FREE VIBRATION ANALYSIS OF THE HORIZONTAL AXIS WIND TURBINE TOWER

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Abstract:

A rigid multibody method for free vibration analysis of horizontal axis wind turbine tower is proposed. The considerations are performed in the frame of Euler-Bernoulli beam theory. There are two basic steps of this method. In the first step, the shape of the tower is approximated by the n cylindrical flexible segments. Then, every of the n cylindrical flexible segments is replaced with the three rigid bodies connected by the corresponding joints and springs in them. Finally, the rigid multibody model of the tower is provided. The results of the proposed model are compared with the similar one from the literature.

Key words: Free vibration, natural frequencies, wind turbine tower, Euler-Bernoulli beam theory.

1. Introduction

A wind turbine is a device that produces electricity from the energy of wind. During the last decade the use of the wind turbines as a renewable energy device has been increased. The main structural element of the wind turbine is the tower. The approximatively model of the wind turbine can be represented as the turbine tower with a tip mass. It is important to carry out a modal analysis of the tower to avoid a resonance in the structure. The fundamental frequency of the tower must be different from the frequency of rotation of the turbine blades.

Many approaches are available for the dynamic analysis of the wind turbines. Analytical methods are often limited by the possibility of solving the partial differential equations with variable coefficients. On the other hand, there are an approximatively discretization methods that has no restrictions of this kind. The main method of this group is the finite element method. However, there is a method of rigid elements, too. We proposed a rigid elements method, based on considerations from [1], for the free vibration analysis of the wind turbine tower with a tip mass. The method is verified thought the comparison with similar one from the literature and the finite element method.

2. A rigid multibody model of the wind turbine tower

Let us consider the wind turbine tower shown in Fig. 1(a). The cross-section of the tower is tubular, where the outer diameter and the wall thickness of the tower are linearly changed according to the laws $d(z) = (1/h)(d_h - d_0)z + d_0$ and $\delta(z) = (1/h)(\delta_h - \delta_0)z + \delta_0$, $0 \le z \le h$ respectively. The outer diameter of the tower at the base and the top is d_0 and d_h, and the wall

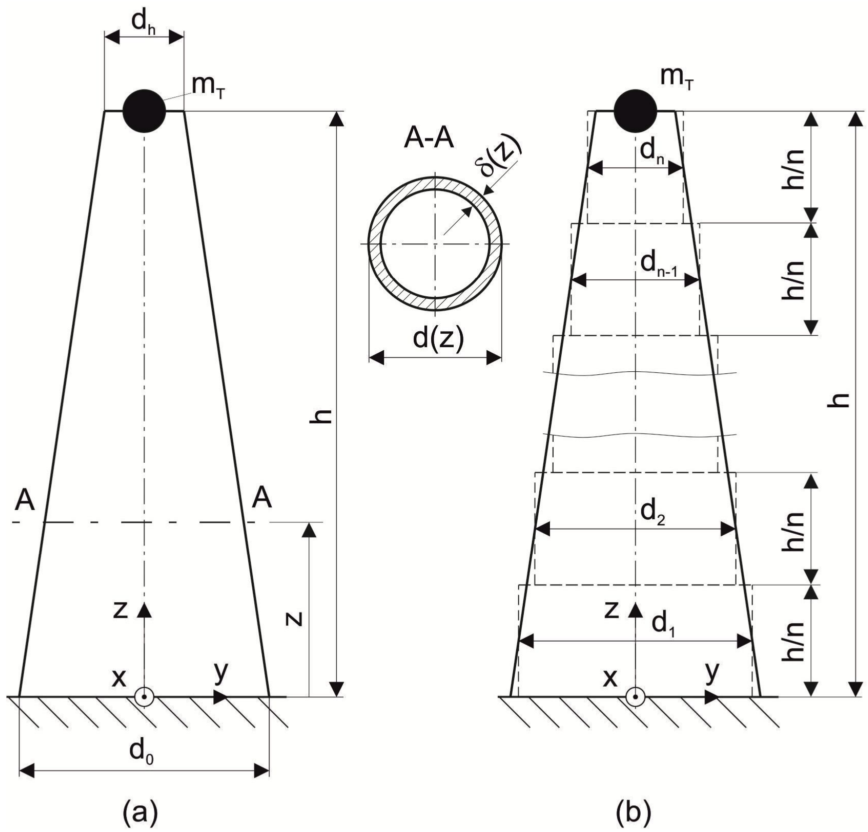

Figure 1. (a) The wind turbine tower, (b) An approximation of the wind turbine tower by a stepped structure

thickness of the tower at the base and the top is δ_0 and δ_h, respectively. The height of the tower is h, and the tip mass at the top of the tower is m_T.

The process of forming a rigid multibody model of the flexible wind turbine tower will be carried out in two steps. In the first step, the tower is divided into n flexible beams of equal length h/n, with the constant cross-sectional area A_i, rigidly connected to each other, as is shown in Fig. 1(b). The first flexible beam at the bottom is clamped, while the tip mass m_T is fixed to the end of the last flexible beam. Similar as in [1], every of the flexible beam is replaced by a rigid multibody joint element RMJE.

The RMJE consist of a three rigid bodies. The first and second rigid bodies are connected by universal joint with four degrees of freedom, whereas the second and third rigid bodies are connected by cardanic joint with two degrees of freedom (see Fig. 2). By using the technique of modelling by fictitious bodies [2], the universal and cardanic joint can be replaced by a system of bodies as shown in Fig. 3. The fictitious bodies are denoted by (v_{6i-k}), k=1,3,4,5. The fictitious body means a dimensionless and massless body.

After the proposed decomposition of joints, the rigid-multibody model of tower consist of $6n+1$ rigid beams interconnected by joints with one degree of freedom and the corresponding springs placed in them.

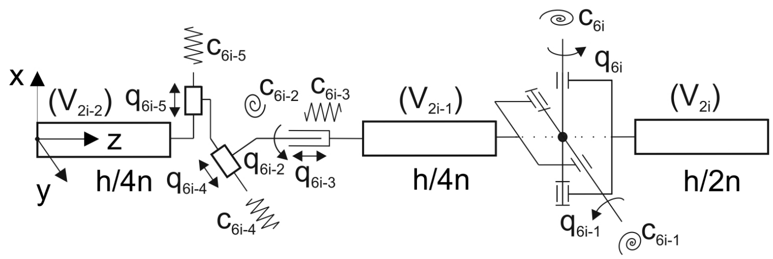

Figure 2. Rigid multibody joint element (RMJE) of the i - th flexible beam

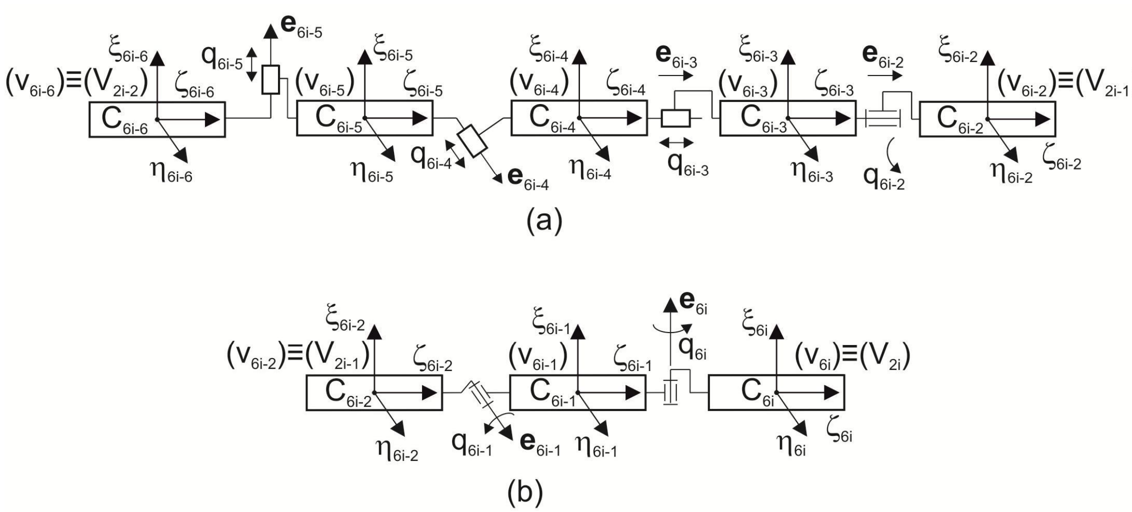

Figure 3. Decomposition of joints: (a) universal joint between the bodies (V_{2i-2}) and (V_{2i-1}), (b) cardanic joint between the bodies (V_{2i-1}) and (V_{2i}).

The length of the introduced rigid bodies read:

$$l_{6i-k} = \begin{cases} 0, & k = 1,3,4,5, \\ \dfrac{h}{4n}, & k = 2, \\ \dfrac{3h}{4n}, & k = 0 \wedge i < n, \\ \dfrac{h}{2n}, & k = 0 \wedge i = n, \end{cases} \tag{1}$$

The mass of the rigid bodies read:

$$m_{6i-k} = \begin{cases} \rho A_i l_{6i-k}, & k = 1,\dots,5, \\ \dfrac{\rho h}{4n}(2A_i + A_{i+1}), & k = 0 \wedge i < n, \\ \rho A_n l_{6n} + m_T, & k = 0 \wedge i = n, \end{cases} \tag{2}$$

where ρ is the mass density, $A_i = (d_i - \delta_i)\pi \cdot \delta_i$ is the cross-sectional area of the i-th flexible beam, and $d_i = d\left(\frac{2i-1}{2}\frac{h}{n}\right)$ and $\delta_i = \delta\left(\frac{2i-1}{2}\frac{h}{n}\right)$ are the outer diameter and the wall thickness of the i-th flexible beam, respectively.

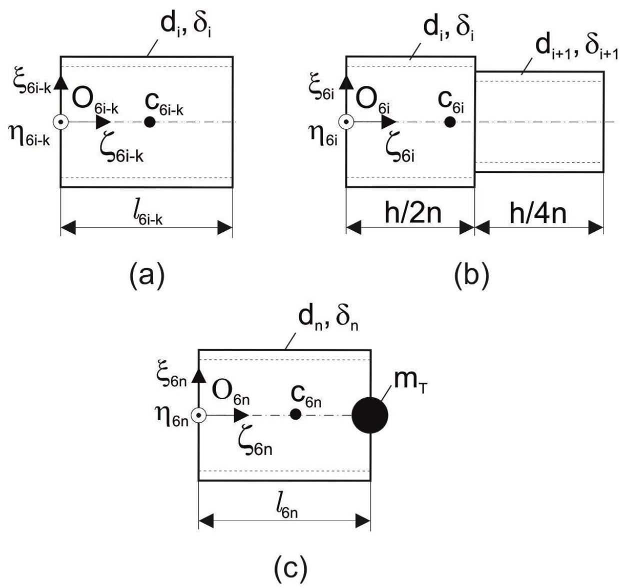

Figure 4. Rigid body (V_{6i-k}) : (a) $k = \overline{1,5}$, (b) $k = 0 \wedge i < n,$ (c) $k = 0 \wedge i = n,$.

The position of the center of mass of each rigid body reads:

$$\overline{O_{6i-k}C}_{6i-k} = \begin{cases} \dfrac{l_{6i-k}}{2}, & k = \overline{1,5}, \\ \dfrac{h}{8n}\dfrac{4A_i + 5A_{i+1}}{2A_i + A_{i+1}}, & k = 0 \wedge i < n, \\ \dfrac{l_n}{2}\dfrac{\rho A_n l_n + 2m_T}{\rho A_n l_n + m_T}, & k = 0 \wedge i = n, \end{cases} \tag{3}$$

The inertia tensor for the principal axes of the each rigid body reads:

$$\mathbf{J}_{C_u} = diag\left(J_{C_u\xi}, J_{C_u\eta}, J_{C_u\zeta}\right),\ u = \overline{1,6n}, \tag{4}$$

where

$$J_{C_{6i-k}\xi} = \begin{cases} J_{C_{6i-k}\xi}', & k = \overline{1,5}, \\ J_{C_{6i}\xi}' + m_{6i}'\left(\overline{O_{6i}C}_{6i} - \dfrac{h}{4n}\right)^2 + J_{C_{6i}\xi}'' + m_{6i}''\left(\dfrac{5h}{8n} - \overline{O_{6i}C}_{6i}\right)^2, & k = 0 \wedge i < n, \\ J_{C_{6n}\xi} + m_{6n}'\left(\overline{O_{6n}C}_{6n} - \dfrac{l_{6n}}{2}\right)^2 + m_T\left(l_{6n} - \overline{O_{6n}C}_{6n}\right)^2, & k = 0 \wedge i = n, \end{cases} \tag{5}$$

$$J_{C_{6i-k}\eta} = J_{C_{6i-k}\xi}, \tag{6}$$

$$J_{C_{6i-k}\zeta} = \begin{cases} m_{6i-k}\left(\dfrac{d_i-\delta_i}{2}\right)^2, & k=\overline{1,5}, \\ \rho\dfrac{h}{2n}\left(2A_i\left(\dfrac{d_i-\delta_i}{2}\right)^2 + A_{i+1}\left(\dfrac{d_{i+1}-\delta_{i+1}}{2}\right)^2\right), & k=0 \wedge i<n, \\ m_{6n}\left(\dfrac{d_n-\delta_n}{2}\right)^2, & k=0 \wedge i=n, \end{cases} \tag{7}$$

and where

$$J_{C_{6i-k}\xi}{}' = \frac{m_{6i\text{-}k}}{12}\left(3\left(\frac{d_i}{2}\right)^2 + 3\left(\frac{d_i-2\delta_i}{2}\right)^2 + l_{6i-k}{}^2\right),\ J_{C_{6i}\xi}{}' = \frac{m_{6i}{}'}{12}\left(3\left(\frac{d_i}{2}\right)^2 + 3\left(\frac{d_i-2\delta_i}{2}\right)^2 + \left(\frac{h}{2n}\right)^2\right), \tag{8}$$

$$J_{C_{6i}\xi}{}'' = \frac{m_{6i}{}''}{12}\left(3\left(\frac{d_{i+1}}{2}\right)^2 + 3\left(\frac{d_{i+1}-2\delta_{i+1}}{2}\right)^2 + \left(\frac{h}{4n}\right)^2\right),\ m_{6i}{}' = \rho A_i\frac{h}{2n},\ m_{6i}{}' = \rho A_{i+1}\frac{h}{4n}, \tag{9}$$

Based on [1], for the tubular cross section of the tower, the stiffnesses of the introduced springs in the joints read:

$$c_{6i-5} = c_{6i-4} = n^3\frac{12E\cdot I_{\xi_i}}{h^3(1+\phi)},\ c_{6i-3} = n\frac{A_i\cdot E}{h},\ c_{6i-2} = n\frac{G\cdot J_i}{h},\ c_{6i-1} = c_{6i} = n\frac{E\cdot I_{\xi_i}}{h},\ i=\overline{1,n}, \tag{10}$$

where E is the Young's modulus, G is the shear modulus, ϕ is the shear coefficient, $I_{\xi_i} = (\pi/64)\left(d_i^4 - (d_i-2\delta_i)^4\right)$ and $J_i = 2I_{\xi_i}$ are the area moments of inertia along the ξ_i principal axis and the polar moment of inertia of the i-th segment cross-section, respectively and n is the number of divisons. Note that in our analysis shear deformation of the beam is neglected ($\phi = 0$).

3. Potential energy due to springs in the joints

The potential energy of the multibody system due to the springs in the joints reads

$$\Pi = \frac{1}{2}\sum_{u=1}^{6n} c_u\cdot q_u{}^2, \tag{11}$$

or, in the concise matrix form

$$\Pi = \frac{1}{2}\mathbf{q}^T\mathbf{K}\mathbf{q}, \tag{12}$$

where $\mathbf{q} = \begin{bmatrix} q_1 & q_2 & \dots & q_{6n} \end{bmatrix}^T$ is the vector of generalized coordinates, and $\mathbf{K} = diag(c_1, c_2, ..., c_{6n})$ is the stiffness matrix of the system.

4. Kinetic energy of the system

The kinetic energy of the multibody system considered reads [3]:

$$T = \frac{1}{2} \sum_{\alpha=1}^{6n} \sum_{\beta=1}^{6n} m_{\alpha\beta}(\mathbf{q}) \dot{q}_\alpha \, \dot{q}_\beta, \tag{13}$$

or, in the concise matrix form

$$T = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}}, \tag{14}$$

where $\mathbf{M}(\mathbf{q}) \in R^{6n \times 6n}$ is the mass matrix whose components are determined by [3]:

$$m_{\alpha\beta}(\mathbf{q}) = \sum_{u=\max(\alpha,\beta)}^{6n} \left(m_u \frac{\partial \mathbf{r}_{C_u}}{\partial q_\alpha}^T \frac{\partial \mathbf{r}_{C_u}}{\partial q_\beta} + \overline{\chi}_\alpha \overline{\chi}_\beta \mathbf{e}_\alpha{}^T \mathbf{J}_{C_u} \mathbf{e}_\beta \right), \tag{15}$$

5. Eigenvalue problem

The linearized differential equations of motion of the considered multibody system about the equilibrium position $\mathbf{q}_0 = \begin{bmatrix} q_1 = 0 & \dots & q_{6n} = 0 \end{bmatrix}^T$ read [4]:

$$\mathbf{M}(\mathbf{q}_0)\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0}_{6n \times 1}, \tag{16}$$

The eigenvalue problem which corresponds to the previous equations of motion reads:

$$\left(\mathbf{K} - \omega^2 \mathbf{M}(\mathbf{q}_0)\right)\mathbf{q} = \mathbf{0}_{6n \times 1}, \tag{17}$$

where ω is the natural frequency of free vibration of the wind turbine tower, and $\mathbf{q} \in R^{6n \times 1}$ represents the eigenvector which corresponds to the given frequency. Approximate values of natural frequencies of the considered tower are obtained by solving the eigenvalue problem (8).

5. Numerical examples

5.1. Tower without a tip mass

Let us consider the wind turbine tower without a tip mass with the following characteristics [5]: $h = 87.6\,\text{m}$, $d_0 = 6\,\text{m}$, $d_h = 3.87\,\text{m}$, $\delta_0 = 0.027\,\text{m}$, $\delta_h = 0.019\,\text{m}$, $m_T = 0$, $\rho = 8500\,\text{kg/m}^3$, $E = 2.1 \cdot 10^{11}\ \text{N/m}^2$, $G = 8.08 \cdot 10^{10}\ \text{N/m}^2$. In the Table 1, a comparison of results of the BModes[6], Adams[6], Zhao et. al. [7] and results of our approach is shown. The Bmodes and Adams are the various approaches to the finite element method. It can be seen that results of our approach and approach from Zhao et. al. [7] converge to the results of BModes and Adams [6] as the number of divisions n increase. In the case of bending frequencies, for the same values of divisions (e.g. $n = 20$), the results of our approach are closer to the BModes results than the results from Zhao et. al. [7]. For the case of torsional and axial frequencies the results of the approach from Zhao et. al. [7] are better than ours.

	Our approach			Zhao et. al. [7]		BModes	Adams
	$n = 20$	$n = 40$	$n = 60$	$n = 10$	$n = 20$	[6]	[6]
Bending	0.8905	0.8911	0.8912	0.8833	0.8857	0.8913	0.8904
	0.8905	0.8911	0.8912	0.8833	0.8857	0.8913	0.8904
	4.3427	4.3510	4.3525	4.2825	4.2929	4.3743	4.3437
	4.3427	4.3510	4.3525	4.2825	4.2929	4.3743	4.3435
	11.1776	11.2295	11.2388	10.9813	11.0090	11.3911	11.1856
	11.1776	11.2295	11.2388	10.9813	11.0090	11.3911	11.1843
Torsional	11.7211	11.8434	11.8834	11.9013	11.9463	11.9656	11.4448
Axial	16.2587	16.3924	16.4363	16.4770	16.5114	16.5217	16.5222

Table 1. Natural frequencies (Hz) of the wind turbine tower without a tip mass

5.2. Tower with a tip mass

Let us consider the wind turbine tower with a tip mass $m_T = 350 \cdot 10^3\,\text{kg}$. Other parameters of the tower are the same as in the previous example. In the Table 2, a comparison of results of the Zhao et. al. [7] and our results is shown. Both the results from Zhao et. al. [7] and our approach are close to each other. It is obvious that with the adding a tip mass all frequencies of the tower decrease.

	Our approach			Zhao et. al. [7]	
	$n = 20$	$n = 40$	$n = 60$	$n = 10$	$n = 20$
Bending	0.3001	0.3002	0.3002	0.2989	0.2996
	0.3001	0.3002	0.3002	0.2989	0.2996
	3.0394	3.0438	3.0446	3.0088	3.0469
	3.0394	3.0438	3.0446	3.0088	3.0469
	9.0486	9.0850	9.0917	8.9179	9.0347
	9.0486	9.0850	9.0917	8.9179	9.0347
Torsional	7.0888	7.1018	7.1060	7.1095	7.1130
Axial	11.7211	11.8434	11.8832	11.9013	11.9463

Table 2. Natural frequencies (Hz) of the wind turbine tower with a tip mass

6. Conclusions

The paper presents a new approach to the free vibration analysis of the wind turbine tower with a tip mass. The results in both examples converge to those from [6]. It is important to note that in Bmodes [6] and Adams [6] is used 50 and 99 finite elements, respectively. The each finite element has 15 degrees of freedom. This is a very large number of degrees of freedom. In the approach from Zhao et. al. [7] and our approach is required significantly fewer degrees of freedom to achieve the same accuracy. Also, our model is simpler than the model from Zhao et. al. [7]. It has the one rigid body and two rotational joints less. In our model, prismatic joints are used instead of rotating joints. This makes the proposed method computationally efficient in comparison to approach from Zhao et. al. [7].

The proposed method can be used for analysis of more complex models of wind turbine, with eccentrically mounted tip mass, the soil influence, etc. This analysis will be the subject of further research by authors.

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