the rating of certain criteria, there is a significant influence of the individual or group subjectivism of the decision-makers of certain decisions. It is necessary, first and foremost, to improve the quality of the analysis, to minimize this subjectivism.

In the final phase of each MCDM procedure, the stability of the solution (choosing one or more alternatives) on changes in input data, as well as changes in relative weighting coefficients of the criteria can be investigated.

Linear normalization of the value of $x_{ij}$ in $r_{ij}$ is carried out, depending on the type of criteria in the following way:

$$\text{for } \mathbf{max} \text{ criteria: } r_{ij} = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}, \tag{1}$$

$$\text{for } \mathbf{min} \text{ criteria: } r_{ij} = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}}, \tag{2}$$

Both of these methods for normalizing data reduce the value of alternatives by criteria between zero and one .

SAW (Simple Additive Weighting Method) [7] is one of the most well-known and widely used methods, which provides similar results as well as significantly more complex methods of multi-criteria decision making. This method takes into account the weight coefficients of the criteria. By each criterion, it is necessary to join the weight factor assigned directly by the decision maker, or obtained by applying some of the known methods for determining the weight coefficients of the criteria. For each of the considered alternatives, the aggregate characteristic, or the value of the sum of multiplication of relative weight factors and normalized performance values, is calculated according to all criteria. Quotient an alternative with the highest value is the best of the offered solutions:

$$A^* = \left\{ A_i \left| \max_i \sum_{j=1}^{n} W_j' r_{ij} \right. \right\} \tag{3}$$

$W_j$` represents the normalized value of the weight coefficient $W_j$:

$$W_j' = \frac{W_j}{\sum_{j=1}^{n} W_j} \tag{4}$$

Values $r_{ij}$ are obtained by the aforementioned linear normalization process.

*2.1. Normalization the value of selected parameters and determining weight coefficients*

In this case, four different materials (see Table 2) for the production of shafts were analysed the influence of six characteristic values (performance) that have the role of criteria in the MCDM process. In the Table 2. the values of the $x_{ij}$ are given and in this way the Matrix of the decision is formed.

**Table 2.** Values of $x_{ij}$.

| Material | Young's Modulus $E_1$ (MPa) | Young's Modulus $E_2$ (MPa) | Shear Modulus $G_{12}$ (MPa) | Density ρ (kg/m$^3$) | Max. deflection f (mm) | Max bending stress σ (MPa) |
|---|---|---|---|---|---|---|
| Steel | 210000 | 210000 | 83000 | 7830 | 0.087 | 7.589 |
| Carbon fibers / epoxy resin | 131600 | 8200 | 4500 | 1550 | 0.591 | 6.31 |
| Fiberglass / epoxy resin | 43300 | 14700 | 4400 | 2100 | 0.691 | 7.95 |
| Aramid fibers / epoxy resin | 81800 | 51000 | 1510 | 1380 | 1.15 | 9.7 |
| Type of criteria | max | max | max | min | min | min |

Linear data normalization from the Table 2. was made on the basis of the equation (1) and (2) depending on whether the maximization or minimization type is the criterion. Results of normalization are presented in the Table 3.

**Table 3.** Values of $r_{ij}$.

| Material | Young's Modulus $E_1$ (MPa) | Young's Modulus $E_2$ (MPa) | Shear Modulus $G_{12}$ (MPa) | Density ρ (kg/m$^3$) | Max. deflection f (mm) | Max bending stress σ (MPa) |
|---|---|---|---|---|---|---|
| Steel | 1 | 1 | 1 | 0 | 1 | 0.62 |
| Carbon fibers / epoxy resin | 0.53 | 0 | 0.04 | 0.97 | 0.53 | 1 |
| Fiberglass / epoxy resin | 0 | 0.03 | 0.04 | 0.87 | 0.43 | 0.52 |
| Aramid fibers / epoxy resin | 0.23 | 0.21 | 0 | 1 | 0 | 0 |

In tables 4, 5 and 6 the values of weight coefficients (three variants), which are calculated on the basis of the expression (4) and applying the Seaty procedure. Additionally, for the sake of transparency, the diagram of comparative values of weight coefficients is given six criteria for all three considered variants (Figure 1).

**Table 4.** Determination of weight coefficients (Seaty scale – procedure), variant 1.

| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | | |
|---|---|---|---|---|---|---|---|---|
| $k_1$ | 1 | 7 | 5 | 0 | 3 | 3 | 19 | 0.260274 |
| $k_2$ | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0.013699 |
| $k_3$ | 0 | 3 | 1 | 0 | 0 | 0 | 4 | 0.054795 |
| $k_4$ | 3 | 9 | 7 | 1 | 4 | 5 | 29 | 0.39726 |
| $k_5$ | 0 | 4 | 4 | 0 | 1 | 3 | 12 | 0.164384 |
| $k_6$ | 0 | 4 | 3 | 0 | 0 | 1 | 8 | 0.109589 |
| | | | | | | | 73 | 1 |

**Table 5.** Determination of weight coefficients (Seaty scale – procedure), variant 2.

| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | | |
|---|---|---|---|---|---|---|---|---|
| $k_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 6 | 0.166667 |
| $k_2$ | 1 | 1 | 1 | 1 | 1 | 1 | 6 | 0.166667 |
| $k_3$ | 1 | 1 | 1 | 1 | 1 | 1 | 6 | 0.166667 |
| $k_4$ | 1 | 1 | 1 | 1 | 1 | 1 | 6 | 0.166667 |
| $k_5$ | 1 | 1 | 1 | 1 | 1 | 1 | 6 | 0.166667 |
| $k_6$ | 1 | 1 | 1 | 1 | 1 | 1 | 6 | 0.166667 |
| | | | | | | | 36 | 1 |

**Table 6.** Determination of weight coefficients (Seaty scale – procedure), variant 3.

| | $k_1$ | $k_2$ | $k_3$ | $k_4$ | $k_5$ | $k_6$ | | |
|---|---|---|---|---|---|---|---|---|
| $k_1$ | 1 | 3 | 3 | 0 | 1 | 0 | 8 | 0.115942 |
| $k_2$ | 0 | 1 | 1 | 5 | 3 | 5 | 15 | 0.217391 |
| $k_3$ | 0 | 1 | 1 | 0 | 0 | 0 | 2 | 0.028986 |
| $k_4$ | 3 | 5 | 5 | 1 | 3 | 1 | 18 | 0.26087 |
| $k_5$ | 1 | 3 | 3 | 0 | 1 | 0 | 8 | 0.115942 |
| $k_6$ | 3 | 5 | 5 | 1 | 3 | 1 | 18 | 0.26087 |
| | | | | | | | 69 | 1 |

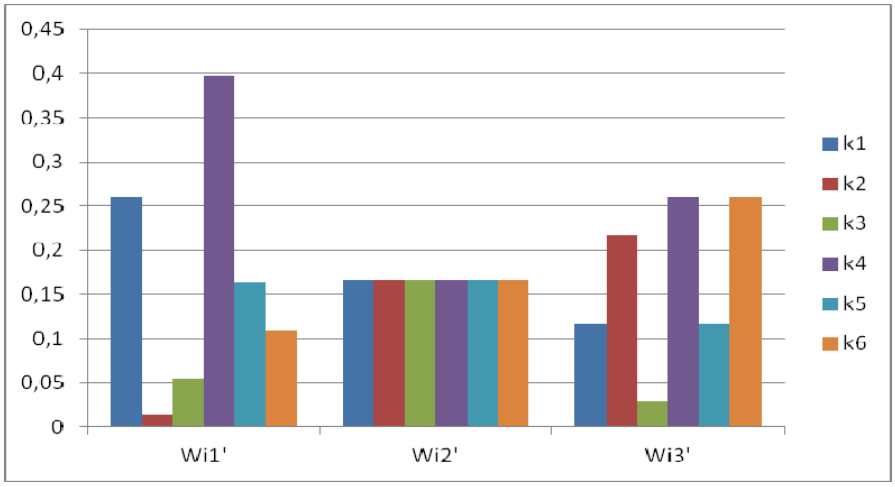


**Figure 1.** Weight coefficients by variants.

*2.2. Results of ranking selected materials*

As the final result of the analysis, using the expression (3), the values of the aggregate characteristics for the three variants of the weight coefficients of the performances - criteria were obtained. In Table 7 and diagram (Figure 2) it is noted that for the values of the first variant of weight coefficients, the carbon fiber composite and the epoxy resin are best chosen. For the second group of weight coefficients, steel was shown as the most favorable material, while in the third case, these two materials were practically identical. The remaining two materials received less or lesser grades in all three variants.

**Table 7.** Collective characteristics for different values of weight coefficients.

| | S | C | F | A |
|---|---|---|---|---|
| $A_{i1}$ | 0.493151 | 0.722192 | 0.47589 | 0.46 |
| $A_{i2}$ | 0.666667 | 0.511668 | 0.315006 | 0.24 |
| $A_{i3}$ | 0.64 | 0.637972 | 0.420144 | 0.333189 |

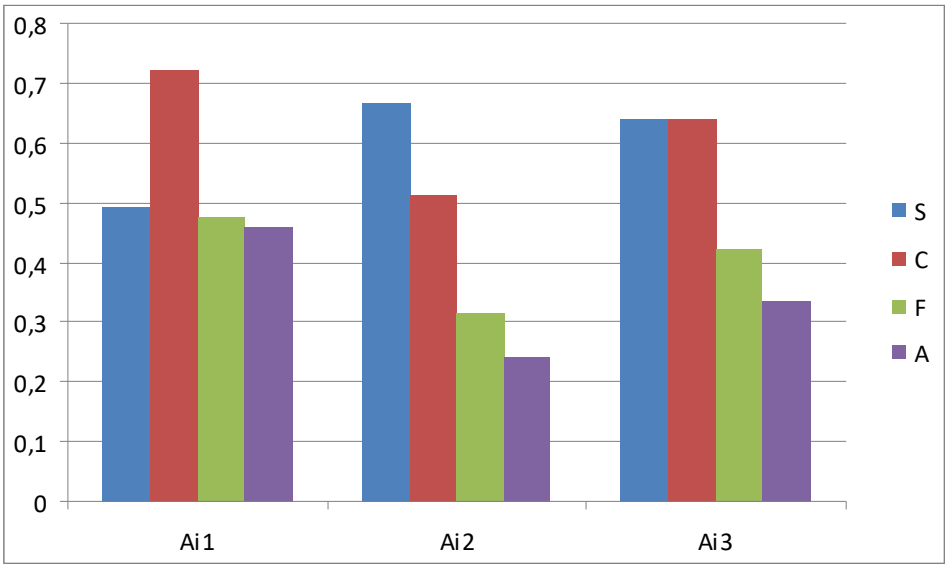


**Figure 2.** Diagram showing aggregate characteristics for three variants of weight coefficients of preformance - criteria.

## 3. Conclusions

The phase of selecting adequate material within the product design process is a task for the designer who has to make the appropriate decisions. Based on the characteristics of the various available materials and certain constructional requirements, it is often not easy to effectively optimize the choice between possible options only based on the subjective assessment of the designer. Solving such and similar problems is greatly facilitated by the use of one of the multi-criteria decision-making methods (MCDM). Based on the analysis presented for the four materials considered and the evaluations of the six selected characteristics, it can be concluded that the steel and carbon fiber composites with epoxy resins are the best estimate of the potential solution. The analysis was made for three variants of the weight coefficients of the criterion - characteristics. In each of the variants, to a greater or lesser extent, these two materials received the highest marks in the multi-criteria decision-making process.

## Acknowledgment

This is result of the TR33015 project, which is investigation of the Technological Development of the Republic of Serbia. The project is titled “Research and development of a Serbian net-zero energy house”. We would like to thank to the Ministry of Education, Science and Technological Development of the Republic of Serbia for their financial support during this investigation.

**References**

[1] Chatterjee P, Athawale V M and Chakraborty S A 2009 Selection of Materials Using Compromise Ranking and Outranking Methods, *Materials & Design* **30**(10) 4043-4053
[2] Jahan A, Edwards K L and Bahraminasab M 2016 *The Importance of Decision Support in Materials Selection*, In book: Multi-Criteria Decision Analysis for Supporting the Selection of Engineering Materials in Product Design 1-23
[3] Jahan A, Ismail M Y, Mustapha F and Sapuan S M 2010 Material Selection Based on Ordinal Data, *Materials & Design* **31**(7) 3180-3187
[4] Chakraborty S and Chatterjee P 2013 Selection of Materials Using Multi-Criteria Decision-Making Methods with Minimum Data, *Decision Science Letters* **2** 135-148
[5] Huang H, Zhang L and Liu Z 2011 Multi-Criteria Decision Making and Uncertainty Analysis for Materials Selection in Environmentally Conscious Design, *Int J Adv Manuf Technol* **52** 421-432
[6] Costa B and Pirlot M 1997 *Thoughts on the Future of Multicriteria Field: Basic Convictions and Outline for General Methodology*, Springer
[7] Fishburn P 1967 *Additive Utilities with Incomplete Product Set: Applications to Priorities and Assignments*, ORSA Publication