Boris Jerman
Assoc. Professor
University of Ljubljana
Faculty of Mechanical Engineering

Miha Peruš
Student
University of Ljubljana
Faculty of Mechanical Engineering

Jurij Hladnik
Teaching Assistant
University of Ljubljana
Faculty of Mechanical Engineering

Mile Savković
Professor
University of Kragujevac
Faculty of Mechanical and Civil Engineering in Kraljevo

Franc Majdič
Assist. Professor
University of Ljubljana
Faculty of Mechanical Engineering

Buckling of the multistage hydraulic cylinders

Buckling of multistage telescopic hydraulic cylinders (THC) is still a current research topic. The problem is particularly important for longer multistage THCs, where the cylinder dimensions with regard to the internal pressure of the hydraulic fluid are not enough to avoid the risk of buckling, and where failures occur in daily use. In the paper, the relevant scientific publications and discussed solutions are introduced together with a corresponding theory. Further, a new method for fast determination of critical buckling load of multistage THCs in practice is proposed, which is based on calculation of theoretical buckling force and the correction factor for encountering the clearance between adjacent cylinder stages. For verification of the proposed method, the comparison of the measured and calculated critical buckling forces of the real-life multistage THC is conducted and good agreement of the results is shown.

Keywords: *multistage hydraulic cylinders, buckling, analytical approach, measurements, individual telescopic stages, overall radial clearance.*

1. INTRODUCTION

Buckling of multistage telescopic hydraulic cylinders (THC), consisting of several stages in form of round tubes of different diameters and filled with hydraulic fluid under pressure, is still a current research topic, as the phenomenon has not yet been fully investigated.

The problem is particularly important for longer, multistage THCs, where the dimensions of the cylinder stages, calculated for the stresses caused by the internal pressure of the hydraulic fluid, are not large enough to avoid the risk of buckling due to the external compressive force, and where failures occur in daily use.

In [1] and [2], examples of multistage THC failure in dump trucks are considered. In both cases, the buckling failure occurred with the THCs almost fully extended. The deformations caused by the elastic buckling resulted in local plasticization of the cylinder walls at the points of contact between the individual cylinder stages, which in turn led to the failure of the hydraulic seal between these cylinder stages and to the leakage of the pressurized hydraulic fluid.

Multistage THCs can be supported in various ways, but in practice, pin and rigid supports at the ends of the cylinder are mainly used. The quality of the supports is also of great importance, as shown in the article by Ohtomo et al [3], where the discrepancies between calculations and measurements were established. Although the hydraulic cylinder was supported by pins, the friction between the pin and the bore had a significant effect on the measurement result, leading to a deviation of up to 80 % from the calculated values. The investigation also showed that the limiting buckling force of the multistage THCs is also significantly influenced by the type of mutual guidance of adjacent telescopic elements, with the overall radial clearance between these elements being the most influential parameter.

In the literature, the problem of overall radial clearance is treated in different ways. The first method was presented by Gamez-Montero et al [4], where the influence of the backlash is considered by the so-called imperfection angle, which is influenced by the manufacturing tolerances, the oil compressibility and the wall deformations of the telescopic segments of the cylinder. The manufacturing tolerances depend on the size of THCs and bearing slip rings, and the current oil compression and deformation of the cylinder depend on the current hydraulic pressure in the system. Such an approach is complex. Its application requires a large amount of data and is therefore demonstrated only for a single-stage hydraulic cylinder.

The methods [3, 5, 6] are derived from the basic buckling differential equation. Their solutions are similar to Euler's equation for determining the theoretical critical buckling force with an additional factor due to the radial clearance. They are adapted for multistage THCs and assume that the hydraulic operating pressure has no effect on the critical buckling force and that the adjacent telescope stages are rigidly connected in a way that there is no longitudinal displacement between them.

Ohtomo et al [3] considered the radial clearance with the factor φ, while Ramasamy and Junaid Basha [5] considered it with the initial imperfection δ. Sugiyama et al [6] described the radial clearances using elastic supports (springs) between each stage of the multistage THC and considered them with factor λ in the final calculation. The equations for all pairs of adjacent THC stages form a system of equations, whose solution is complex and for which certain data are not directly available - e.g., stiffnesses of the added springs, which must be calculated additionally.

In the paper, a new method for fast determination of critical buckling load of multistage THCs in practice is proposed, which is based on calculation of theoretical

Correspondence to: Dr Boris Jerman, associate professor
Faculty of Mechanical Engineering,
Aškerčeva cesta 6, Si-1000 Ljubljana, Slovenia
E-mail: boris.jerman@fs.uni-lj.si

buckling force without encountering the clearance between adjacent cylinder stages, and corresponding correction factor for encountering that clearance, which is determined using the experimental data.

2. THE METODOLOGY

2.1 Euler buckling theory

Euler buckling theory [7] can be simply presented by noting that the internal bending moment in a compression loaded and slightly deformed column is $-F \cdot y$ where F is the compressive force and y is the column transversal deflection along the column length. When $-F \cdot y$ is inserted instead of M in the beam bending equation, $E \cdot I_z \cdot y'' = M$, the differential equation of column buckling (1) is formed:

$$E \cdot I_z \cdot y'' + F \cdot y = 0 \quad (1)$$

Its solution for pinned and roller supports at the column ends is given in (2):

$$F_{cr} = \frac{\pi^2 \cdot E \cdot I_z}{{L_b}^2} \quad (1)$$

which is the Euler column formula, which predicts the critical buckling load of a long column, where E is the modulus of elasticity, I_z is the moment of inertia and L_b is the buckling length of the column which, in this case, corresponds to actual column length.

2.2 The selected theoretical model of buckling of multistage THC

In the paper, the base critical buckling force of the multistage THC is determined according to the procedure from the article [5] by Ramasamy and Junaid Basha.

To determine the critical buckling force, they proceed from the differential equation (1), which can be written also in the form of (3):

$$E \cdot I_z \cdot \frac{d^4 y}{dx^4} + F \cdot \frac{d^2 y}{dx^2} = 0 \quad (3)$$

When a new variable $k^2 = \frac{F}{E \cdot I_z}$ is introduced, the (3) is rewritten as presented in (4):

$$\frac{d^4 y}{dx^4} + k^2 \cdot \frac{d^2 y}{dx^2} = 0 \quad (4)$$

The clearance between adjacent cylinder stages is neglected at this time. The solution of the homogeneous differential equation (4) is presented by (5).

$$y = A \cdot \sin\frac{\pi \cdot x}{L} \quad (5)$$

The problem is solved using the energy method, where the deformation energy (ΔU) and the work of the external forces (ΔT) are defined in (6) and (7).

$$\Delta U = \sum_{i=1}^{n} \int_{L_{i-1}}^{L_i} \frac{M^2 dx}{2 \cdot E \cdot I_i} \quad (6)$$

$$\Delta T = F \int_{L_0}^{L_n} \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx \quad (7)$$

In (6), M represents the bending moment due to consideration of the 2nd order theory, I_i the moment of inertia of the individual THC section, and L_i the distance from the beginning of the THC to the end of the individual THC section. In equation (7), F is the external axial pressure load, dy is the differential of transverse displacement, and dx is the differential of length of the longitudinal axis.

When the deformation energy and the work of external forces are equalized, the critical buckling force F_{cr} can be expressed as shown in (8):

$$F_{cr} = \frac{\pi^2 \cdot E}{2 \cdot L_n \cdot \left\{\sum_{i=1}^{n} \frac{Z_i}{I_i}\right\}} \quad (8)$$

Here, L_n is the total length of THC, Z_i are adjusted buckling lengths (see (9)) and I_i cross-section moments of inertia (see (10)).

$$Z_i = \frac{L_i - L_{i-1}}{2} - \frac{L_n \cdot \left(\sin\frac{2 \cdot \pi \cdot L_i}{L_n} - \sin\frac{2 \cdot \pi \cdot L_{i-1}}{L_n}\right)}{4 \cdot \pi} \quad (9)$$

$$I_i = \frac{\pi}{64} \cdot [D_i^4 - d_i^4] \quad (10)$$

Here, D_i and d_i are the outer and the inner diameters of the individual THC section.

2.3 Consideration of clearance in determining the critical buckling force

Because the clearance between adjacent cylinder stages do exists and do have significant influence on the critical buckling force, they have to be considered. We decided to take it into account by simplified, more practical approach. We took the measurement results from [5] and defined the reduction formula (12) with reduction coefficient RC for reduction of the theoretical critical buckling force without encountering the clearances to the value, which takes the clearances into account. Doing so we assumed that THCs considered in [5] are representative.

The results of the measurements from [5] are shown in Figure 1 and Table 1 for cases with different clearances between adjacent cylinder stages.

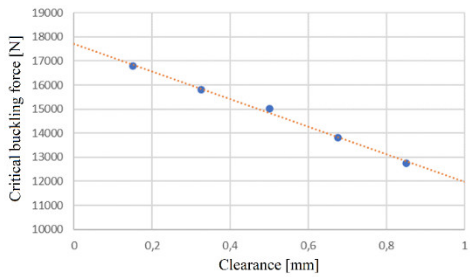

Figure 1. Critical buckling force for cases with different clearances between adjacent cylinder stages (see Table 1)

It can be seen that the relationship between critical buckling force F_{cr} in [N] and clearance c in [mm] is almost linear, therefore the linear regression is considered, as shown in (11).

$$F_{cr} = -5741{,}1 \cdot c + 17709 \quad (21)$$

From (11) the reduction formula (12) is derived:

$$F_{cr.cl} = F_{cr} \cdot RC = F_{cr} \cdot (1 - 0{,}324 \cdot c) \quad (32)$$

for calculation of critical buckling force $F_{cr.cl}$ with taking clearances between adjacent cylinder sections into account. In (12) $F_{cr.cl}$ is in [N] and the internal clearance c is in [mm]. For multi-stage THCs, the average clearance of all stages must be calculated.

2.4 Confirmation of the proposed method

In order to validate the proposed method of determining the critical buckling force of multistage THCs, the comparison of the results of measurements summarized according to [3] and calculations according to the proposed method (see chapters 2.2 and 2.3) is implemented and introduced in 4.3.

3. INPUT DATA

The essential data from [5] is listed in Table 1 and used in chapter 2.3 for derivation of (12).

Table 1. Measured critical buckling force for different internal clearances of THCs for examples from [5]

F_{cr} [N]	Clearance c [mm]
16781	0.150
15815	0.325
15022	0.500
13816	0.675
12757	0.850

The essential data from [3] is listed in Tables 2 and 3.

Table 2. Specifications of test three-stage THC from [3]

	Length of individual section, L_i [mm]	Outer diameter, D_i [mm]	Inner diameter, d_i [mm]
THC housing	1531	120	105
First stage	1336	80	70
Second stage	1307	56	45
Third stage	1426	40	0
Overall length	5600 mm		
Young's mod.	206 000 MPa		

Table 3. Test results for the three-stage THC from [3]

Measurement number	Measured critical buckling force F_{cr} [kN]
1	19.41
2	19.85

4. RESULTS AND DISCUSSION

4.1 Reduction coefficient for practical use

In Table 4 the reduced buckling forces are calculated for different clearances for example from [5], using newly developed reduction formula (12).

Table 4. Reduction of the critical buckling force due to the internal clearances. Forces are calculated using (12) for THC from [5]

Critical buckling force $F_{cr.cl}$ [N]	Clearance c [mm]	Reduction of the critical buckling force [%]
17709	0.0	0.00
17135	**0.1**	**3.24**
16561	**0.2**	**6.48**
15987	0.3	9.73
15413	0.4	13.0
14838	0.5	16.2

During manufacturing of THCs, clearances between 0.15 mm and 0.20 mm are usually realized. According to this data and the Table 3, we can estimate that the critical buckling force, calculated without considering the clearance, needs to be reduced by about 5 % to 6.5 %. Therefor in practice, when the exact values of individual clearances are not known, the 6 % reduction can be used. The corresponding reduction coefficient for practical usage is $RC_P = 0{,}94$. Of course, when clearances are known the usage of reduction formula (12) is encouraged.

4.2 The critical buckling force

In Table 5 the reduced buckling lengths and cross-section moments of inertia for individual THC segments are calculated for example from [3], using (9) and (10) and data from Table 2.

Table 5. Reduced buckling lengths and cross-section moments of inertia of individual segments of THC from [3]

	Reduced buckling length, Z_i [mm]	Section moment of inertia I_i [mm^4]
THC housing	325	4212158
First stage	1142	832031
Second stage	1065	281461
Third stage	268	85903

From data in Table 4 the theoretical buckling force without taking clearances into account can be calculated, using (8). The result is $F_{cr} = 21.74$ kN.

Further, the reduced buckling force with taking clearances into account can be calculated, using (12) and reduction coefficient for practical usage $RC_P = 0{,}94$.

The result is $F_{cr.cl} = 20.44$ kN.

4.3 Verification of the proposed method

The proposed method is verified using experimental results from [3], listed in Table 3. The average measured critical buckling forces is calculated as $F_{cr.cl.meas} = 19.63$ kN. The calculated critical buckling forces from 4.2 is $F_{cr.cl.calc} = 20.44$ kN. Using these data, the deviation between calculations and measurements can be expressed as:

$$d = \frac{F_{cr.cl.calc} - F_{cr.cl.meas}}{F_{cr.i.meas}} \cdot 100\ \% \quad (13)$$

$$d = \frac{20.44 - 19.63}{19.63} \cdot 100\ \% = 4{,}1\ \% \quad (14)$$

Since the deviation between calculated and measured results is less than 5 %, the proposed procedure for calculation of the critical buckling force for multistage THCs seem fine. In order to reliably verify the proposed method, more comparisons would be necessary.

5. CONCLUSION

In the paper the method for fast calculation of critical buckling force of multistage THCs in practice is proposed.

First, the calculation of theoretical buckling force without considering the clearances between adjacent cylinder stages is performed. Further, for taking the clearances into account, the correction factor RC is introduced.

Formula for calculation of the correction factor is developed using the experimental data.

For fast calculation of critical buckling force of multistage THCs in practice the correction factor $RC_{\mathrm{P}} = 0{,}94$ is suggested.

The proposed method is validated by comparison of analytical and experimental results. A good agreement between the results was determined, as the deviations do not exceed 5 %.

REFERENCES

[1] Trucks hydraulics system blowing out. URL: https://www.youtube.com/watch?v=xrWkrc1GXhg, viewed: 3.2.2020.

[2] V. D. da Silva: Mechanics and Strength of Materials. Springer – Verlag Berlin Heidelberg, 2006.

[3] T. Ohtomo, Y. Sugiyama, T. Katayama and K. A. Mladenov: Euler buckling test of four-staged, 6 m long telescopic cylinder-columns. Experimental Techniques, pp. 24-28, 2002.

[4] P. J. Gamez-Montero, E.Salazar, R.Castilla, J.Freire, M.Khamashta, E.Codina: Misalignment effects on the load capacity of a hydraulic cylinder. International Journal of Mechanical Sciences 51, pp. 105-113, 2009.

[5] V. Ramasamy in A.M. Junaid Basha: Effect of Internal Clearance on Buckling of Multistage Hydraulic Cylinder. Defence Science Journal, Vol. 68, No. 2, pp. 167-174, 2018.

[6] Y. Sugiyama, T. Otomo: Buckling Load and Slenderness Ratio of Columns Having Piece-Wire Constant Cross Sections. Bulletin of Osaka Prefecture University Series A, Vol. 45, No. 2, pp. 111-117, 1997.

[7] P. Kelly: Solid mechanics part I: An introduction to solid mechanics: Applications of Elasticity. University of Auckland, pp. 167-228, 2015.