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CONTRIBUTION TO THE DYNAMIC MODELING OF A SINGLE LINK FLEXIBLE MANIPULATOR

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Abstract: *This paper presents a rigid multibody approach for dynamic analysis of a single link flexible revolute manipulator. The multibody approach is based on the use of the modified Hencky bar-chain model of flexible beams. The flexible link of the manipulator is considered as a Timoshenko beam. According to this approach, the flexible link is replaced by a system consisting of massless rigid beams carrying lumped masses. The massless beams are connected through frictionless two-degrees of freedom joints with appropriate lateral and rotational springs in them. In the paper, the influence of the payload carrying by the manipulator on the system dynamics is also considered. The validity and accuracy of the approach presented are proven through the comparison with the results previously reported in the literature. It is shown that the rigid multibody system, by which the flexible link is replaced, allows the analysis of the coupling between rigid motion and small elastic deformatin of the manipulator link.*

Keywords: *flexible revolute manipulator,Timoshenko beam, multibody, Hencky bar-chain model.*

1. INTRODUCTION

Single link flexible manipulators are widely present in many industrial applications. Often such type of manipulators is modeled as a hub-flexible beam system with large angular motion of the hub and small elastic vibration of the flexible beam which is attached to the hub. This is a typical example of rigid flexible coupling system.

Various methods for the dynamic analysis of flexible manipulators are important for the precision in positional accuracy as well as vibration control of flexible manipulators. The detailed survey of methods related to dynamic analyses of flexible robotic manipulators was given in [1]. Also, at high angular velocities of the flexible link manipulators the dynamic stiffening effect ocurs [2-6].

In this paper, the modified Hencky bar-chain method developed in [7] is used to analyse the dynamic of single link flexible manipulators. Through the numerical examples given in Section 4 it is demonstrated that the modified Hencky bar-chain method can be used for the approximate determination of dynamic responses of the single link flexible manipulators.

2. LUMPED PARAMETER MODEL OF THE FLEXIBLE LINK MANIPULATOR

A single link flexible manipulator shown in Fig. 1 consists of a rotating rigid hub, which is driven with an applied torque $\tau(t)$, and a flexible link fixed to the hub. The hub has the rotary inertia J_H and the radius r_H whereas the flexible link carries a payload modeled as a particle whose mass and mass moment of inertia are, respectively, m_P and J_P. In general case, the flexible link is assumed to be a uniform homogeneous Timoshenko beam of the length L, the mass m, the Young's modulus E, the area moment of inertia of the beam cross-section I_z, the cross-section area A, and the mass density ρ. Only the transverse bending deformation $w(\xi,t)$ of the link is considered. In Fig. 1 $Oxyz$ represents the stationary inertial coordinate frame whereas $O_1\xi\eta\zeta$ is the floating coordinate frame, where O_1 lies at the junction of the hub and the link and where the axis ξ coincides with the neutral axis of the undeformed manipulator link. In the further considerations the effect of gravity of the hub and beam is neglected.

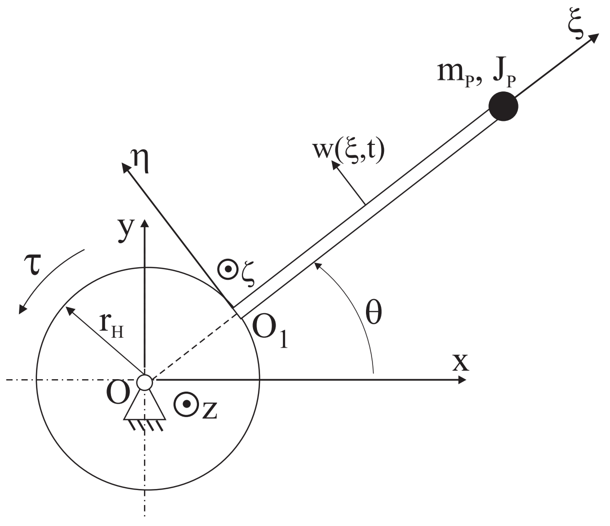

Figure 1. A single link flexible manipulator

According to modified variant of the classical Hencky bar-chain model developed in [7], the manipulator link may be divided into n equal segments of length L/n and then each of the segments are replaced by a rigid multibody system composed of massless rigid beams carrying corresponding lumped masses. In this paper $n=2$ will be taken (see Fig. 2(a)).

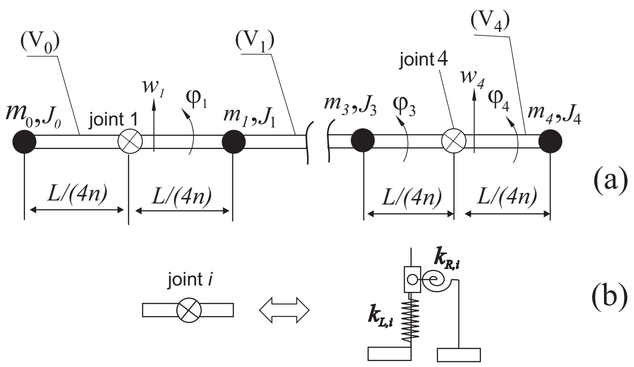

Figure 2. The modified Hencky bar-chain model of the flexible manipulator link

The introduced lumped masses have masses:

$$m_1 = m_3 = \frac{2\rho AL}{3n}, \quad m_2 = \frac{\rho AL}{3n}, \tag{1}$$

$$m_0 = \frac{\rho AL}{6n}, \quad m_4 = \frac{\rho AL}{6n} + m_P, \tag{2}$$

and rotary inertias:

$$J_1 = J_3 = \frac{2\rho I_z L}{3n}, \quad J_2 = \frac{\rho I_z L}{3n}, \tag{3}$$

$$J_0 = \frac{\rho I_z L}{6n}, \quad J_4 = \frac{\rho I_z L}{6n} + J_P. \tag{4}$$

The massless rigid beams $(V_i)(i=1,\dots,4)$ are connected to each other by frictionless two-degrees of freedom joints (see Fig. 2(b)) with appropriate lateral and rotational springs. The joints introduced allow relative rotation and relative translation of the beams in the coordinate plane $O_1\xi\eta$. The stiffnesses of springs in the joints are determined by the following expressions:

$$k_{R,i} = \frac{EI_z}{L/(2n)}, \quad k_{L,i} = \frac{12EI_z}{(L/(2n))^3(1+\Phi_y)}, \ i=1,\dots,4, \tag{5}$$

where Φ_y is the quantity determined by the following expression:

$$\Phi_y = \frac{12EI_z}{GAk(L/(2n))^2}, \tag{6}$$

where k is the shear coefficient and G is the shear modulus. The generalized coordinates by which the positions of the massless beams $(V_i)(i=1,\dots,4)$ with respect to the floating frame are determined in a slightly different manner in comparison to the corresponding generalized coordinates used in [7]. Namely, in this paper, the generalized coordinates w_i and φ_i determining the position of the massless beam $(V_i)(i=1,\dots,4)$ with respect to the frame $O_1\xi\eta\zeta$ (see Fig. 3) represent, respectively, the relative linear displacement in the ith joint and the angular displacement of beam (V_i) relative to the floating frame $O_1\xi\eta\zeta$. Note that the rigid beam (V_0) is fixed to the hub so that it is unmovable with respect to the frame $O_1\xi\eta\zeta$ while its position with respect to the frame $Oxyz$ is described by the generalized coordinate θ, which represents the angle of rotation of the hub about the axis z. Such choice of generalized coordinates enables the formulation of the corresponding differential equations of motion of the manipulator link modeled as an Euler-Bernoulli beam without using the inverse matrices (see Eqs. (33-37) in [7]). Note that if the flexible manipulator link is modeled in the frame of Euler-Bernoulli beam theory [8], then should be taken that $J_i = 0(i=0,\dots,4)$ and $\Phi_y = 0$. Also, in the case of the Rayleigh beam theory [8] one has that $\Phi_y = 0$.

3. FORMULATION OF THE DIFFERENTIAL EQUATIOS OF MOTION OF THE MANIPULATOR

The kinetic energy of the multibody system shown in Fig. 2(a) can be written in a similar manner as in [9] as follows:

$$T = \frac{1}{2}\dot{\mathbf{y}}^T \mathbf{M} \dot{\mathbf{y}}, \tag{7}$$

where $\mathbf{M} = \mathrm{diag}(J_H^*, m_1, m_1, J_1, \dots, m_4, m_4, J_4)$ is the inertia matrix, $J_H^* = J_H + J_0 + m_o r_H^2$, $\mathbf{y} = [\theta, x_1, y_1, \theta_1, \dots, x_4, y_4, \theta_4]^T$ is the vector of absolute Cartesian generalized coordinates of the system [10], $x_i(i=1,\dots,4)$, $y_i(i=1,\dots,4)$ are the Cartesian coordinates of point masses with respect to the frame $Oxyz$ determined by the following expressions (see Fig.3):

$$x_1 = (r_H + \frac{L}{4n})\cos\theta - w_1 \sin\theta + \frac{L}{4n}\cos(\theta + \varphi_1), \tag{8}$$

$$y_1 = (r_H + \frac{L}{4n})\sin\theta + w_1\cos\theta + \frac{L}{4n}\sin(\theta+\varphi_1), \quad (9)$$

$$x_j = (r_H + \frac{L}{4n})\cos\theta - \sum_{s=1}^{j} w_s \sin\theta$$

$$+\sum_{s=1}^{j-1}\frac{L}{2n}\cos(\theta+\varphi_s) + \frac{L}{4n}\cos(\theta+\varphi_j),\ j=\{2,3,4\}, \quad (10)$$

$$y_j = (r_H + \frac{L}{4n})\sin\theta + \sum_{s=1}^{j} w_s \cos\theta$$

$$+\sum_{s=1}^{j-1}\frac{L}{2n}\sin(\theta+\varphi_s) + \frac{L}{4n}\sin(\theta+\varphi_j),\ j=\{2,3,4\}, \quad (11)$$

and

$$\theta_i = \theta + \varphi_i,\ i=1,\ldots,4, \quad (12)$$

are the angular displacements of the beams $(V_i)(i=1,\ldots,4)$ relative to the inertial frame $Oxyz$, and an overdot denotes the derivative with respect to time.

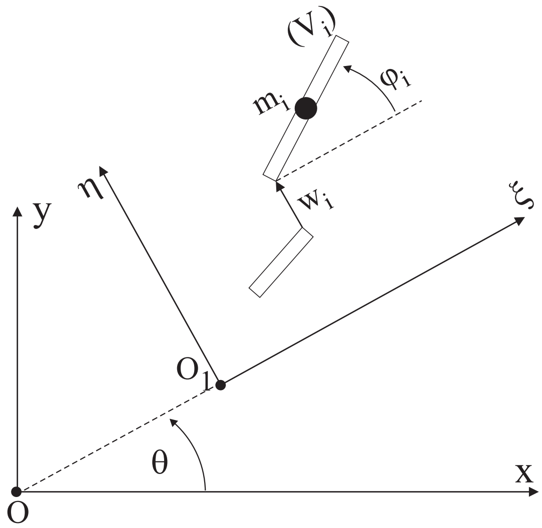

Figure 3. Generalized coordinates of the system

Taking the time derivatives of Eqs. (8)-(12) and using the approximations $\sin(\theta+\varphi_i)\approx\sin\theta$ and $\cos(\theta+\varphi_i)\approx\cos\theta$ yields the following relations:

$$\dot{x}_1 = -\left[(r_H + \frac{L}{2n})\sin\theta + w_1\cos\theta\right]\dot{\theta}$$

$$-\frac{L}{4n}\dot{\varphi}_1\sin\theta - \dot{w}_1\sin\theta, \quad (13)$$

$$\dot{y}_1 = \left[(r_H + \frac{L}{2n})\cos\theta - w_1\sin\theta\right]\dot{\theta}$$

$$+\frac{L}{4n}\dot{\varphi}_1\cos\theta + \dot{w}_1\cos\theta, \quad (14)$$

$$\dot{x}_j = -\left[(r_H + \frac{j}{2}\frac{L}{n})\sin\theta + \sum_{s=1}^{j} w_s\cos\theta\right]\dot{\theta} - \sum_{s=1}^{j}\dot{w}_s\sin\theta$$

$$-\sum_{s=1}^{j-1}\frac{L}{2n}\dot{\varphi}_s\sin\theta - \frac{L}{4n}\dot{\varphi}_j\sin\theta,\ j=\{2,3,4\}, \quad (15)$$

$$\dot{y}_j = \left[(r_H + \frac{j}{2}\frac{L}{n})\cos\theta - \sum_{s=1}^{j} w_s\sin\theta\right]\dot{\theta} + \sum_{s=1}^{j}\dot{w}_s\cos\theta$$

$$+\sum_{s=1}^{j-1}\frac{L}{2n}\dot{\varphi}_s\cos\theta + \frac{L}{4n}\dot{\varphi}_j\cos\theta,\ j=\{2,3,4\}, \quad (16)$$

$$\dot{\theta}_i = \dot{\theta} + \dot{\varphi}_i,\ i=1,\ldots,4. \quad (17)$$

Based on the relations (13-17) the following matrix relation can be formed:

$$\dot{\mathbf{y}} = \mathbf{B}\dot{\mathbf{q}}, \quad (18)$$

where $\mathbf{B}\in R^{13\times 9}$ is the velocity transformation matrix which is a function of the generalized coordinates of the system chosen as follows (see Fig. 3):

$$\mathbf{q} = [\theta, w_1, \varphi_1, \ldots, w_4, \varphi_4]^T \quad (19)$$

and $\dot{\mathbf{q}} = [\dot{\theta}, \dot{w}_1, \dot{\varphi}_1, \ldots, \dot{w}_4, \dot{\varphi}_4]^T$ is the vector of generalized velocities. The potential energy of the multibody system considered reads:

$$\Pi = \frac{1}{2}k_{R,1}\varphi_1^2 + \frac{1}{2}\sum_{s=2}^{4}k_{R,s}(\varphi_s - \varphi_{s-1})^2$$

$$+\frac{1}{2}\sum_{i=1}^{4}k_{L,i}w_i^2. \quad (20)$$

Also, the virtual work of the torque $\tau(t)$ is given as:

$$\delta W(\tau) = \tau\,\delta\theta. \quad (21)$$

Finally, similarly as in [9], using the Lagrange equations of the second kind [10] and the velocity transformation method [11] yields the differential equations of motion of the considered system:

$$\overline{\mathbf{M}}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} = \mathbf{Q}, \quad (22)$$

where $\overline{\mathbf{M}} = \mathbf{B}^T\mathbf{M}\mathbf{B}$, $\mathbf{C}\in R^{9\times 9}$ is the matrix whose components are given as [12]:

$$C_{ij} = \sum_{k=1}^{9}\frac{\dot{q}_k}{2}\left(\frac{\partial \overline{M}_{ij}}{\partial q_k} + \frac{\partial \overline{M}_{ik}}{\partial q_j} - \frac{\partial \overline{M}_{jk}}{\partial q_i}\right) \quad (23)$$

and $\mathbf{Q}$ is the vector of generalized forces associated with the generalized coordinates $\mathbf{q}$, which is given by:

$$\mathbf{Q} = \left[\tau, -\frac{\partial \Pi}{\partial w_1}, -\frac{\partial \Pi}{\partial \varphi_1}, \ldots, -\frac{\partial \Pi}{\partial w_4}, -\frac{\partial \Pi}{\partial \varphi_4}\right]^T. \quad (24)$$

4. NUMERICAL EXAMPLES

4.1. Dynamic behavior of the flexible manipulator link in the floating coordinate frame with the prescribed motion of the hub

To show that the presented approach captures the dynamic stiffening effect, let us consider the following parameters of the system [4]: $L = 8\,\text{m}$, $\rho = 2.7667\times 10^3\ \text{kg/m}^3$, $A = 7.2968\times 10^{-5}\ \text{m}^2$, $I_z = 8.2189\times 10^{-9}\ \text{m}^4$, $E = 6.8952\times 10^{10}\ \text{N/m}^2$, $r_H = 0$, $J_H = 0$, $m_P = 0.085\,\text{kg}$, and $J_P = 0$, where the prescribed motion of the hub is written as [2]:

$$\theta(t) = \begin{cases} \dfrac{\omega_0}{T}\left[\dfrac{t^2}{2} + \left(\dfrac{T}{2\pi}\right)^2\left(\cos\left(\dfrac{2\pi t}{T}\right) - 1\right)\right], & t < T, \\ \omega_0\left(t - \dfrac{T}{2}\right), & t \geq T, \end{cases} \quad (25)$$

where the spin up time T is set to $15\,\text{s}$. For various values of the steady-state rotation speed ω_0, the corresponding tip transverse deformations of the manipulator link are shown in Figs. 4-7. The corresponding numerical simulations are performed by applying the built-in function NDSolve[] in the Mathematica programming package [13].

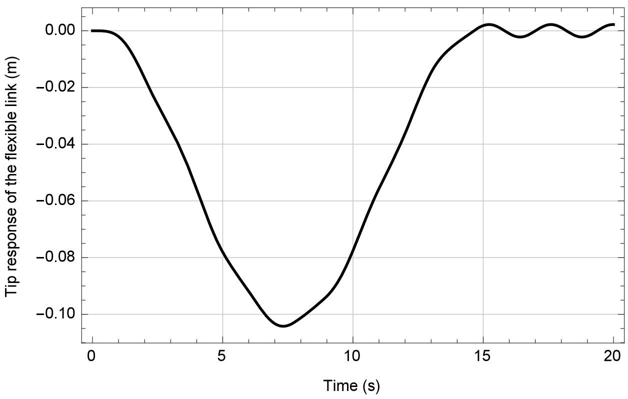

Figure 4. Tip response of the flexible link when $\omega_0 = 0.6\,\text{rad/s}$

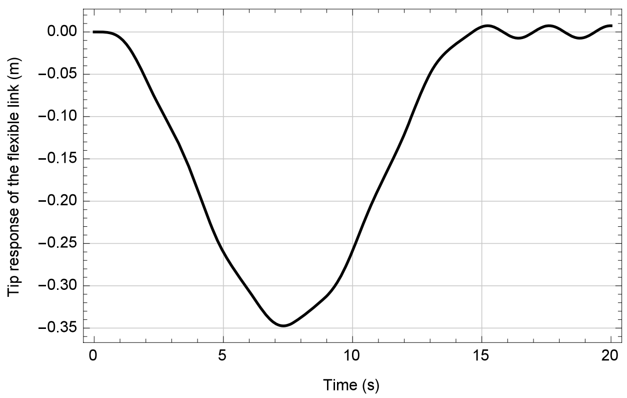

Figure 5. Tip response of the flexible link when $\omega_0 = 2\,\text{rad/s}$

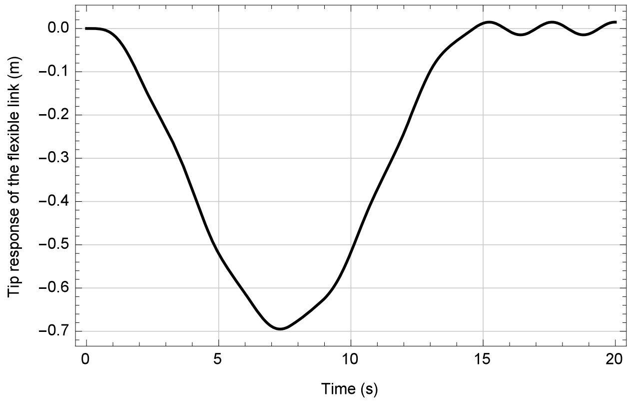

Figure 6. Tip response of the flexible link when $\omega_0 = 4\,\text{rad/s}$

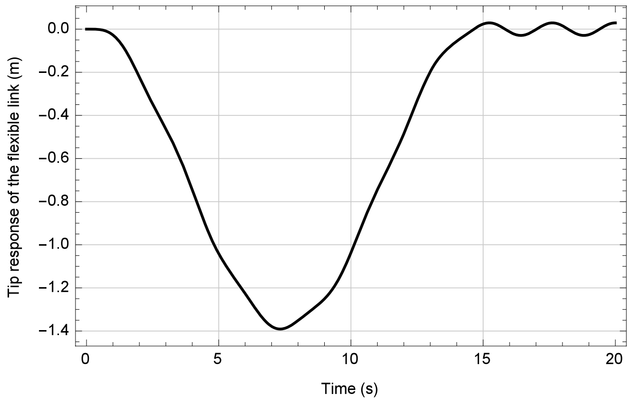

Figure 7. Tip response of the flexible link when $\omega_0 = 8\,\text{rad/s}$

By observing the results shown in Figs. 4-7 it can be concluded that the application of the modified Hencky bar-chain model [7] produces convergent results in the case of large angular velocity of the hub.

4.2. Dynamic behavior of the flexible manipulator link with the prescribed driving torque acting on the hub

In this subsection the interaction between the elastic vibration of the flexible link and the angular motion, $\theta(t)$, of the hub will be examined. For this purpose, consider the following parameters of the system [4]: $L = 1.8\,\text{m}$, $\rho = 2.766\times 10^3\ \text{kg/m}^3$, $A = 2.5\times 10^{-4}\ \text{m}^2$, $I_z = 1.3021\times 10^{-10}\ \text{m}^4$, $E = 6.90\times 10^{10}\ \text{N/m}^2$, $r_H = 0.05\,\text{m}$, $J_H = 0.30\,\text{kg}\,\text{m}^2$, $m_P = 0.085\,\text{kg}$, and $J_P = 0$, where it is assumed that the following driving torque is put on the hub:

$$\tau(t) = \begin{cases} \tau_0 \sin\left(\dfrac{2\pi}{T} t\right), & 0 \leq t \leq T, \\ 0, & t > T, \end{cases} \quad (26)$$

where $T = 2\,\text{s}$. For $\tau_0 = 1\,Nm$ the corresponding results are shown in Figs. 8 and 9. The results obtained are close to those obtained in [4].

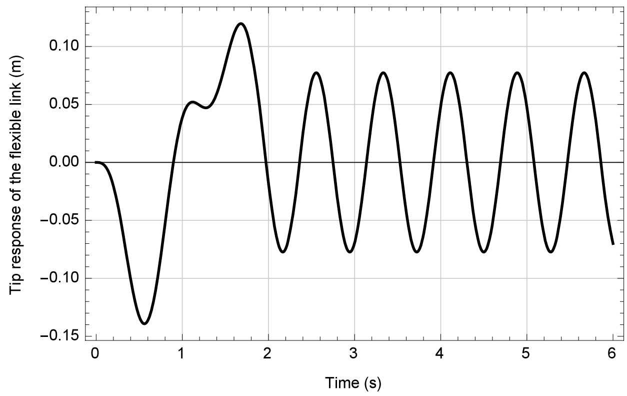

Figure 8. Tip response of the flexible link when $\tau_0 = 1\,\mathrm{Nm}$

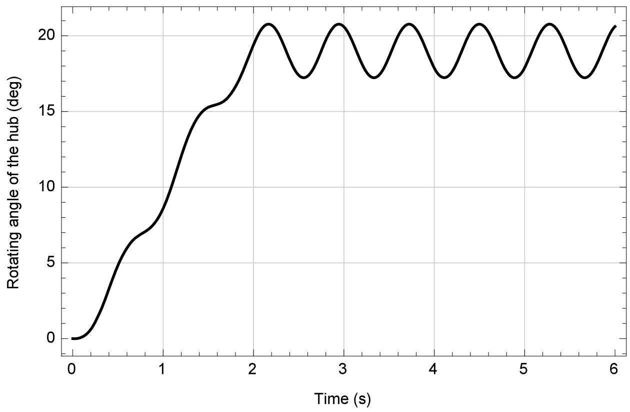

Figure 9. Rotating angle of the hub when $\tau_0 = 1\,\mathrm{Nm}$

5. CONCLUSION

In this paper, the new approach based on the modified Hencky bar-chain model [7] for dynamic analysis of flexible single link manipulators has been proposed. The flexible link has been modeled as a Timoshenko beam. Moreover, the approach allows that the flexible link to be considered in the frame of both the Euler-Bernoulli and the Rayleigh beam theory. In contrary to the rigid multibody approach presented in [14], our approach gives convergent results at high angular velocities of the hub. For more accurate results, the higher values of the parameter must be used.

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