EXERGY EFFICIENCY OF A RADIATOR HEATING SYSTEM

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Abstract: *This paper wants to answer the following question– what is the change of exergy for a given radiator heating system? For this purpose a mathematical model in Matlab software was made and was simulated in Simulink.*
As a variable in the model is used environment temperature, as constants:
-Indoor temperature (in this paper is used $t_u = 20\ ^0C$)
-Geometrical and construction characteristics of the model (house)
-Radiator type(constant exponent for the radiator heating power for different temperature differences between the radiator's mean temperature and the room temperature)
The model allows the change of the next parameters between simulations:
-To change a type heating fluid (in this paper is used exclusively warm water)
-Difference of temperature between radiator's input and output
-A middle temperature of heating fluid

Key words: *Exergy efficiency, radiator heating system, heat requirement*

1. INTEGRATED ENERGY, ENTROPY, AND EXERGY ANALYSIS

Exergy is defined as the maximum useful work that could be obtained from a system at a given state with respect to a reference environment (dead state) [4]. In a process or a system, the total amount of exergy is not conserved but destroyed due to internal irreversibilities.
In a thermodynamic system, exergy can be transferred to or from a system in three forms:
heat, work and mass flow, which are recognized at the system boundaries.

The exergy transfer by heat is expressed as [4]:

$$\dot{X}_{heat} = \dot{Q}(1 - \frac{T_0}{T})\ (kW) \tag{1}$$

Where:
$\dot{Q}$ = rate of heat transfer crossing the system boundaries [kW];
T_o = environment temperature [K];
T = temperature of heat source [K].

In the case of mechanical work or electricity crossing the system boundaries, exergy transfer $\dot{X}_{work}$ [kW] equals the electricity or mechanical work itself $\dot{W}$ [kW].

In the case of mass flow crossing the system boundaries, exergy transfer by mass $\dot{X}_{mass}$ is:

$$\dot{X}_{mass} = \dot{m}x\ [kW] \tag{2}$$

Where:
$\dot{m}$ = mass flow rate crossing the system boundaries [kg/s];
x = exergy per unit mass [kJ/kg].

For a flow stream, the unit mass exergy can be expressed as:

$$x = (h - h_0) - T_0(s - s_0)\ [kJ/kg] \tag{3}$$

The exergy change of a flow stream is [5]:
$\Delta x = x_2 - x_1$, or in the developed form

$$\Delta x = (h_2 - h_1) - T_0(s_2 - s_1)\ [kJ/kg] \tag{4}$$

Where:

T =temperature, [K];
h =enthalpy, [kJ/kg];
s =entropy [kJ/kg·K].

The subscript "0" indicates the environmental dead state and subscripts "1" and "2" indicate different states of the flow stream.
For the steady-state flow process, there is no storage of energy, entropy as well as exergy within the system. The energy balance equation is:

$$E_{in} = E_{out}\ [kW] \tag{5}$$

The entropy balance equation is:

$$\dot{S}_{in} + \dot{S}_{generated} = \dot{S}_{out}\ [kW/K] \tag{6}$$

The exergy balance equation is:

$$\dot{X}_{in} = \dot{X}_{out} + \dot{X}_{destroyed}\ [kW] \tag{7}$$

Where the subscript "in" indicates the flow (energy flow, entropy flow or exergy flow) entering the system and the subscript "out" indicates the flow leaving the system.

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Exergy destruction $\dot{X}_{destroyed}$ in a process is the product of entropy generation $\dot{S}_{generated}$ in the same process and the reference environment temperature T_0 :

$$\dot{X}_{destroyed} = T_0 \dot{S}_{generated} \ [\text{kW}] \tag{8}$$

Wepfer et al. [6] stated that for a system, such as a heating system, the steady-flow exergy balance can also be expressed as:

$$\dot{X}_{sup\,plied} = \dot{X}_{useful} + \dot{X}_{destroyed} + \dot{X}_{lost} \ [\text{kW}] \tag{9}$$

The exergy supplied to the system is partially destroyed inside the system due to the irreversibilies, partially delivered to the outside with the effluents and partially used by the system.
The energy efficiency of a heating system is defined as:

$$\eta_1 = \frac{\dot{E}_{useful}}{\dot{E}_{sup\,plied}} \tag{10}$$

The exergy efficiency, which provides the realistic measure of performance of engineering system [5], can be expressed in the following forms [6]:

$$\eta_2 = \frac{\dot{X}_{useful}}{\dot{X}_{sup\,plied}} \tag{11}$$

$$\eta_2 = 1 - \frac{\dot{X}_{destroyed} + \dot{X}_{lost}}{\dot{X}_{sup\,plied}} \tag{12}$$

In order to improve the exergy efficiency η_2, the amount of exergy destroyed inside a system plus the amount lost through the effluents should be reduced.

2. EXERGY EFFICIENCY OF THE RADIATOR WITH WARM WATER

In the radiator arrives, in the steady state, warm water with enthalpy H_1 and it gives off heat to a room (Figure 1).
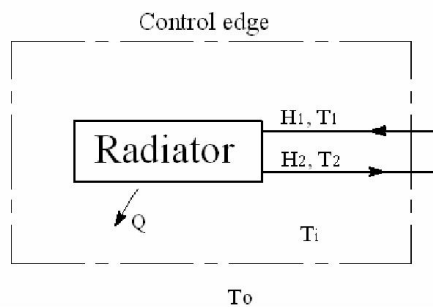

Fig.1. The scheme of the process
Where

T_i - room temperature [K];

T_o -environment temperature, [K];

T_1 -temperature of the water in the supply pipe [K];

T_2 -temperature of the water in the return pipe [K];

Q -heat which is liberated to the room [W];

The calculation is done on the following manner [1]: Warm water leaves the radiator with enthalpy H_2. According to the first low of thermodynamics, it will be

$$Q = H_1 - H_2 = \dot{m}(h_1 - h_2) \ \Rightarrow$$

$$Q = \dot{m}c(T_1 - T_2) \tag{13}$$

Exergy that enters the system in the supply pipe with the temperature T_1 will be:

$$E_{in} = \dot{m}\left|h_1 - h_2 - T_o(s - s_o)\right| \ \Rightarrow$$

$$E_{in} = \dot{m}c\left|T_1 - T_o - T_o \ln \frac{T_1}{T_o}\right| \tag{14}$$

Analogous to (14) output exergy in the return stream with the temperature T_2 will be:

$$E_{out} = \dot{m}c\left|T_2 - T_o - T_o \ln \frac{T_2}{T_o}\right| \tag{15}$$

Preserved exergy in the form of heat which is given to the room on the temperature T_i is:

$$E_{xg} = Q\frac{T_i - T_o}{T_i} = Q \cdot \left(1 - \frac{T_o}{T_i}\right) \tag{16}$$

When we include (13), previous equation will be:

$$E_{xg} = \dot{m}c(T_1 - T_2)\left(1 - \frac{T_o}{T_i}\right) \tag{17}$$

We mark Q_l as the heat which is lost passing through non–heating rooms. It was assumed that 1,75% of the heating need of the house was lost in for example unheating rooms and cellar which is a common case. For the house used in the model that is:

$$Q_l = 230 \ [\text{W}] \tag{18}$$

Its corresponding exergy is:

$$E_l = Q_l\left(\frac{T_m - T_o}{T_m}\right) = Q_l\left(1 - \frac{T_o}{T_m}\right) \tag{19}$$

Where
$T_m = (T_1 + T_2)/2$ - middle water temperature in the radiator

Finally, after neglecting the work of pump, exergy efficiency of the radiator with warm water will:

$$\eta_{ex} = \frac{E_{in} + E_{xg}}{E_{out} + E_l} \tag{20}$$

The pump work is neglected for this system, because there are a lot of gravitational systems in the houses of this kind. In addition, the work required for circulation pumps could be neglected.

3. CASE STUDY

In this paper, a two-floor house was taken into consideration (Figure 2), and is shown as a box (without inside walls), or other words, as a single room.
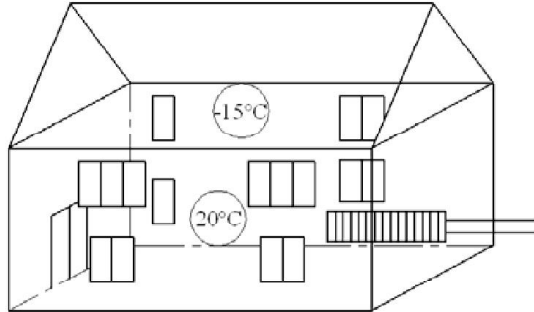

Fig. 2. The scheme of the house model

A radiator, with next characteristics, is used in the heating model [3]:

- $q_n = 146 \; [W / element]$, the power of a heating element at the nominal temperature
- $\Delta t_n = 50°C$, is the nominal temperature difference between the radiator's middle temperature and the room.
- $m = 1{,}31199$, value of exponent

Outdoor temperature is shown as linear function of time, which is changed from $T_0 = -20°C$ to $T_0 = 12°C$, what is justified, because when environmental temperature exceeds $T_0 = 12°C$, the heating stops. Inside temperature is kept constant at $T_i = 20°C$, and the attic temperature is $T_a = -15°C$.

A total necessary heat per unit time, Q, contains from transmission and ventilation heat losses:

$$Q = Q_T + Q_V \quad [W] \tag{21}$$

In calculation of heat losses we neglect the floor losses. The calculation of heat losses is lead at the following manner [2]:

1) A transmission losses are calculated as

$$Q_T = Ak(T_i - T_o) \tag{22}$$

where
- transmission heating coefficients:

$$k = 0{,}8 \left[\frac{W}{m^2 K} \right] \text{-through walls ;}$$

$$k = 3 \left[\frac{W}{m^2 K} \right] \text{-through windows and}$$

the main door;

$$k = 1{,}21 \left[\frac{W}{m^2 K} \right] \text{-through ceiling;}$$

- and house characteristics are:

The dimensions of the house are 12m×6m×5,44m.
The main door dimension (with two parts)- 1,5m×2,42m

The Windows dimensions:
single frame double glazed windows: 0,69m×0,9m
double winged windows: 1,45m×1,5m
triple winged windows -2,2m×1,5m

2) A ventilation losses is calculated as:

$$Q_V = 0{,}65 \sum (al)_S \cdot (T_i - T_o) \cdot R \cdot H \tag{23}$$

$R = 0{,}9$ - room feature,

$H = 3{,}09$ - building feature,

$\sum ls$ - lengths sum all of fissures,

with a is marked permeable coefficient, which is in our case $0{,}4 \; [m^3 / mhPa^{2/3}]$;

Let us remark that 65% of the house's external surface is under influence of wind at a time instant.

Maximal heating demand per unit time is at the minimal outdoor temperature $T_o = -20°C$, and is

$$Q_{max}(-20°C) = 13080 \; [W] \tag{24}$$

This is shown on Fig. 5.

Now, for different middle temperature of the heating fluid, in this case water, we can calculate number of elements from which our imaginary radiator consists.

- $T_m = 80°C$

The temperature difference, is
$$\Delta t = T_m - T_i \; \Rightarrow$$

$$\Delta t = 80° - 20° = 60°C \tag{25}$$

Calculating the heating power of the radiator at the projected conditions is [2]:

$$q = q_n \cdot \left(\frac{\Delta t}{\Delta t_n} \right)^m \tag{26}$$

or with our values

$$q = 185{,}45 \; [W / element] \tag{27}$$

Finally, the number of radiator elements is:

$$n = \frac{Q_{max}}{q} = 70{,}54, \tag{28}$$

Well, under this condition, we adopt a radiator with 71 elements.

Similarly, for different mean radiator temperatures we obtain

$\qquad \succ \quad T_m = 70°C \implies 90$ elements

$\qquad \succ \quad T_m = 60°C \implies 121$ elements

If we multiple (26) with n, we give equation for the heating power which give our radiator gives off to the room:

$$Q = Q_n \cdot \left(\frac{\Delta t}{\Delta t_n} \right)^m \qquad (29)$$

where Q_n marks the heat power which gives the radiator under nominal conditions,

$$Q_n = n \cdot q_n \quad [W] \qquad (30)$$

From equation (29) we, also, can obtain T_1 in function of temperature difference, $\Delta T_r = T_1 - T_2$,

environmental temperature, T_o, and middle radiator temperature, T_m.

In this paper, we considered $\Delta T_r = 20°, 18°, 15°$ and $10°C$ to be the temperature difference between supply and return lines.

4. SIMLATION RESULTS

For the purpose of the paper, we made simulation in Matlab – Simulink, what is shown in Figure 3. In this program we can see next behaviours:

✓ Heating requirements of the house in function of the change of environment temperature;

✓ A mass flow in function of the change of environment temperature and difference between input and output fluid temperature;

✓ Exergy efficiency in function of the change of environment temperature, middle temperature in the radiator and difference between input and output fluid temperatures;

✓ Fluid temperature on input in function of the change of environment temperature, middle temperature in the radiator and difference between input and output fluid temperature;
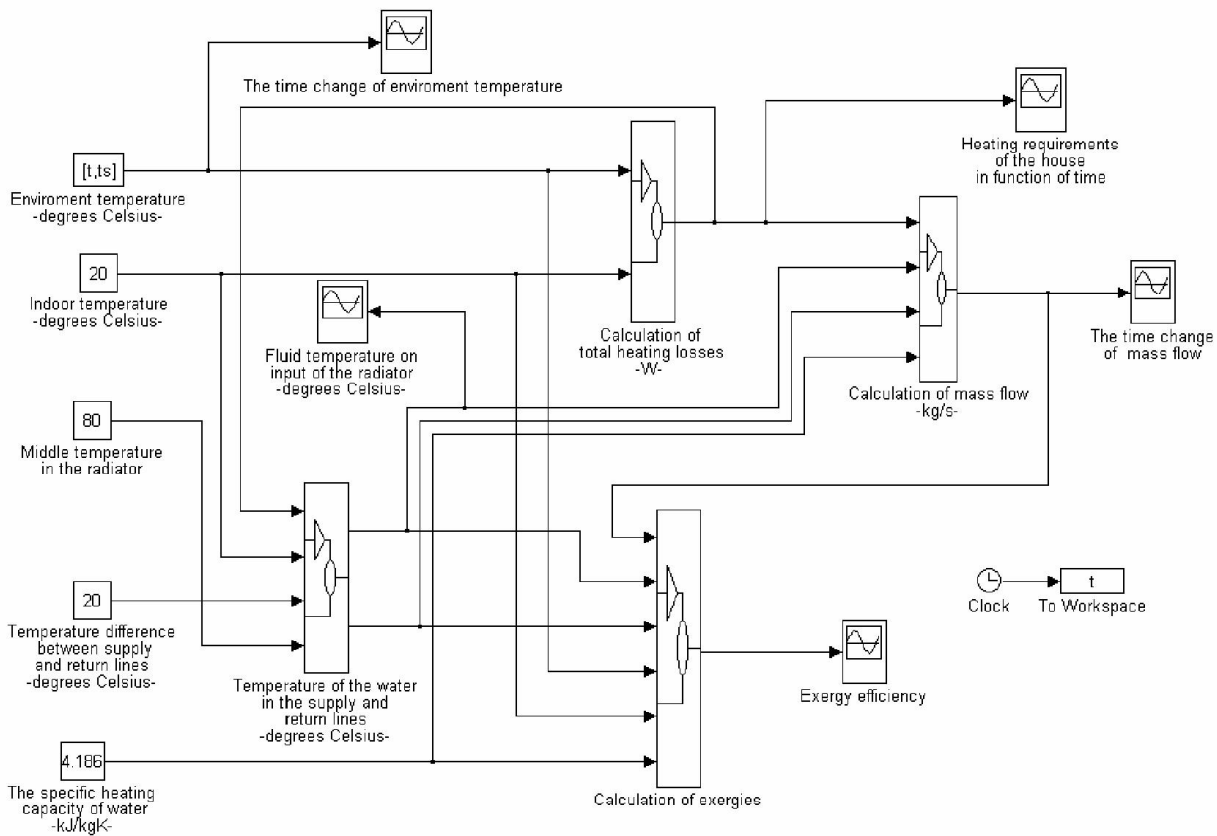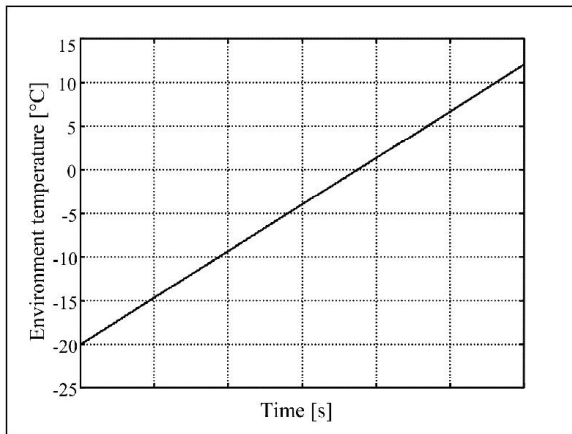
Fig. 3. The model scheme in Matlab - Simulink

Now, let's show the result of simulations.
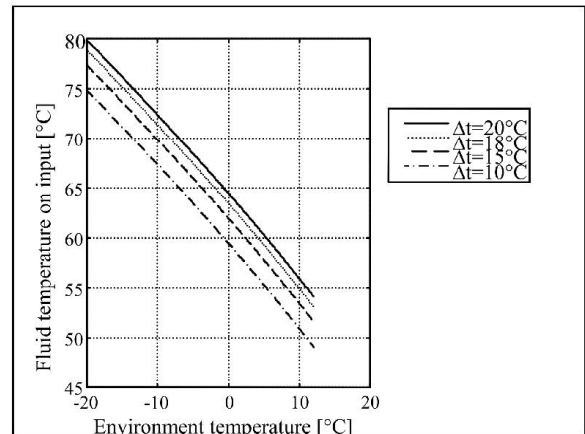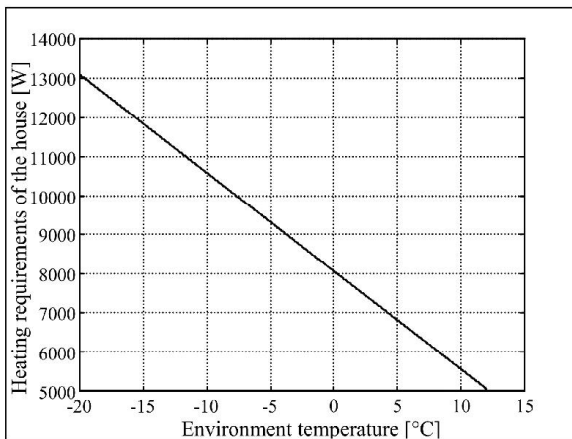
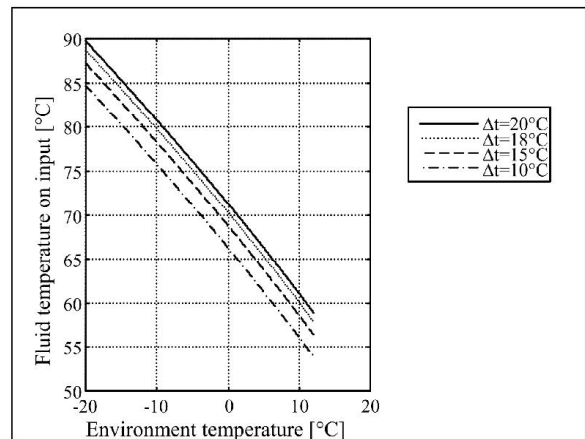Fig. 4. Time change of environment temperature
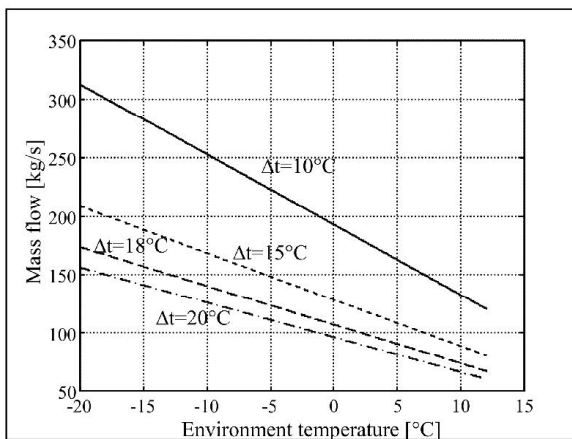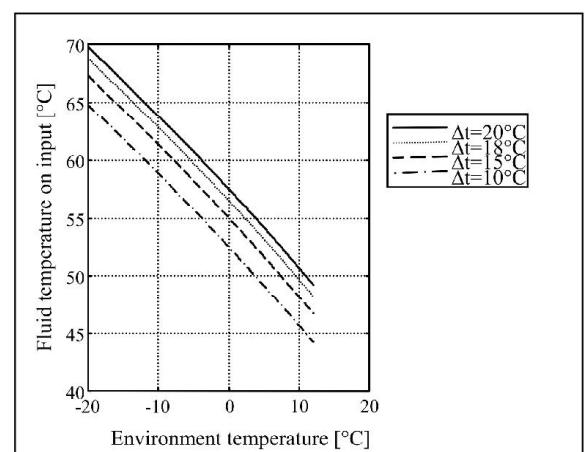

Fig. 8. Fluid temperature in function of the change of environment temperature, for different Δt; tm=70°C

Fig. 5. Heating requirements of the house in function of the change of environment temperature

Fig. 7. Fluid temperature in function of the change of environment temperature, for different Δt; tm=80°C

Fig. 6. Mass flow in function of the change of environment temperature, for different Δt

Fig. 9. Fluid temperature in function of the change of environment temperature, for different Δt; tm=60°C
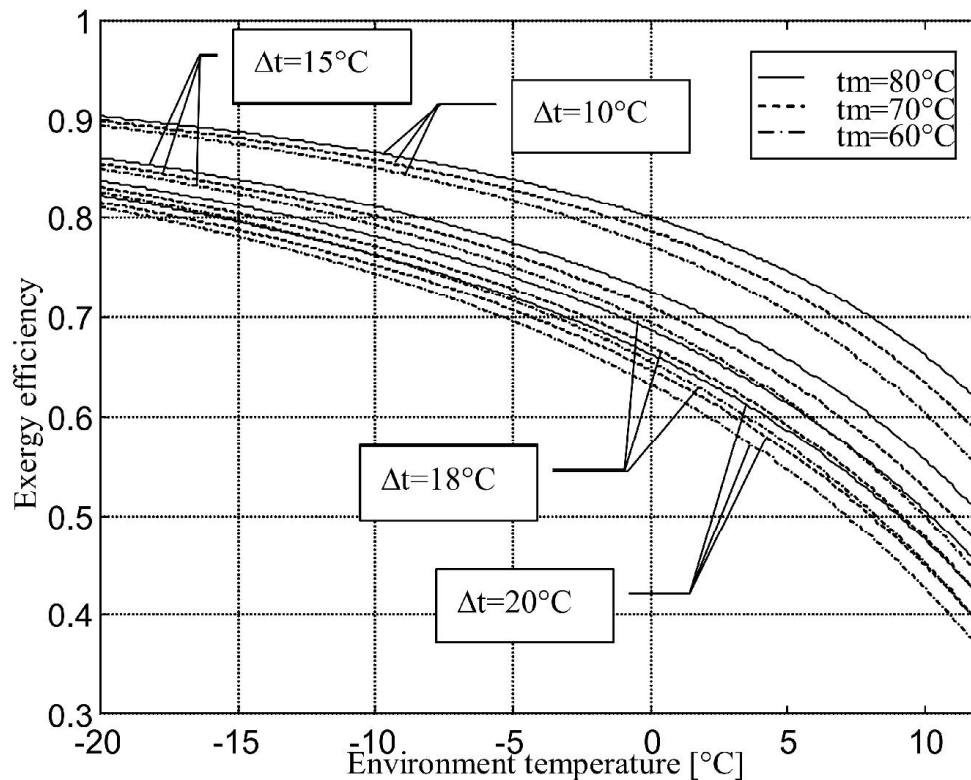
Fig. 10. Exergy efficiency in function of the change environment temperature, for different Δt and t

5. CONCLUSION

From the Figure 10. can be concluded that the temperature difference between supplying and returning lines has greater influence on the exergy efficiency of the radiator heating system than mean radiator temperature. The main reason lies in the fact that with the smaller temperature differences between supplying and returning temperatures preserved exergy in returning lines has larger value. The implementation of the smaller temperature differences between supplying and returning lines is favourable from the exergy efficiency point of view, however in practice it needs larger fluid flows, and because of those larger diameters of supplying lines or with the same piping system the price is paid in larger pressure losses.

6. REFERENCES

[1] D. Milincic, D. Voronjec, "*Thermodynamics*", Faculty of Mechanical Engineering,University of Belgrade, 2000, pp. 146-150.

[2] B. Todorovic, "*Projecting systems for central heating*", Belgrade, 2000.

[3] Catalog "Global", Vox radiators which made on Aluminium

[4] Cegel, Y. A. and Boles, M. A. (2002), "Thermodynamics: an engineering approach", 4th edition, McGraw-Hill.

[5] Rosen, M.A., Leong, W.H. and Le, M.N. (2001), "Modeling and analysis of building systems that integrate cogeneration and district heating and cooling", Proceedings of eSim 2001 Conference.

[6] Wepfer, W.J., Gaggioli, R.A. and Obert, E.F. (1979), "Proper evaluation of available energy for HVAC", ASHRAE Transactions, 85(1), 214-230.